1769

Dioptricae pars prima, continens librum primum, de explicatione principiorum ex quibus constructio tam telescopiorum quam microscopiorum est petenda

Leonhard Euler

DIOPTRICAE

PARS PRIMA

CONTINENS

LIBRVM PRIMVM,

DE

EXPLICATIONE

PRINCIPIORVM,

EX QVIBVS

CONSTRVCTIO TAM TELESCOPIORVM

QVAM

MICROSCOPIORVM

EST PETENDA.

AVCTORE

LEONHARDO EVLERO

ACAD. SCIENT. BORVSSIAE DIRECTORE VICENNALI ET SOCIO

ACAD. PETROP. PARISIN. ET LOND.

PETROPOLI

Impensis Academiae Imperialis Scientiarum

1769.

INDEX CAPITVM.

In Tomo I. contentorum.

LIBER PRIMVS

CONTINENS

EXPLICATIONEM PRINCIPIORVM,

EX QVIBVS CONSTRVCTIO

TAM TELESCOPIORVM

QVAM

MICROSCOPIORVM

EST PETENDA.

CAPVT I.

DE DIFFVSIONE IMAGINIS PER VNICAM LENTEM REPRAESENTATAE.

Definitio I.

1.

Imago principalis vocatur ea, quae a radiis axi lentis proximis per lentem refractis repraesentatur. Tab. I. Fig. 1.

Scilicet si lentis PP axis sit EF in eaque tantum spatium minimum aAa sit apertum, per quod radiis transitus concedatur, radii a puncto lucido E emissi in vno puncto F colligentur, quod punctum imago principalis vocatur.

Coroll. 1.

2. Cum nempe apertura aAa sit minima, omnes radii, qui a puncto quocunque in eam incidunt, ita aequabilem patiuntur refractionem, vt omnes iterum in vnum punctum colligantur, id quod non solum de puncto lucido E in axe lentis sito, sed etiam de quibusuis aliis extra axem positis est intelligendum.

Coroll. 2.

3. Quodsi ergo lentis apertura fuerit minima, singula cuiusuis obiecti puncta post refractionem iterum singulis punctis referentur, sicque imago principalis erit distincta et confusione carebit: siquidem confusio tum demum oritur, quando radii ex vno puncto emissi non iterum in vno puncto colliguntur.

Coroll. 3.

4. Et quamdiu apertura lentis aAa est minima, nihil interest, secundum quamnam figuram facies lentis sint elaboratae; quaecumque enim earum figura fuerit, quoniam tantum portiuncula minima in computum venit, ea semper vt sphaerica spectari poterit.

Coroll. 4.

5. Pendet ergo locus imaginis principalis primum a loco puncti lucidi E, siue id in axe siue extra axem fuerit situm; deinde a sphaericitate vtrius-

vtriusque faciei aAa et bBb refringentis; tertio ab earum diftantia AB feu lentis craffitie, et quarto a ratione refractionis, quam radii in tranfitu per lentem patiuntur.

Scholion 1.

6. Refractio radiorum per huiusmodi lentium aperturas minimas transmifforum, ideoque determinatio imaginum principalium fatis accurate in elementis Dioptricis tradi folet, quod igitur negotium hic fufius non profequar; fed potius in eam refractionis rationem hic inquirere conftitui, quando lentium apertura eft modicae quantitatis, in quo imprimis ad vtriusque faciei figuram eft fpectandum. Hic autem perpetuo lentium figuras fphaericas affumo, propterea quod haec figura vulgo lentibus induci, vel faltem, nifi forte accurate fucceffferit, intendi folet. In praxi certe nulla adhuc alia figura lentibus commode et accurate tribui poterit, atque adeo a fphaerica figura, etfi ad praxin maxime eft accommodata, ab artificibus frequenter aberrari folet. A follertioribus autem talia vitia iam plerumque fatis feliciter euitantur, vnde non adeo erit verendum, ne ea, quae per calculum ex hypothefi fphaericae figurae elicientur, experientiae non confentanea fint futura. Hanc ob rem hic perpetuo poftulo, vt ambae facies lentium exactiffime fecundum fphaericam figuram fint elaboratae.

Scholion 2.

7. Sphaerica autem figura hoc laborat incommodo, quod ſtatim ac lenti maior apertura tribuatur, non amplius omnes radii, qui quidem ab vno obiecti puncto ſunt profecti, poſt refractionem ad vnum punctum dirigantur radiique EM longius ab axe lentis transmiſſi non amplius in puncto F concurrant. Vnde confuſionem eo maiorem naſci neceſſe eſt, quo magis hi radii remotiores ab iis, qui prope axem tranſeunt, declinauerint; et quoniam talis declinatio a figura ſphaerica originem ducit, eo maior euadet, quo maior lenti apertura tribuatur. Quanta igitur quouis caſu futura ſit haec confuſio, hoc capite definire conſtitui; quae cum ceteris paribus a quantitate aperturae pendeat, hic in perpetuum moneo, me cuiusuis lentis aperturam circularem aſſumere, per cuius centrum axis lentis tranſeat: ita vt ſemidiameter huius circuli ſimul menſuram aperturae exhibeat. Ita ſi in ſuperficie lentis PP ſpatium MAM apertum relinquatur, reliqua parte MP velamine opaco obducta, punctorum extremorum MM diſtantia diametrum aperturae eiusque ſemiſſis ſemidiametrum aperturae praebebit.

Definitio 2.

8. *Imago extrema eſt ea, quam radii per extremitatem aperturae tranſmiſſi exhibent.*

Ita

Ita si MM sit apertura lentis, radiique EM, EM a puncto lucido E circa oram aperturae transmissi conueniant in puncto f, in hoc ipso puncto f erit imago extrema.

Coroll. 1.

9. Si punctum lucidum E est in ipso axe lentis, nullum est dubium, quin radii inde per marginem circularem MM transeuntes iterum in vno axis puncto f concurrant, imaginemque distinctam repraesentent, quae imago extrema vocatur.

Coroll. 2.

10. Verum si punctum lucidum non esset in axe lentis, hoc neutiquam eueniet, radiique per marginem illum circularem transmissi non amplius in vno puncto colligentur; vnde hoc casu imago extrema eo magis erit confusa, quo magis punctum lucidum ab axe fuerit remotum.

Scholion.

11. Quomodo se habeat refractio radiorum, quando punctum lucidum extra axem lentis fuerit constitutum, quaestio est non solum difficillima, sed etiam ita prolixis calculis inuoluitur, vt inde vix quicquam concludi possit. Ceterum in vsu, ad quem lentes accommodantur, nunquam obiecta ab axe remotiora spectari solent, atque contentos esse nos oportet, dummodo obiecta in ipso axe lentis sita distin-

cte

cte repraesententur, neque etiam confusio, qua obiecta axi proxima afficiuntur, sensibilis esse potest; nam cum imago extrema puncti E in ipso lentis axe siti sit punctum f, nulla confusione inquinatum; etiamsi id parumper esset remotum ab axe, vix sensibilis confusio se immiscere poterit. Quam ob causam investigationes sequentes tantum ad obiecta in ipso lentis axe sita adstringam.

Definitio 3.

12. *Spatium diffusionis vocatur interuallum, inter imaginem principalem et extremam interceptum.*

Ita si imago principalis sit in F, extrema vero in f, interuallum Ff appellatur spatium diffusionis.

Coroll. 1.

13. Si ergo apertura lentis MM euanescat, spatium simul diffusionis euanescit, tum enim tantum radii axi proximi transmittuntur, quibus imago distincta in F effingitur. Ex quo intelligi licet, quo maior fuerit apertura lentis, eo maius fore spatium diffusionis Ff.

Coroll. 2.

14. Cum in F imago a radiis axi proximis, in f autem imago a radiis circa marginem circularem MM transmissis formetur; si totam aperturam lentis infinitis circulis concentricis diuisam concipiamus, radii

radii per ſingulos circulos transmiſſi imagines intermedias exhibebunt, quibus interuallum Ff replebitur.

Coroll. 3.

15. Si enim apertura primum nulla, tum vero continuo increſcens ſtatuatur, imago extrema primum cum principali congruet; tum vero continuo magis ab ea diſcedet, ſicque cum vsque ad MM fuerit aucta, omnes illae imagines etiam nunc ſubſiſtent, ſpatiumque Ff implebunt.

Scholion 1.

16. Spatium hoc diffuſionis cauſam continet confuſionis, qua repraeſentatio imaginis perturbatur; cum enim eiusdem puncti lucidi E infinitae imagines per interuallum Ff diſpoſitae exhibeantur; earum commiſtio confuſionem pariat neceſſe eſt, quae eo maior erit, quo maius fuerit ſpatium diffuſionis Ff. Quemadmodum enim ad repraeſentationem diſtinctam requiritur, vt omnes radii ex ~~eodem obiecti~~ puncto emiſſi iterum in vnico puncto colligantur; ita ſi hi radii in plura puncta coeant, pluresque eiusdem puncti imagines referant, prout hae magis minusue inter ſe diſcrepant, maior inde minorue confuſio naſcetur. Quemadmodum autem hanc confuſionem aeſtimari oporteat, deinceps demum explicare licebit, cum ante ſpatium diffuſionis accurate definire docuerimus: quo circa in hoc capite, cum propoſita fuerit lens quaecunque vitrea faciebus ſphaericis terminata

pro quauis puncti lucidi E ab ea distantia et quauis apertura spatium diffusionis Ff inuestigare const tui. Quo facto eandem inuestigationem pro duabus pluribusue lentibus inter se coniunctis suscipi conueniet, vt tandem inde confusionem in quibusuis instrumentis dioptricis assignare valeamus.

Scholion 2.

17. Quaestio igitur principalis huius capitis in hoc versatur, vt proposita lente PP in eiusque axe puncto lucido E, radius quicunque incidens EM consideretur, eiusque per lentem refractio definiatur, vnde punctum f, vbi iterum in axem incidat, assignari queat. Cum enim in eodem puncto f omnes radii per totam peripheriam circularem MM transmissi concurrant; ab his imago quaepiam puncti E in f exprimetur, quae erit extrema, si circulus MM in lente eius aperturam determinet; sin autem capiatur minor, imago quaedam intermedia habebitur. Cum autem hic duplex refractio eueniat, altera in ingressu radii EM in vitrum, altera in egressu eiusdem e vitro, quemadmodum eius directio in vtraque inflectatur, seorsim est inuestigandum; vnde duo nascuntur problemata quasi praeliminaria, ex quorum combinatione deinceps negotium conficietur. Verum quo haec problemata commodius calculo expediri queant, quaedam lemmata ex doctrina angulorum petita praemitti oportet.

Lem-

Lemma I.

18. *Si angulus Φ triginta gradus non excedat eius sinus satis accurate erit* sin. $\Phi = \Phi - \frac{1}{6}\Phi^3$, *si quidem in circulo, cuius radius est* $= 1$, *arcus* Φ, *qui pro illius anguli mensura habetur, in partibus radii exprimatur.*

Demonstratio.

Quantuscunque fuerit angulus Φ, notum est, eius sinum hac serie infinita exprimi:

$$\text{sin. } \Phi = \Phi - \tfrac{1}{6}\Phi^3 + \tfrac{1}{120}\Phi^5 - \tfrac{1}{5040}\Phi^7 + \text{etc.}$$

sumtis igitur tantum binis primis terminis, error committitur reliquis neglectis aequalis, si ergo statuamus sin. $\Phi = \Phi - \frac{1}{6}\Phi^3$, hincque pro variis angulis Φ sinus colligamus, eorum comparatio cum tabulis sinuum errores manifestabit. Ita si sumatur $\Phi = 30^\circ$, quia arcus 180° valet 3, 14159265, erit in partibus radii $\Phi = 0,52359877$, et $\frac{1}{6}\Phi^3 = 0,0239246$, hincque $\Phi - \frac{1}{6}\Phi^3 = 0,4996741$. at est reuera

sin. $\Phi = 0,5000000$ vnde habetur

error $= 0,0003259$, qui ergo ne ad $\frac{1}{3000}$ partem radii quidem assurgit. At si angulus Φ caperetur duplo minor, scilicet $\Phi = 15^\circ$, reperiretur

$$\Phi - \tfrac{1}{6}\Phi^3 = 0,2588088$$

cum tamen sit sin. $\Phi = 0,2588190$

errore existente $= 0,0000102$

qui tricies bis minor est quam casu praecedente. Cum ergo in praxi error partem adeo termillesimam radii adaequans facile tolerari possit, multo magis, si angulus Φ fuerit 30° minor, expreſſio $\Phi - \frac{1}{6}\Phi^3$ verum eius ſinum exhibere cenſenda erit.

Lemma 2.

19. *Viciſſim ſi anguli triginta gradibus minoris detur ſinus* $= s$, *ex eo ipſe angulus ita proxime definitur, vt ſit in circulo, cuius radius* $= 1$, *arcus eum metiens* $= s + \frac{1}{6}s^3$.

Demonſtratio.

Si enim Φ deſignet iſtum angulum, cuius ſinus proponitur $= s$; modo vidimus, eſſe ſatis exacte $s = \Phi - \frac{1}{6}\Phi^3$, hinc autem per conuerſionem oritur proxime $\Phi = s + \frac{1}{6}s^3$, quae expreſſio quantum peccet in angulo triginta graduum, vt videamus, ſit $s = \frac{1}{2}$ eritque $s + \frac{1}{6}s^3 = 0,5208333$. Cum autem numerus $3,14159265$ reſpondeat angulo 180°, hic numerus $0,5208333$ praebet angulum $29^\circ\ 50'\ 30''$: verus autem angulus eſt 30°, ita vt error ſit $9'\ 30''$. Sit autem $s = \frac{1}{4}$ cui ſinui reſpondet angulus $= 14^\circ\ 28'\ 39''$, erit $s + \frac{1}{6}s^3 = 0,2526041 = 14^\circ\ 28'\ 23''$, ita vt hoc caſu error non excedat $16''$. Patet ergo dum angulus minor ſit 30°, hoc modo ſatis exacte ex dato ſinu reperiri angulum.

Problema 1.

Tab. I. Fig. 2. 20. Si ex puncto lucido E in ſuperficiem ſphaericam reuolutione arcus circularis AM circa axem EC genitam incidat radius EM, definire eius, poſtquam fuerit refractus, concurſum cum axe EC.

Solutio.

Solutio.

Sit C centrum superficiei sphaericae, eiusque radius $CA = CM = f$, et distantia puncti lucidi E a superficie refringente $EA = a$, tum vero sit ratio refractionis radiorum ex aere in vitrum $= n : 1$. Vocetur pro radio incidente CM angulus $ACM = \Phi$, eritque huius anguli sinus $= \Phi - \frac{1}{6}\Phi^3$. Ponatur iam breuitatis gratia distantia $EC = a + f = c$, et quia in triangulo ECM dantur latera $CM = f$, $EC = c$ cum angulo $CEM = \Phi$, fiet producta CM in c

$$CM\ (f) : \text{sin. } CEM = CE\ (c) : \text{sin. } EMc$$

ideoque

$$\text{sin. } EMc = \frac{c}{f}\,\text{sin. } \Phi = \frac{c}{f}(\Phi - \tfrac{1}{6}\Phi^3);$$

vnde ipse angulus EMc elicitur, neglectis tertia altioribus potestatibus ipsius Φ:

$$EMc = \frac{c}{f}\Phi - \frac{c}{6f}\Phi^3 + \frac{c^3}{6f^3}\Phi^3 = \frac{c}{f}\Phi + \frac{c(cc-ff)}{6f^3}\Phi^3$$

qui cum sit $= ECM + CEM$, erit

$$ECM = \frac{c-f}{f}\Phi + \frac{c(cc-ff)}{6f^3}\Phi^3$$

Verum angulus EMc est angulus incidentiae, ac si MO referat radium refractum, erit CMO angulus refractionis, sicque per hypothesin

$$\text{sin. } EMc : \text{sin. } CMO = n : 1.$$

vnde colligitur

$$\text{sin. } CMO = \frac{c}{nf}(\Phi - \tfrac{1}{6}\Phi^3)$$

hincque ipse angulus

$$CMO = \frac{c}{nf}\Phi + \frac{c(cc-nnff)}{6n^3f^3}\Phi^3$$

 quo

quo ablato ab angulo ECM relinquitur angulus

$$COM = \frac{(n-1)c-nf}{nf}\Phi + \frac{c((n^3-1)cc-nn(n-1)ff)}{6n^3f^3}\Phi^3$$

cuius sinus propterea erit

$$\text{sin. } COM = \frac{(n-1)c-nf}{nf}\Phi + \frac{3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3}{6nnf^3}\Phi^3$$

Iamuero ob

sin. $COM : CM\,(f) =$ sin. $CMO : CO$, obtinebitur

$$CO = \frac{\frac{c}{n} - \frac{c}{6n}\Phi\Phi}{\frac{(n-1)c-nf}{nf} + \frac{3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3}{6nnf^3}\Phi^2}$$

cuius formulae euolutio praebet:

$$CO = \frac{cf}{(n-1)c-nf} - \frac{cf\Phi\Phi}{6((n-1)c-nf)} - \frac{c(3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3)}{6nf((n-1)c-nf)^2}\Phi\Phi$$

quae porro reducitur ad hanc formam:

$$CO = \frac{cf}{(n-1)c-nf} - \frac{(n-1)cc(c-f)(c+nf)}{2nf((n-1)c-nf)^2}\Phi\Phi$$

cui si addamus $CA = f$ prodibit

$$AO = \frac{nf(c-f)}{(n-1)c-nf} - \frac{(n-1)cc(c-f)(c+nf)}{2nf((n-1)c-nf)^2}\Phi\Phi$$

atque hinc positio radii refracti MO primo per interuallum AO modo inuentum definitur, tum vero insuper ex angulo AOM, qui erit:

$$AOM = \frac{(n-1)c-nf}{nf}\Phi + \frac{c((n^3-1)cc-nn(n-1)ff)}{6n^3f^3}\Phi^3$$

siue

$$AOM = \frac{(n-1)c-nf}{nf}\Phi + \frac{(n-1)c((nn+n+1)cc-nnff)}{6n^3f^3}\Phi^3$$

Coroll. I.

21. Cum posuerimus $c = a+f$ erit

$c-f = a$, et $(n-1)c-nf = (n-1)a-f$,

atque

atque

$$(nn+n+1)cc-nnff=(nn+n+1)aa+2(nn+n+1)af+(n+1)ff$$

quibus valoribus ſubſtitutis habebimus

$$AO=\frac{naf}{(n-1)a-f}-\frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\Phi\Phi \text{ et}$$

$$AOM=\frac{(n-1)a-f}{nfa}\Phi+\frac{(n-1)(a+f)((nn+n+1)(a+f)+(n+1)ff)}{6n^3f^3}\Phi^3$$

Coroll. 2.

22. Si M ſit in extremitate aperturae, erit ſemidiameter aperturae $=f$ ſin. ECM $=a\Phi$ ſatis exacte: neque enim neceſſe eſt, aperturam tam exacte noſſe. Vnde ſi ſemidiameter aperturae ponatur $=x$, erit prope $x=a\Phi$, ideoque $\Phi=\frac{x}{a}$.

Coroll. 3.

23. Si accuratius rem definire velimus, quia eſt

$$\text{ſin. } ECM=\frac{c-f}{f}\Phi+\frac{(c-f)(3c-f)}{6ff}\Phi^3=\frac{a}{f}\Phi+\frac{a(3a+2f)}{6ff}\Phi^3$$

erit $x=a\Phi+\frac{a(3a+2f)}{6f}\Phi^3$

ideoque $\Phi=\frac{x}{a}-\frac{(3a+2f)x^3}{6a^3f}$

ſed quia in valore ipſius AO non vltra ſecundam dimenſionem ipſius Φ aſcendimus, hac expreſſione non indigemus.

Coroll. 4.

24. Etſi hae expreſſiones tantum ſunt prope verae; tamen in praxi ſine errore adhiberi poterunt, dummodo anguli, qui in calculum ſunt ingreſſi, infra

30°

30° ſubſiſtant. Non ſolum ergo neceſſe eſt, vt angulus $AEM = \Phi$ ſed etiam angulus EMc ſeu $\frac{c}{f}\Phi = \Phi + \frac{a}{f}\Phi$ minor ſit 30 gradibus.

Scholion.

25. Summo quidem rigore geometrico diſtantiam EO definire potuiſſemus, neque opus fuiſſet ad approximationes confugere: ſcilicet ſi poſuiſſemus angulum $ECM = \omega$, et diſtantiam $CO = u$; deueniſſemus ad hanc determinationem.

$$\frac{cf}{u} = -c\,\text{coſ.}\,\omega + \sqrt{(nn(c\,\text{coſ.}\,\omega - f)^2 + (nn-1)cc\,\text{ſin.}\,\omega^2)}$$

foretque

$$\text{coſ.}\,\omega = \frac{c}{f}\,\text{ſin.}\,\Phi^2 + \text{coſ.}\,\Phi\sqrt{(1 - \frac{cc}{ff}\,\text{ſin.}\,\Phi^2)} \text{ et}$$

$$\text{ſin.}\,\omega = \frac{c}{f}\,\text{ſin.}\,\Phi\,\text{coſ.}\,\Phi - \text{ſin.}\,\Phi\sqrt{(1 - \frac{cc}{ff}\,\text{ſin.}\,\Phi^2)}$$

hisque valoribus ſubſtitutis

$$\frac{cf}{u} = \frac{-cc\,\text{ſin.}\,\Phi^2}{f} - c\,\text{coſ.}\,\Phi\sqrt{(1 - \frac{cc}{ff}\,\text{ſin.}\,\Phi^2)}$$
$$+ (c\,\text{coſ.}\,\Phi - \sqrt{(ff - cc\,\text{ſin.}\,\Phi^2)})\sqrt{(nn - \frac{cc}{ff}\,\text{ſin.}\,\Phi^2)}$$

vnde ponendo angulo Φ valde paruo per aproximationem ſuperior expreſſio eliceretur. Verum praecedens analyſis magis ad praeſens inſtitutum videtur accommodata. Interim ſi quis propius ad veritatem accedere voluerit, ex formula vera hic exhibita adipiſcetur.

$$\frac{cf}{u} = (n-1)c\,\text{coſ.}\,\Phi - nf + \frac{(n-1)cc}{2nff}((n-1)f + c\,\text{coſ.}\,\Phi)\,\text{ſin.}\,\Phi^2$$
$$+ \frac{c^4}{8n^3f^4}((nn-1)^2 f + (n^3-1)c\,\text{coſ.}\,\Phi)\,\text{ſin.}\,\Phi^4$$

ſeu

seu

$$\frac{cf}{u}=(n-1)c-nf+\frac{(n-1)c(c-f)(c+nf)}{2nff}\Phi\Phi$$
$$+\frac{(n-1)(c-f)((nn+n+1)c^3+2nn(n+1)ccf+2n(n-1)cff-n^3f^3)}{24n^3f^4}\Phi^4.$$

Problema 2.

26. Si prout in problemate praecedente inuenimus, radius MO in vitrum miſſus per ſuperficiem ſphaericam BN iterum in aerem erumpat, definire punctum V, vbi is cum axe concurret.

Solutio.

Ponatur interuallum $AB=d$; ſitque ſuperficiei ſphaericae BN centrum in D eiusque radius $DB=DN=g$. Quoniam igitur poſitio radii incidentis MNO datur, ponamus $BO=b$, et angulum $BON=\psi$, et breuitatis ergo interuallum $DO=b+g=e$. Cum nunc in triangulo DON dentur latera $DN=g$, $DO=e$ cum angulo $BON=\psi$, reperitur

$$\text{ſin. } DNM=\frac{e}{g}\text{ ſin. }\psi=\frac{e}{g}\psi-\frac{e}{6g}\psi^3,\text{ hincque}$$

$$DNM=\frac{e}{g}\psi+\frac{e(ee-gg)}{6g^3}\psi^3\text{ et}$$

$$ODN=\frac{e-g}{g}\psi+\frac{e(ee-gg)}{6g^3}\psi^3$$

Sed hic ſin. DNM eſt ſinus incidentiae, cui reſpondet angulus refractionis VNd, cuius ſinus propterea eſt ad illum vt $n:1$, vnde fit

$$\text{ſin. } VNd=\frac{ne}{g}\psi-\frac{ne}{6g}\psi^3$$

 hinc-

hincque

$$VNd = \frac{ne}{g}\psi + \frac{ne(nnee-gg)}{6g^3}\psi^3$$

a quo ablato angulo ODN relinquitur angulus

$$DVN = \frac{(n-1)e+g}{g}\psi + \frac{e((n^3-1)ee-(n-1)gg)}{6g^3}\psi^3$$

ergo

$$\text{fin. } DVN = \frac{(n-1)e+g}{g}\psi + \frac{3n(n-1)e^3-3(n-1)^2eeg-4(n-1)egg-g^3}{6g^3}\psi^3$$

Cum nunc fit

fin. $DVN : g =$ fin. $VNd : DV$ erit

$$\frac{g}{DV} = \frac{(n-1)e+g+\frac{1}{6gg}(3n(n-1)e^3-3(n-1)^2eeg-4(n-1)egg-g^3)\psi^2}{ne-\frac{1}{6}ne\psi^2}$$

quae expreffio reducitur ad hanc formam :

$$\frac{g}{DV} = \frac{(n-1)e+g}{ne} + \frac{(n-1)(e-g)(ne+g)\psi^2}{2ngg}$$

vnde reciproce oritur

$$DV = \frac{neg}{(n-1)e+g} - \frac{n(n-1)ee(e-g)(ne+g)}{2g((n-1)e+g)^2}\psi^2$$

et

$$BV = \frac{g(e-g)}{(n-1)e+g} - \frac{n(n-1)ee(e-g)(ne+g)}{2g((n-1)e+g)^2}\psi^2$$

Puncto autem V inuento notandum eft effe angulum

$$BVN = \frac{(n-1)e+g}{g}\psi + \frac{e((n^3-1)ee-(n-1)gg)}{6g^3}\psi^3;$$

Coroll. I.

27. Cum fit

$e = b+g$ erit $(n-1)e+g = (n-1)b+ng$,

quo

quo valore restituto habebimus.

$$BV = \frac{bg}{(n-1)b+ng} - \frac{n(n-1)b(b+g)^2(nb+(n+1)g)}{2g((n-1)b+ng)^2}\psi^2 \quad \text{et}$$

$$BVN = \frac{(n-1)b+ng}{g}\psi + \frac{(b+g)((n^2-1)(b+g)^2-(n-1)gg)}{6g^3}\psi^3$$

Coroll. 2.

28. Hae formulae etiam ex praecedentibus erui possunt, si pro litteris n, a, f scribantur $\frac{1}{n}$, $-b$, et $-g$, quoniam hoc modo casus praecedentis problematis ad hunc reducitur. Praeterea autem, qui angulus ibi erat Φ hic est ψ.

Coroll. 3.

29. Quia in praecedente problemate inuenimus tam lineam AO quam angulum AOM erit hoc problemate ad radium illum refractum MO accommodato

$$BO = b = \frac{naf}{(n-1)a-f} - d - \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\Phi\Phi$$

et

$$\psi = \frac{(n-1)a-f}{nf}\Phi + \frac{(n-1)(a+f)((nn+n+1)a(a+2f)+(n+1)ff)}{6n^3f^2}\Phi^3$$

Problema 3.

30. Proposita lente vitrea MABN faciebus Tab. I. Fig. 2.
sphaericis AM et BN terminata, si a puncto quocunque E in eius axe posito incidat in eam radius EM, definire punctum V, in quo is post geminam refractionem iterum cum axe lentis sit concursus.

Solutio.

Confideremus lentem vt vtrinque conuexam, fitque faciei anterioris AM radius $AC = f$, pofterioris vero BN radius $BD = g$, ipfius lentis autem craffities $AB = d$. Lens porro fit vitrea, ita vt fi in eam radius lucis ex aere incidat, fit finus incidentiae ad finum refractionis vt n ad 1. Iam recta iungens centra vtriusque faciei C et D erit axis lentis, in quo reperiatur punctum lucidum E ante lentem in diftantia $AE = a$, vnde fub angulo $AEM = \Phi$ in lentem incidat radius EM, qui prima refractione ita inflectatur, vt productus cum axe concurrat in O. Quodfi iam ex iis, quae problemate primo funt inuenta, ponamus

$$BO = \frac{naf}{(n-1)a-f} - d - \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\Phi\Phi = b$$

et ang. $BON = \frac{(n-1)a-f}{nf}\Phi = \psi$

in valore enim anguli ψ negligere licet terminum Φ^3 inuoluentem, quoniam calculum tantum ad fecundam poteftatem ipfius ψ extendimus; his pofitis in problemate fecundo inuenimus fore.

$$BV = \frac{bg}{(n-1)b+ng} - \frac{n(n-1)b(b+g)^2(nb+(n+1)g)}{2g((n-1)b+ng)^2}\psi\psi$$

et

$$BVN = \frac{(n-1)b+ng}{g}\psi.$$

Totum ergo negotium huc redit, vt ifthic pro b et ψ valo-

valores aſſignatos ſubſtituamus, quod quo facilius fieri poſſit, ſtatuamus

$$b = \mathrm{P} - \mathrm{Q}\Phi\Phi \text{ et } \psi = \mathrm{R}\Phi \text{ vt ſit}$$

$$\mathrm{P} = \frac{naf}{(n-1)a-f} - d = \frac{naf-(n-1)ad+df}{(n-1)a-f}$$

$$\mathrm{Q} = \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}, \text{ et } \mathrm{R} = \frac{(n-1)d-f}{nf}$$

Hinc erit:

$$\frac{bg}{(n-1)b+ng} = \frac{\mathrm{P}g-\mathrm{Q}g\Phi\Phi}{(n-1)\mathrm{P}+ng-(n-1)\mathrm{Q}\Phi\Phi} = \frac{\mathrm{P}g}{(n-1)\mathrm{P}+ng} - \frac{n\mathrm{Q}gg\Phi\Phi}{((n-1)\mathrm{P}+ng)^2}$$

at in altero membro ſufficit pro b ſcribere P: ex quo obtinebimus

$$\mathrm{BV} = \frac{\mathrm{P}g}{(n-1)\mathrm{P}+ng} - \frac{n\mathrm{Q}gg\Phi\Phi}{((n-1)\mathrm{P}+ng)^2} - \frac{n(n-1)\mathrm{PRR}(\mathrm{P}+g)^2(n\mathrm{P}+(n+1)g)}{2g((n-1)\mathrm{P}+ng)^3}\Phi\Phi$$

$$\text{et } \mathrm{BVN} = \frac{(n-1)\mathrm{P}+ng}{g}\mathrm{R}\Phi$$

Pro his autem ſubſtitutionibus notandum eſt fore:

$$(n-1)\mathrm{P}+ng = \frac{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}{(n-1)a-f}$$

$$\mathrm{P}+g = \frac{naf+(n-1)ag-fg-(n-1)ad+df}{(n-1)a-f}$$

$$n\mathrm{P}+(n+1)g = \frac{nnaf+(nn-1)ag-(n+1)fg-n(n-1)ad+ndf}{(n-1)a-f}$$

vnde concluditur:

$$\mathrm{BV} = \frac{nafg-(n-1)adg+dfg}{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}$$

$$- \frac{(n-1)agg(a+f)^2(a+(n+1)f)}{2f(n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df)^2}\Phi\Phi$$

$$- \frac{(n-1)(naf-(n-1)ad+df)(naf+(n-1)ag-fg-(n-1)ad+df)^2(nnaf+(nn-1)ag-(n+1)fg-n(n-1)ad+ndf)}{2nffg(n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df)^3}\Phi\Phi$$

$$\text{et } \mathrm{BVN} = \frac{n(n-1)a(f+g)-nfg+(n-1)df-(n-1)^2ad}{nfg}\Phi.$$

Coroll. 1.

31. Si angulus Φ prorſus euaneſcat, punctum V cadet in imaginem principalem, cuius ſi a lente diſtantia dicatur $= \alpha$ erit

$$\alpha = \frac{nafg - (n-1)adg + dfg}{n(n-1)a(f+g) - nfg - (n-1)^2 ad + (n-1)df}$$

quae ſi vt data ſpectetur, hac aequatione relatio inter f et g definitur, vt haec diſtantia principalis oriatur.

Coroll. 2.

32. Quare ſi diſtantia obiecti ante lentem ſit $= a$, eiusque imago principalis poſt lentem ad diſtantiam $= \alpha$ proiici debeat huic aequationi ſatisfieri oportet.

$$\begin{aligned} n(n-1)a\alpha(f+g) &- nafg + (n-1)adg - (n-1)^2 a\alpha d = 0 \\ &- n\alpha fg + (n-1)\alpha df \quad - dfg \end{aligned}$$

Coroll. 3.

33. Hanc autem imaginis principalis diſtantiam α in calculum introducendo expreſſiones noſtrae inuentae haud mediocriter contrahentur. Cum enim ſit $\alpha = \frac{Pg}{(n-1)P + ng}$, erit hinc $P = \frac{n\alpha g}{g - (n-1)\alpha}$

et porro

$$(n-1)P + ng = \frac{ngg}{g-(n-1)\alpha}; \quad P + g = \frac{Pg + \alpha g}{g-(n-1)\alpha} = \frac{g(\alpha+g)}{g-(n-1)\alpha}$$

$$nP + (n+1)g = \frac{g(\alpha + (n+1)g)}{g-(n-1)\alpha}$$

vnde

vnde facta substitutione sequentes determinationes simpliciores assequemur.

$$BV = \alpha - \frac{(n-1)a(a+f)^2(g-(n-1)\alpha)^2(\alpha+(n+1)f)}{2nnffgg((n-1)a-f)^2}\Phi\Phi - \frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2}\Phi\Phi$$

et

$$BVN = \frac{((n-1)a-f)g}{(g-(n-1)\alpha)f}\Phi.$$

Scholion.

34. Non solum hae formulae multo sunt breuiores et concinniores, quam primo inuentae, sed etiam perspicuus ordo in iis obseruatur quo litterae α et g cum litteris a et f ita connexae sunt, vt permutationem admittant. Nullum igitur est dubium, quin si statim distantiam α in calculum introduxissemus, via breuiori ad eas peruenire licuisset. Ceterum quia hae formulae posteriores non amplius crassitiem lentis $AB = d$ inuoluunt, euidens est eas ex primo inventis nasci, si introducta distantia α crassicies d eliminetur, seu eius loco hic valor surrogetur.

$$d = \frac{n(n-1)a\alpha(f+g) - n(a+\alpha)fg}{(n-1)^2a\alpha - (n-1)(ag+\alpha f) + fg} = n\frac{(n-1)a\alpha(f+g)-(a+\alpha)fg}{((n-1)a-f)((n-1)\alpha-g)}$$

qui labor autem ne suscipi quidem mereretur, nisi iam ante de eximio eius vsu certiores essemus facti. In sequentibus igitur his elegantioribus formulis vtamur, quoad negotium adhuc succinctius expedire didicerimus.

Proble-

Problema 4.

35. Propoſita lente quacunque faciebus ſphaericis terminata, ſi obiectum in data ab ea diſtantia ſit conſtitutum, pro data lentis apertura ſpatium diffuſionis aſſignare.

Solutio.

Tab. I. Fig. 1. Concipiamus lentem vtrinque conuexam, ſitque faciei anterioris MAM radius $=f$, poſterioris NBN$=g$, lentisque craſſities AB$=d$. Sit porro MM lentis huius apertura, cuius ſemidiameter ſit $=x$; atque in axe lentis expoſitum ſit obiectum vel ſaltem punctum lucidum E, cuius a lente ponatur diſtantia AE $=a$. Iam primo quaeramus eius imaginem principalem, quae cadat in F atque ſupra (31) invenimus fore

$$BF = \frac{nafg - (n-1)adg + dfg}{n(n-1)a(f+g) - nfg - (n-1)^2ad + (n-1)df}$$

Vocetur ergo haec diſtantia $=\alpha$, et ſi radii EM circa oram aperturae tranſeant, erit angulus AEM $= \Phi = \frac{x}{a}$, quo valore in ſuperioribus formulis ſubſtituto prodibit diſtantia imaginis extremae f a lente ſcilicet:

$$Bf = \alpha - \frac{(n-1)\alpha(a+f)^2(g-(n-1)\alpha)^2(a+(n+1)f)}{2nnfgg((n-1)a-f)^2}\cdot\frac{xx}{aa}$$
$$-\frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2}\cdot\frac{xx}{aa}$$

ex quo colligitur ſpatium diffuſionis quaeſitum:

$$Ff = +\frac{(n-1)a(a+f)^2(g-(n-1)\alpha)^2(a+(n+1)f)}{2nnfgg((n-1)a-f)^2}\cdot\frac{xx}{aa}$$
$$+\frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2}\cdot\frac{xx}{aa}$$

ac

ac praeterea angulus BfN erit

$$BfN = \frac{((n-1)a-f)g}{(g-(n-1)\alpha)f} \cdot \frac{x}{a}.$$

Coroll. 1.

36. Quoties ergo ſpatium diffuſionis hoc modo expreſſum eſt poſitiuum, imago extrema propius ad lentem cadit quam principalis ſeu eſt $Bf < BF$. Contra autem ſi iſta expreſſio valorem obtineat n gatiuum, imago extrema a lente longius erit remota principali.

Coroll. 2.

37. Patet hinc etiam ſpatium diffuſionis cum apertura ita creſcere, vt ſit quadrato ſemidiametri aperturae proportionale, ſequetur ergo ipſam aperturae rationem.

Coroll. 3.

38. Spatium diffuſionis etiam hoc modo exprimi poteſt

$$Ff = \begin{cases} + \dfrac{(n-1)a\left(1+\frac{a}{f}\right)^2\left(1-\frac{(n-1)\alpha}{g}\right)^2\left(n+1+\frac{a}{f}\right)}{2nn\left(\frac{(n-1)a}{f}-1\right)^2} \cdot \dfrac{xx}{aa} \\ + \dfrac{(n-1)\alpha\left(1+\frac{\alpha}{g}\right)^2\left(\frac{(n-1)a}{f}-1\right)^2\left(n+1+\frac{\alpha}{g}\right)}{2nn\left(1-\frac{(n-1)\alpha}{g}\right)^2} \cdot \dfrac{xx}{aa} \end{cases}$$

et angulus

$$BfN = \frac{\frac{(n-1)a}{f}-1}{1-\frac{(n-1)\alpha}{g}} \cdot \frac{x}{a}.$$

Coroll. 4.

39. Quia hae formulae introducendis litterarum valoribus reciprocis redditae sunt simpliciores, in hunc modum etiam aequatio (§. 32) exhibita tractetur quae per $a\alpha dfg$ diuisa abit in hanc formam.

$$n(n-1)\tfrac{1}{d}\left(\tfrac{1}{f}+\tfrac{1}{g}\right)-n\tfrac{1}{d}\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)+(n-1)\left(\tfrac{1}{\alpha f}+\tfrac{1}{a g}\right)-(n-1)^2\tfrac{1}{fg}-\tfrac{1}{a\alpha}=0$$

seu

$$\tfrac{n}{d}\left((n-1)a\alpha\left(\tfrac{1}{f}+\tfrac{1}{g}\right)-a-\alpha\right)=\left(\tfrac{(n-1)\alpha}{f}-1\right)\left(\tfrac{(n-1)a}{g}-1\right)$$

quae commodior erit tam ad relationem inter a et α, quam inter f et g definiendam.

Scholion.

40. Scilicet si proposita lente obiectum in variis distantiis exponatur ac pro singulis distantiam imaginis principalis definire velimus, aequatio hoc modo referatur:

$$\tfrac{1}{a\alpha}-(n-1)\left(\tfrac{1}{\alpha f}+\tfrac{1}{a g}\right)+n\left(\tfrac{1}{d a}+\tfrac{1}{d\alpha}\right)-n(n-1)\left(\tfrac{1}{d f}+\tfrac{1}{d g}\right)+(n-1)^2\tfrac{1}{fg}=0$$

quae per factores ita adornari poterit

$$\left(\tfrac{1}{a}-\tfrac{(n-1)}{f}+\tfrac{n}{d}\right)\left(\tfrac{1}{\alpha}-\tfrac{(n-1)}{g}+\tfrac{n}{d}\right)=\tfrac{nn}{dd}$$

Vnde patet productum ex his duobus factoribus semper esse idem. Cuius resolutio quo facilius instituatur, capiantur numeri μ et ν ita, vt eorum summa sit $=1$, scilicet $\mu+\nu=1$ ac statuatur

$$\tfrac{1}{a}-\tfrac{(n-1)}{f}+\tfrac{n}{d}=\tfrac{-\mu n}{\nu d} \text{ erit } \tfrac{1}{a}=\tfrac{n-1}{f}-\tfrac{n}{\nu d}$$

$$\tfrac{1}{\alpha}-\tfrac{(n-1)}{g}+\tfrac{n}{d}=\tfrac{-\nu n}{\mu d} \text{ erit } \tfrac{1}{\alpha}=\tfrac{n-1}{g}-\tfrac{n}{\mu d}$$

Quare

Quare si distantia obiecti $AE = a$ ita capiatur, vt sit $\frac{1}{a} = \frac{n-1}{f} - \frac{n}{\nu d}$, distantia imaginis principalis $BF = \alpha$ ita se habebit, vt sit $\frac{1}{\alpha} = \frac{n-1}{g} - \frac{n}{\mu d}$.

Eodem modo si dentur binae distantiae $AE = a$ et $BF = \alpha$ cum crassitie lentis $AB = d$, radii facierum f et g ita debent esse comparati, vt sit

$$\frac{1}{f} = \frac{1}{(n-1)a} + \frac{n}{\nu(n-1)d} \text{ et } \frac{1}{g} = \frac{1}{(n-1)\alpha} + \frac{n}{\mu(n-1)d}$$

id quod infinitis modis praestari potest, cum numeri μ et ν pro arbitrio accipi queant, dummodo eorum summa $\mu + \nu$ aequetur vnitati. Si hoc modo etiam in formulis pro spatio diffusionis inuentis omnes litterae in denominatores detrudantur, reperietur:

$$Ff = \frac{(n-1)\alpha\alpha xx}{2nn}\left\{\frac{(\frac{1}{a}+\frac{1}{f})^2(\frac{1}{\alpha}-\frac{(n-1)}{g})^2(\frac{n+1}{a}+\frac{1}{f})}{(\frac{n-1}{f}-\frac{1}{a})^2} + \frac{(\frac{1}{\alpha}+\frac{1}{g})^2(\frac{n-1}{f}-\frac{1}{a})^2(\frac{n+1}{\alpha}+\frac{1}{g})}{(\frac{1}{\alpha}-\frac{(n-1)}{g})^2}\right\}$$

et

$$\text{angulus } BfN = \frac{\frac{n-1}{f}-\frac{1}{a}}{\frac{1}{\alpha}-\frac{(n-1)}{g}} \cdot \frac{x}{\alpha} = \frac{\frac{n}{\nu d}}{\frac{-n}{\mu d}} \frac{x}{\alpha} = -\frac{\mu}{\nu} \cdot \frac{x}{\alpha}$$

Cum autem sit

$$\frac{n-1}{f} - \frac{1}{a} = \frac{n}{\nu d} \text{ et } \frac{1}{\alpha} - \frac{(n-1)}{g} = \frac{-n}{\mu d} \text{ erit}$$

$$Ff = \frac{(n-1)\alpha\alpha\, xx}{2nn}\left(\frac{\nu\nu}{\mu\mu} \cdot \frac{1}{a} + \frac{1}{f}\right)^2\left(\frac{n+1}{a} + \frac{1}{f}\right) + \frac{\mu\mu}{\nu\nu} \cdot \frac{1}{\alpha} + \frac{1}{g}\Big)^2\left(\frac{n+1}{\alpha} + \frac{1}{g}\right)$$

et

$$\text{angulus } BfN = -\frac{\mu x}{\nu\alpha} = \frac{\mu x}{(\mu-1)\alpha}.$$

Proble-

Problema 5.

Tab. I. Fig. 1. 41. Datis diſtantiis obiecti ante lentem $AE = a$ et imaginis principalis poſt lentem $BF = \alpha$ vna cum lentis craſſitie $AB = d$, definire omnes lentes ſatisfacientes, ſimulque pro ſingulis ſpatium diffuſionis Ff.

Solutio.

Modo vidimus ſi lentis faciei anterioris AM radius ponatur $= f$, poſterioris $BN = g$, lente vt conuexa vtrinque ſpectata, hos duos radios ita comparatos eſſe debere, vt ſit

$$\frac{n-1}{f} = \frac{1}{a} + \frac{n}{\nu d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} + \frac{n}{\mu d}$$

ſumtis pro μ et ν numeris quibuscunque, vt ſit $\mu + \nu = 1$; vnde infinitae lentes quaeſito ſatisfacientes obtinentur. Deinde ſi huius lentis ſemidiameter aperturae ponatur $= x$, ſpatium diffuſionis Ff ita exprimi poteſt vt ſit

$$Ff = \frac{\alpha\alpha xx}{2nn(n-1)^2}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{n-1}{a} + \frac{n-1}{f}\right)^2\left(\frac{nn-1}{a} + \frac{n-1}{f}\right) + \frac{\mu\mu}{\nu\nu}\left(\frac{n-1}{\alpha} + \frac{n-1}{g}\right)^2\left(\frac{nn-1}{\alpha} + \frac{n-1}{g}\right)\right)$$

et angulus $BfN = \frac{\mu x}{(\mu - 1)\alpha}$.

Quod ſi iam hic pro $\frac{n-1}{f}$ et $\frac{n-1}{g}$ valores aſſignati ſubſtituantur ſpatium diffuſionis erit

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{1}{a} + \frac{1}{\nu d}\right)^2\left(\frac{n}{a} + \frac{1}{\mu d}\right) + \frac{\mu\mu}{\nu\nu}\left(\frac{1}{\alpha} + \frac{1}{\mu d}\right)^2\left(\frac{n}{\alpha} + \frac{1}{\mu d}\right)\right)$$

ſeu

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)}\left\{\begin{array}{l} + \frac{\nu\nu}{\mu\mu}\left(\frac{n}{a^3} + \frac{2n+1}{\nu aad} + \frac{n+2}{\nu\nu add} + \frac{1}{\nu^3 d^3}\right) \\ + \frac{\mu\mu}{\nu\nu}\left(\frac{n}{\alpha^3} + \frac{2n+1}{\mu\alpha\alpha d} + \frac{n+2}{\mu\mu\alpha dd} + \frac{1}{\mu^3 d^3}\right) \end{array}\right\}$$

Si

Si craſſities lentis fuerit valde parua, ne confuſio fiat enormis, neceſſe eſt pro μ et ν ſumi numeros vehementer magnos, alterum ſcilicet poſitiuum alterum negatiuum. Cum igitur ſit $\mu d + \nu d = d$, ſtatuatur $\mu\, d = \frac{d-k}{2}$ et $\nu d = \frac{d+k}{2}$, vt ſit

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-d} \text{ ſeu}$$

$$f = \frac{(n-1)a(k+d)}{2na+k+d} \text{ et } g = \frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

hincque obtinetur ſpatium diffuſionis

$$Ff = \frac{n\alpha a x x}{2(n-1)^2}\left\{\begin{array}{l} +\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2 \\ +\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2 \end{array}\right\}$$

in quibus formulis paruitas craſſitiei lentis $AB = d$ nullum negotium faceſſit: tum vero eſt

angulus $BfN = \frac{k-d}{k+d} \cdot \frac{x}{a}$.

Coroll. 1.

41. Propoſitis ergo binis diſtantiis $AE = a$ et $BF = \alpha$ vna cum craſſitie lentis $AB = d$, infinitis modis lentes idoneae parari poſſunt cum pro k quantitates pro lubitu ſiue poſitiuae ſiue negatiuae aſſumi queant.

Coroll. 2.

43. Cum ſemidiameter aperturae x faciem anteriorem reſpiciat, et angulus BfN ſeu BFN ſit $= \frac{k-d}{k+d} \cdot \frac{x}{\alpha}$, exiſtente diſtantia $BF = \alpha$, manifeſtum eſt ſemidiametrum

trum aperturae posterioris faciei esse debore $= \frac{k-d}{k+d} \cdot x$ vel saltem non minorem.

Coroll. 3.

44. Quodsi lentis crassities tam sit parua, vt ea prae k contemni queat, formulae nostrae fient simpliciores

$$f = \frac{(n-1)\alpha k}{k+2n\alpha} \text{ et } g = \frac{(n-1)\alpha k}{k-2n\alpha}$$

et spatium diffusionis

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^3}\left(\left(\frac{n}{\alpha}+\frac{2}{k}\right)\left(\frac{1}{\alpha}+\frac{2}{k}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k}\right)\left(\frac{1}{\alpha}-\frac{2}{k}\right)^2\right)$$

angulusque $BfN = \frac{x}{\alpha}$.

Scholion.

45. Sufficiat haec de spatio diffusionis in genere quaecunque fuerit lentis crassities, tradidisse, cum non obstante his satis concinnis transformationibus calculus nimium fieret molestus, si in determinatione spatii diffusionis rationem crassitiei lentis habere vellemus. Licebit autem, vt modo vidimus, crassitiem lentis negligere non solum, quando ipsa est per se valde exigua verum etiam dummodo prae quantitate k fuerit perparua. Atque hinc etiam in sequentibus facile iudicare poterimus, vtrum crassitiem lentis recte in calculo contemserimus, nec ne? quouis enim casu consideretur quantitas pro k assumta, quae si multoties fuerit maior quam d error nullus erit pertimescendus;

fcendus; contra vero fi k non multum fuperet d multum aberrabitur, quantumuis exigua fuerit craffities ipfa per fe.

Vt fiat Ff minimum, definiri poteft conueniens valor ipfius k hoc modo. Ponatur $\frac{k-d}{k+d} = z$ et pro minimo peruenitur ad hanc aequationem:

$$0 = 2z^4\left(\tfrac{n}{\alpha}+\tfrac{1}{d}\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{d}\right)^2$$
$$-\tfrac{3nz^3}{\alpha d}\left(\tfrac{1}{\alpha}+\tfrac{1}{d}\right)-\tfrac{z^3}{d}\left(\tfrac{1}{\alpha}+\tfrac{2}{d}\right)^2$$
$$+\tfrac{2nz}{\alpha d}\left(\tfrac{1}{\alpha}+\tfrac{1}{d}\right)+\tfrac{z}{d}\left(\tfrac{1}{\alpha}+\tfrac{2}{d}\right)^2$$
$$-2\left(\tfrac{n}{\alpha}+\tfrac{1}{d}\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{d}\right)^2$$

vnde valor ipfius z erui debet.

Problema 6.

46. Neglecta lentis craffitie, fi detur cum obiecti ante lentem diftantia $AE = a$, tum imaginis principalis poft lentem diftantia $BF = \alpha$, eam definire lentem quae pro data apertura minimam pariat diffufionem.

Solutio.

Pofitis radiis faciei anterioris $AM = f$ et pofterioris $BN = g$ vtraque vt connexa fpectata, vidimus omnes lentes datis diftantiis a et α conuenientes his formulis determinari: fi fcilicet pro k fcribamus $2k$

$$\tfrac{n-1}{f} = \tfrac{1}{a}+\tfrac{n}{k} \text{ et } \tfrac{n-1}{g} = \tfrac{1}{\alpha}-\tfrac{n}{k}$$

deno-

denotante k quantitatem quamcunque. Tum autem ſpatium diffuſionis ita exprimi eſt repertum

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{1}{k}\right)\left(\frac{1}{a}+\frac{1}{k}\right)^2+\left(\frac{n}{\alpha}-\frac{1}{k}\right)\left(\frac{1}{\alpha}-\frac{1}{k}\right)^2\right)$$

quae expreſſio ad hanc reducitur formam:

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{2n+1}{k}\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{n+1}{kk}\right)$$

Quaeſtio igitur huc reducitur, vt definiatur quantitas k qua huic expreſſioni valor minimus concilietur; cui quidem requiſito ſatisfacit valor

$$\frac{1}{k} = \frac{-(2n+1)}{2(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right) \text{ ſeu } \frac{1}{k} = \frac{2n+1}{2(n+2)}\left(\frac{1}{\alpha}-\frac{1}{a}\right)$$

quo ſubſtituto habebitur:

$$\frac{n-1}{f} = \frac{4+n-2nn}{2(n+2)a}+\frac{n(2n+1)}{2(n+2)\alpha} \text{ et } \frac{n-1}{g} = \frac{4+n-2nn}{2(n+2)\alpha}+\frac{n(2n+1)}{2(n+2)a}.$$

et ſpatium diffuſionis quod eſt minimum:

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)-\frac{(2n+1)^2}{4(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2\right)$$

At eſt

$$n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right) = n\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{n}{a\alpha} \text{ ideoque}$$

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\frac{4n-1}{4(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{n}{a\alpha}\right)$$

quae expreſſio etiam in hanc formam transfunditur

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\frac{4n-1}{4(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{(n-1)^2}{(n+2)a\alpha}\right)$$

ſeu

$$Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

Coroll.

Coroll. 1.

47. Vt igitur lens minimum ſpatium diffuſionis producat, eam ita formari neceſſe eſt vt ſit

$$f = \frac{2(n-1)(n+2)a\alpha}{n(2n+1)\alpha + (4+n-2nn)a} \quad \text{et}$$

$$g = \frac{2(n-1)(n+2)a\alpha}{n(2n+1)a + (4+n-2nn)\alpha}$$

neglecta ſcilicet lentis craſſitie, quae quidem recte negligitur, ſi modo fuerit vehementer parua prae quantitate $k = \frac{2(n+2)a\alpha}{(2n+1)(a-\alpha)}$.

Coroll. 2.

48. Si igitur ſit $a = \alpha$, ſeu $BF = AE$ craſſities lentis, quantacunque fuerit, nihil turbat in ſpatio diffuſionis. Pro hoc autem caſu erit $f = g = (n-1)a$ et ſpatium diffuſionis ipſum $Ff = \frac{nnxx}{(n-1)^2 a}$. At quo magis diſtantiae a et α a ſe inuicem diſcrepant, ſeu minor fuerit quantitas $\frac{a\alpha}{a-\alpha}$, eo magis haec determinatio ob lentis craſſitiem fiet erronea.

Coroll. 3.

49. Spatium autem hoc confuſionis minimum Ff pluribus modis exhiberi poteſt, inter quos commodiſſimum eligi conuenit; ſunt autem praecipui:

$$\text{I. } Ff = \frac{n(4n-1)a\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

$$\text{II. } Ff = \frac{n(4n-1)a\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2 + \frac{4n(n+2)}{(4n-1)a\alpha}\right)$$

$$\text{III. } Ff = \frac{n(4n-1)a\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{aa}+\frac{1}{\alpha\alpha}\right) + \frac{2(2nn+1)}{(4n-1)a\alpha}\right)$$

$$\text{IV. } Ff = \frac{n(4n-1)a\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right) + \frac{(2n+1)^2}{(4n-1)a\alpha}\right),$$

Coroll. 4.

50. Cum ergo hoc ſpatium diffuſionis ſit minimum, ſi lenti alia quaecunque figura tribuatur, ita tamen, vt obiecti ad diſtantiam $AE = a$ remoti imaginem principalem in diſtantia $BF = \alpha$ exhibeat, ſpatium diffuſionis erit maius, quam hic inuenimus.

Scholion.

51. Quia $n : 1$ denotat rationem refractionis ex aere in vitrum, haec autem pro radiorum natura eſt variabilis, conueniet pro n medium valorem aſſumi, qui eſt $n = \frac{31}{20}$; hinc ergo erit

$$n - 1 = \tfrac{11}{20};\ n + 2 = \tfrac{71}{20};\ 2n + 1 = \tfrac{41}{10};\ 4 + n - 2nn = \tfrac{149}{200},$$

hincque

$$\frac{n(2n+1)}{2(n-1)(n+2)} = \frac{1271}{781} = 1,627401$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = \frac{149}{781} = 0,190781$$

vnde lens minimam confuſionem pariens ita definietur

$$\frac{1}{f} = \frac{149}{781a} + \frac{1271}{781\alpha} = \frac{0,190781}{a} + \frac{1,627401}{\alpha}$$

$$\frac{1}{g} = \frac{149}{781\alpha} + \frac{1271}{781a} = \frac{0,190781}{\alpha} + \frac{1,627401}{a}$$

ſeu

$$f = \frac{781a\alpha}{149\alpha + 1271a} = \frac{a\alpha}{0,190781\,\alpha + 1,627401\,a}$$

et

$$g = \frac{781a\alpha}{149a + 1271\alpha} = \frac{a\alpha}{0,190781\,a + 1,627401\,\alpha}.$$

Pro

Pro ſpatio autem diffuſionis ipſo definiendo ob $4n-1=\frac{26}{5}$

$$\text{erit } \frac{n(4n-1)}{8(n-1)^2(n+2)}=\frac{8060}{8591}=0,938191$$

$$\text{et } \frac{4(n-1)^2}{4n-1}=\frac{121}{520}=0,232692$$

hincque

$$Ff=0,938191\,aaxx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{0,11261}{a\alpha}\right)$$

vnde reliquae formulàe facile deducuntur. Ceterum hic monendum eſt, quoniam valor $n=\frac{31}{20}$ ex experimentis eſt concluſus, neque ipſe pro omnibus radiorum generibus valet, ſuperfluum fore in praxi hos numeros inuentos nimis ſtudioſe obſeruare; quin etiam ipſa natura minimi aliquam aberrationem permittit. Nihilo vero minus has fractiones decimales ad tot figuras producere viſum eſt, quo facilius quantum ab hac hypotheſi aberretur, dignoſci queat.

Problema 7.

52. Neglecta lentis craſſitie, ſi detur cum obiecti ante lentem diſtantia $AE=a$, tum imaginis principalis poſt lentem diſtantia $BF=\alpha$, eam definire lentem, quae pro data apertura non minimum, ſed datum pariat ſpatium diffuſionis Ff.

Solutio.

Lente vtrinque vt conuexa ſpectata, ſit faciei anterioris radius $=f$, poſterioris $=g$; atque vt ex data

 diſtan-

diſtantia obiecti $AE = a$, data oriatur imaginis principalis diſtantia $= \alpha$ neceſſe eſt ſit in genere

$$\frac{n-1}{f} = \frac{1}{a} + \frac{n}{k} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{n}{k}$$

vnde ſpatium diffuſionis fit

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa} - \frac{1}{a\alpha} + \frac{1}{\alpha\alpha}\right) + \frac{2n+1}{k}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{n+2}{kk}\right)$$

Cum autem ſpatium diffuſionis minimum repertum ſit

$$\frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

neceſſe eſt vt illud ſit maius: ſtatuatur ergo:

$$Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha} + S\right)$$

et habebitur iſta aequatio:

$$4n(n+2)\left(\frac{1}{aa} - \frac{1}{a\alpha} + \frac{1}{\alpha\alpha}\right) + \frac{4(n+2)(2n+1)}{k}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{4(n+2)^2}{kk}$$
$$= (4n-1)\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{a\alpha} + (4n-1)S$$

quae in hanc formam redigitur.

$$(2n+1)^2\left(\frac{1}{a} - \frac{1}{\alpha}\right)^2 + \frac{4(n+2)(2n+1)}{k}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{4(n+2)^2}{kk} = (4n-1)S$$

vnde radice extracta fit

$$(2n+1)\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{2(n+2)}{k} = \sqrt{(4n-1)S}$$

$$\text{et } \frac{1}{k} = \frac{-(2n+1)}{2(n+2)}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{\sqrt{(4n-1)S}}{2(n+2)}$$

Quare erit

$$\frac{n-1}{f} = \frac{1}{a} - \frac{n(2n+1)}{2(n+2)}\left(\frac{1}{a} - \frac{1}{\alpha}\right) + \frac{n}{2(n+2)}\sqrt{(4n-1)S}$$
$$\frac{n-1}{g} = \frac{1}{\alpha} + \frac{n(2n+1)}{2(n+2)}\left(\frac{1}{a} - \frac{1}{\alpha}\right) - \frac{n}{2(n+2)}\sqrt{(4n-1)S}$$

ſiue

fiue

$$f = \frac{2(n-1)(n+1)a\alpha}{(4+n-2nn)\alpha + n(2n+1)a + na\alpha\sqrt{(4n-1)S}}$$

$$g = \frac{2(n-1)(n+1)a\alpha}{(4+n-2nn)a + n(2n+1)\alpha - na\alpha\sqrt{(4n-1)S}}$$

Oportet ergo pro S fumi quantitatem pofitiuam, atque vt cum reliqua parte, cui jungitur, fit homogenea, ponamus

$$S = (\lambda - 1)\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2$$

vbi λ denotat numerum vnitate maiorem; atque effici poterit, vt fpatium diffufionis ita exprimatur

$$Ff = \frac{n(4n-1)\alpha\alpha\, xx}{4(n-1)^2(n+1)^2}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

quod cum vt datum fpectetur, numerus λ pro dato affumi poterit, ac lens hunc effectum producens ita determinabitur.

$$f = \frac{2(n-1)(n+1)a\alpha}{(4+n-2nn)\alpha + n(2n+1)a + n(a+\alpha)\sqrt{(4n-1)(\lambda-1)}}$$

$$g = \frac{2(n-1)(n+1)a\alpha}{(4+n-2nn)a + n(2n+1)\alpha - n(a+\alpha)\sqrt{(4n-1)(\lambda-1)}}$$

quod ergo cum fignum radicale ambiguo illatum inuoluat, duplici modo fieri poterit.

Coroll. I.

53. Omnes igitur lentes, quae obiecti ad diftantiam $AE = a$ remoti, imaginem principalem in diftantia $BF = \alpha$ repraefentant, hanc habent proprietatem vt fit:

$$\frac{n-1}{f} + \frac{n-1}{g} = \frac{1}{a} + \frac{1}{\alpha} \text{ feu } \frac{fg}{f+g} = \frac{(n-1)a\alpha}{a+\alpha}.$$

et cum sit

$$n=\tfrac{31}{20} \text{ erit } \frac{fg}{f+g}=\frac{11\,a\alpha}{20(a+\alpha)}=\frac{0,55\,a\alpha}{a+\alpha}.$$

Coroll. 2.

54. Si ergo lens sit vtrinque aeque conuexa seu $f=g$, oportet esse $f=g=\frac{2(n-1)a\alpha}{a+\alpha}=\frac{11\,a\alpha}{10(a+\alpha)}$.

sin autem lens desideretur plano-conuexa vt sit $f=\infty$, capi debet

$$g=\frac{(n-1)a\alpha}{a+\alpha}=\frac{11\,a\alpha}{20(a+\alpha)}=\frac{0,55\,a\alpha}{a+\alpha}$$

At si lens debeat esse conuexo-plana seu $g=\infty$, capi oportet

$$f=\frac{(n-1)a\alpha}{a+\alpha}=\frac{11\,a\alpha}{20(a+\alpha)}=\frac{0,55\,a\alpha}{a+\alpha}$$

Scholion 1.

55. Substituamus pro n valorem ipsi conuenientem $\frac{31}{20}$, et iam vidimus fore:

$$\frac{n(4n-1)}{8(n-1)^2(n+2)}=\frac{8060}{8591}=0,938191$$

$$\frac{4(n-1)^2}{4n-1}=\frac{121}{520}=0,232692$$

$$\frac{4+n-2nn}{2(n-1)(n+2)}=\frac{149}{781}=0,190781$$

$$\frac{n(2n+1)}{2(n-1)(n+2)}=\frac{1271}{781}=1,627401$$

nunc vero notari oportet esse:

$$\frac{n\sqrt{(4n-1)}}{2(n-1)(n+2)}=\frac{62\sqrt{130}}{781}=0,905133$$

Quare

Quare si lens ita construatur vt sit

$$f = \frac{a\alpha}{0,190781\, a + 1,627401\, \alpha \pm 0,905133 (a + \alpha) \sqrt{(\lambda - 1)}}$$

$$g = \frac{a\alpha}{0,190781\, \alpha + 1,627401\, a \mp 0,905133 (a + \alpha) \sqrt{(\lambda - 1)}}$$

erit pro eius apertura, cuius semidiameter $= x$, spatium diffusionis

$$Ff = 0,938191\, a\alpha xx \left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)^2 + \tfrac{0,232692}{a\alpha}\right)$$

Cum autem in posterum hi numeri frequentissime occurrant eorum loco ad abbreuiandum certis characteribus vtamur, ponamus ergo

$$\frac{n(4n-1)}{8(n-1)^2(n+2)} = 0,938191 = \mu.$$

$$\frac{4(n-1)^2}{4n-1} = 0,232692 = \nu$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = 0,190781 = \varrho$$

$$\frac{n(2n+1)}{2(n-1)(n+2)} = 1,627401 = \sigma$$

$$\frac{n\sqrt{(4n-1)}}{2(n-1)(n+2)} = 0,905133 = \tau$$

seu

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{1}{8(n+1)^2}$$

$$\varrho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1$$

$$\sigma = 1 + \frac{1}{2(n-1)} - \frac{1}{n+2}$$

$$\tau = \tfrac{1}{3}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right)\sqrt{(4n-1)}$$

vnde pro quouis alio medio pellucido hi valores supputari possunt sicque ficque vt prodeat fpatium diffufionis

$$Ff = \mu\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

lentis conftructio erit

$$f = \frac{a\alpha}{\varrho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}$$

$$g = \frac{a\alpha}{\varrho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}$$

Dummodo igitur λ fuerit numerus pofitiuus vnitate non minor talis lens duplici modo effici poterit. Cafu autem $\lambda=1$, quo fpatium diffufionis eft minimum, vnica lens propofito fatisfaciens conftrui poteft.

Coroll. 3.

56. Si igitur lens fit vtrinque aequaliter convexa, ideoque $f=g=\frac{2(n-1)a\alpha}{a+\alpha}=\frac{11}{10}\cdot\frac{a\alpha}{a+\alpha}$, in expreffione noftra pro fpatio diffufionis inuenta valor ipfius λ ex aequalitate inter f et g ftatuta definietur, vnde fit

$$(\sigma-\varrho)(a-\alpha)=2\tau(a+\alpha)\sqrt{(\lambda-1)} \text{ hincque}$$

$$\lambda=1+\frac{0{,}629795\,\alpha\alpha-1{,}259589\,a\alpha+0{,}629795\,aa}{(a+\alpha)^2}.$$

Coroll. 4.

57. Si lens capiatur plano conuexa vt fit:

$$f=\infty \text{ et } g=\frac{(n-1)a\alpha}{a+\alpha}=\frac{11}{20}\cdot\frac{a\alpha}{a+\alpha}$$

pro fpatio diffufionis habebitur

$$\varrho\alpha+\sigma a=\pm\tau(a+\alpha)\sqrt{(\lambda-1)}$$

vnde in numeris colligitur

$$\lambda=1+\frac{0{,}044427\,\alpha\alpha+0{,}757540\,a\alpha+3{,}252692\,aa}{(a+\alpha)^2}$$

Coroll.

Coroll. 5.

58. Si denique lens adhibeatur conuexo-plana, vt sit:

$$g = \infty \text{ et } f = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11}{20} \cdot \frac{a\alpha}{a+\alpha}$$

pro spatio diffusionis inueniendo poni oportet

$$\varrho a + \sigma \alpha = \pm \tau (a+\alpha) \surd (\lambda - 1)$$

vnde in numeris colligimus:

$$\lambda = 1 + \frac{3,332697\alpha\alpha + 0,757640a\alpha + 0,044427aa}{(a+\alpha)^2}$$

Scholion 2.

59. Quod ad aperturam attinet iam initio animaduertimus, in ea maiores arcus comprehendi non debere, quam qui sint principiis stabilitis conformes. Scilicet vt nullus angulus supra 30° gradus occurrat, anguli ACM et BDN certe minores 30 gradibus esse debent, cum anguli EMc et VNd ipsis sint maiores, quorum alter cum quandoque ad duplum assurgere possit, poterimus hanc regulam statuere, vt aperturae semidiameter x neque $\frac{1}{4} f$ neque $\frac{1}{4} g$ superet. Verum quouis casu ad ipsos angulos EMc et VNd, qui sunt maximi, attendi conueniet, atque tanta apertura admitti poterit, vnde neuter horum angulorum 30 gradus superans oriatur, si cautius procedere velimus, etiam angulos 20 gradibus minores cuitare poterimus, quo pacto apertura magis restringetur. Tab. I. Fig. 2.

Problema 8.

Tab. I. Fig. 3. 60. Non neglecta lentis crassitie AB si pro distantia obiecti $AE = a$ detur distanta imaginis principalis $BF = \alpha$, obiectum autem parumper longius in e remoueatur, definire locum imaginis principalis f.

Solutio.

Posita lentis crassitie $AB = d$, radiisque faciei anterioris $AM = f$ et posterioris $BN = g$, supra vidimus hos radios ita a binis distantiis a et α atque crassitie lentis d pendere debere, vt sit

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-d}$$

denotante k quantitatem quamcunque. Hinc ergo cum sit

$$\frac{k+d}{2n} = \frac{af}{(n-1)a-f} \text{ et } \frac{k-d}{2n} = \frac{\alpha g}{g-(n-1)\alpha}$$

erit eliminando k

$$\frac{d}{n} = \frac{af}{(n-1)a-f} - \frac{\alpha g}{g-(n-1)\alpha}.$$

Ponamus iam distantiam $AE = a$ crescere particula $Ee = da$ ac per differentiationem inueniemus, quantum inde distantia imaginis $BF = \alpha$ immutetur: habebimus scilicet:

$$\frac{-ff\,da}{((n-1)a-f)^2} - \frac{gg\,d\alpha}{(g-(n-1)\alpha)^2} = 0$$

vnde elicimus

$$d\alpha = \frac{-ff(g-(n-1)\alpha)^2}{gg((n-1)a-f)^2}\,da = \frac{-\alpha\alpha}{aa}\,da\left(\frac{k+d}{k-d}\right)^2$$

Quare

Quare obiecto E per ſpatiolum minimum $\mathrm{E}e$ longius a lente remoto, imago principalis ex F propius ad lentem admouebitur per ſpatiolum minimum $\mathrm{F}f$, ita vt ſit

$$\mathrm{F}f = \frac{\alpha\alpha}{a\,a}\left(\frac{k+d}{k-d}\right)^2 . \mathrm{E}e = \frac{ff(g-(n-1)\alpha)^2}{gg((n-1)a-f)^2} . \mathrm{E}e.$$

Coroll. 1.

61. Quia quantitas $\frac{\alpha\alpha}{a\,a}\left(\frac{k+d}{k-d}\right)^2$ neceſſario eſt poſitiua, euidens eſt ſi obiectum longius a lente remoueatur, imaginem ſemper propius ad lentem admoueri. Contra ergo etiam ſi obiectum propius ad lentem accedat, imago longius ab ea recedet.

Coroll. 2.

62. Si craſſities lentis d euaneſcat erit $\mathrm{F}f = \frac{\alpha\alpha}{a\,a} . \mathrm{E}e$, ſin autem ea non euaneſcat fieri poteſt vt ſit vel $\mathrm{F}f > \frac{\alpha\alpha}{a\,a}\,\mathrm{E}e$ vel $\mathrm{F}f < \frac{\alpha\alpha}{a\,a} . \mathrm{E}e$, prius eueniet ſi k ſit quantitas poſitiua, poſterius ſi negatiua. At ſi ſit vel $k = \infty$ vel $k = 0$, vtroque caſu erit $\mathrm{F}f = \frac{\alpha\alpha}{a\,a} . \mathrm{E}e$ etiamſi craſſities lentis non euaneſcat.

Coroll. 3.

63. Cum in diſtantiis a et α mutatio minima fieri concipiatur, ſpatium diffuſionis nullam inde variationem ſubire cenſendum eſt: ſiue ergo obiectum in E ſiue e reperiatur, ac ſemidiameter aperturae

 lentis

lentis in facie anteriore fuerit $= x$, erit spatium diffusionis =

$$\frac{n 1 \alpha x x}{2(n-1)} \left\{ \begin{array}{l} + \left(\frac{k+d}{k-d}\right)^2 \left(\frac{n}{a} + \frac{2}{k+1}\right)\left(\frac{1}{a} + \frac{2}{k+1}\right)^2 \\ + \left(\frac{k-d}{k+d}\right)^2 \left(\frac{n}{\alpha} - \frac{2}{k-d}\right)\left(\frac{1}{\alpha} - \frac{2}{k-d}\right)^2 \end{array} \right\}$$

in facie autem posteriori semidiameter aperturae debet esse $= \frac{k-1}{k+d} \cdot x$.

Problema 9.

Tab. I. Fig. 3. 64. Rationem definire, quam habet magnitudo imaginis ad magnitudinem obiecti non neglecta lentis crassitie.

Solutio.

Sit obiecti ante lentem distantia $AE = a$, imaginis vero $BF = \alpha$ existente lentis crassitie $= d$; vbi quidem tantum imaginem principalem spectamus neglecto spatio diffusionis. Sit iam $E\varepsilon$ obiectum, cui tribuatur magnitudo quam minima $E\varepsilon = z$, normaliter axi lentis insistens; eiusque imago in $F\zeta$ exhibebitur, cuius magnitudo $F\zeta$ quaeritur. Cum igitur punctum ζ a puncto ε oriatur, ita vt radii ab ε emissi in ζ colligantur, consideretur radius quicunque εM, qui productus cum axe in e occurrat: et perinde est, ac si hic radius ex axis puncto e emanaret. Quare is post refractionem cum axe in f concurret, vt sit $Ff = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 \cdot Ee$, indeque ad ζ perget: ex quo magnitudinem $F\zeta$ definire licebit. Statuatur in in hunc finem $AM = x$, erit $BN = \frac{k-d}{k+d} \cdot x$; hincque colligemus has proportiones:

$$Ee : E\varepsilon = eA : AM = a : x$$

$$Ff : F\zeta = fB : BN = \alpha : \frac{k-d}{k+d} x$$

ob ſpatiola Ee et Ff quam minima: vnde habebimus

$$\frac{Ff}{Ee} : \frac{F\zeta}{E\varepsilon} = \frac{\alpha}{a} : \frac{k-d}{k+d} = \frac{\alpha x}{a x} \left(\frac{k+d}{k-d}\right)^2 : \frac{F\zeta}{E\varepsilon}$$

Concluditur ergo $\frac{F\zeta}{E\varepsilon} = \frac{\alpha}{a} \cdot \frac{k+d}{k-d}$, ex quo cum magnitudo obiecti $E\varepsilon$ poſita ſit $= z$, erit magnitudo imaginis $F\zeta = \frac{\alpha(k+d)}{a(k-d)} z$.

Coroll. 1.

65. Secundum hanc ergo rationem diameter obiecti immutatur. Vbi notandum eſt, ſi expreſſio $\frac{\alpha(k+d)}{a(k-d)}$ habeat poſitiuum valorem obiecti imaginem ſitu inuerſo repraeſentari, contra autem ſitu erecto, ſi $\frac{\alpha(k+d)}{a(k-d)}$ negatiuum valorem adipiſcatur.

Coroll. 2.

66. Si craſſities lentis euaneſcat, fit $F\zeta = \frac{\alpha}{a} z$. quo ergo caſu recta iungens puncta extrema ε et ζ tranſit per centrum lentis. At ſi craſſities in computum ducatur, recta $\varepsilon\zeta$ modo intra lentem modo extra, axem lentis ſecare poterit.

Scholion.

67. Sic igitur omnia, quae circa vnam lentem quamcunque noſſe oportet, expediuimus, vt etiam

craſſitiei rationem habuerimus. Ac primo quidem ad binas diſtantias lentis quaſi determinatrices ſpectari conuenit, quae ſunt obiecti diſtantia ante lentem $AE = a$ et imaginis principalis poſt lentem diſtantia $BF = \alpha$, quibus addi debet lentis craſſities $AB = d$. His autem conditionibus infinitae lentes ſatisfaciunt; ſi enim radius faciei anterioris AM dicatur $= f$, et poſterioris $BN = g$, vtraque tanquam conuexa conſiderata, conſtructio lentis his continetur formulis:

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+d} \quad \text{et} \quad \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-d}$$

ſeu

$$f = \frac{(n-1)a(k+d)}{k+d+2na} \quad \text{et} \quad g = \frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

exiſtente $n = \frac{31}{20}$; vbi k eſt quantitas arbitraria, hincque lentes innumerabiles quaeſito ſatisfacientes obtinentur.

Quod ſi iam obiecti diameter ponatur $= z$, erit imaginis principalis repraeſentatae diameter $= \frac{\alpha(k+d)}{a(k-d)} z$, quatenus imago ſitu inuerſo exhibita conſideratur.

Deinde ſi aperturae in facie anterioris lentis ſemidiameter ſit $= x$, ſpatium diffuſionis, quatenus ab imagine principali ad lentem porrigitur, ita exprimetur vt ſit:

$$\frac{na\alpha xx}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k+d}{k-d}\right)^2 \left(\frac{n}{a} + \frac{2}{k+d}\right)\left(\frac{1}{a} + \frac{2}{k+d}\right)^2 \\ +\left(\frac{k-d}{k+d}\right)^2 \left(\frac{n}{\alpha} - \frac{2}{k-d}\right)\left(\frac{1}{\alpha} - \frac{2}{k-d}\right)^2 \end{array} \right\}$$

in facie autem poſteriori aperturae ſemidiameter debet eſſe $= \frac{k-d}{k+d} \cdot x$ vel ſaltem non minor. Quia ſpatium diffu-

diffusionis factorem habet $\alpha\alpha x x$, breuitatis gratia id ita $P\alpha\alpha x x$ indicabimus. Haecque in genere teneantur etiam crassitiei lentis ratione habita. At si crassities lentis euanescat, formulas magis euoluere licuit, scilicet si breuitatis gratia ponatur:

$$\mu = 0,938191; \quad \varrho = 0,190781; \quad \tau = 0,905133$$

$$\nu = 0,232692; \quad \sigma = 1,627401; \quad \lambda > 1 \text{ arbitr.}$$

sumaturque pro formatione lentis:

$$f = \frac{a\alpha}{\varrho\alpha + \sigma a \pm \tau (a + \alpha)\sqrt{(\lambda - 1)}}$$

$$g = \frac{a\alpha}{\varrho a + \sigma\alpha \mp \tau (a + \alpha)\sqrt{(\lambda - 1)}}$$

spatium diffusionis erit pro aperturae semidiametro x

$$\mu\alpha\alpha x x \left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{\nu}{a\alpha}\right)$$

et obiecti diametro existente $= z$ imaginis diameter erit $= \frac{\alpha}{a} z$. His ergo pro vna lente determinatis, videamus quomodo in combinatione duarum pluriumve lentium spatium diffusionis definiatur: vt deinceps in omnis generis instrumentis dioptricis siue Telescopiis siue microscopiis confusionem assignare, modumque eam diminuendi iuuestigare possimus.

CAPVT II.

CAPVT II.

DE DIFFVSIONE IMAGINIS PER PLVRES LENTES REPRAESENTATAE.

Problema I.

68.

Tab. I. Fig. 4. Si loco obiecti adſit imago per ſpatium Ee diffuſa, indeque radii per lentem PP aperturae indefinitae transmittantur, determinare ſpatium diffuſionis Ff per hanc lentem productum.

Solutio.

Loco obiecti veri hic conſideramus imaginem iam per aliam lentem repraeſentatam, quae ſit diffuſa per ſpatium Ee, ita vt in E ſit imago principalis per radios axi proximos formata in e autem imago extrema, radiis ſcilicet per marginem aperturae lentis praecedentis transmiſſis formata; qui radii cum axe conſtituant angulum $=\Phi$. Quare a puncto E nonniſi radii axi proximi in lentem PP emittuntur, a puncto e autem eiusmodi tantum radii, qui ad axem eA ſub angulo $MeA=\Phi$ ſint inclinati. Ponatur iam

iam diſtantia $EA = a$, prae qua ſpatium diffuſionis Ee vt valde paruum ſpectetur, lens autem PP ſit eiusmodi vt obiecti in E exiſtentis imaginem principalem referat in diſtantia $BF = \alpha$, exiſtente lentis craſſitie $AB = d$. Hanc ob rem ſi ponatur faciei anterioris AM radius $= f$, poſterioris $BN = g$, lente vt conuexa vtrinque ſpectata oportet eſſe:

$$f = \frac{(n-1)a(k+d)}{k+d+2na} \text{ et } g = \frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

vbi eſt $n = \frac{31}{20}$ et k quantitas arbitraria. Hinc autem vti demonſtrauimus, pro aperturae ſemidiametro $= x$ naſceretur ſpatium diffuſionis

$$\frac{n\alpha\alpha xx}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2 \\ +\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2 \end{array} \right\}$$

pro quo ſcribamus breuitatis gratia $P\alpha\alpha xx$. Iam puncti E, quia inde nonniſi radii axi proximi ad lentem emittuntur imago exhibebitur in F, vt ſit diſtantia $BF = \alpha$: videamus ergo quorſum imago puncti e debeat cadere. Ac ſi radii ex puncto e emiſſi eſſent axi proximi, quia id a lente magis eſt remotum quam E, eius imago propius ad lentem caderet puta in Φ, vt eſſet, ſicut §. 60 definiuimus

$$F\Phi = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 . Ee.$$

in Φ ſcilicet caderet imago principalis, ſi obiectum eſſet in e. Sed quia ab e tantum radii eM ad axem angulo $AeM = \Phi$ inclinati emittuntur, hi lenti in

 punctis

punctis M ab A interuallo $AO = \alpha A . \Phi = \alpha\Phi$ remotis occurrent, quandoquidem interuallum Ee prae distantia $Ae = a$ contemnimus, perinde igitur est ac si lenti apertura tribueretur, cuius semidiameter esset $= \alpha\Phi$, et obiecti in e existentis imago extrema definiri deberet, quae cadat in f ita vt Φf sit spatium diffusionis obiecto in e existenti, et aperturae lentis, cuius semidiameter $= \alpha\Phi$, conueniens. Hinc ergo erit interuallum $\Phi f = Paa\alpha\alpha\Phi\Phi$; et quia puncti E imago in F, puncti e autem imago in f exhibetur, erit spatium diffusionis per lentem PP productum

$$Ff = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 . Ee + Paa\alpha\alpha\Phi\Phi.$$

In imagine autem extrema f radii Nf ita cum axe concurrent, vt sit

$$\text{angulus } BfN = \frac{k-d}{k+d} \cdot \frac{\alpha\Phi}{a}.$$

Coroll. 1.

69. Si ergo imago iam diffusa per spatium Ee locum obiecti teneat respectu lentis PP, per hanc spatium diffusionis nouum producitur Ff, ita vt imago principalis cadat in F extrema vero in f: fierique poterit, vt hoc nouum spatium diffusionis Ff maius sit vel minus proposito Ee.

Coroll. 2.

70. Aperturam lentis PP vt indefinitam assumsi, manifestum autem est sufficere, dum eius semidiameter non sit minor quam $\alpha\Phi$. Si enim esset minor

minor, radii ex puncto e emiſſi plane non per lentem tranſitum inuenirent, neque imago puncti e exprimeretur, ſed imaginis Ff punctum extremum reſponderet imagini cuipiam intermediae ſpatii Ee.

Coroll. 3.

71. Si diameter obiecti ſeu potius imaginis in E ſitae ſit $= z$, tum imaginis inde per lentem PP in F formatae diameter erit $= \frac{\alpha(b+d)}{a(k-d)} z$ quae expreſſio ſi ſit poſitiua ſimul declarat ſitum imaginis in F eſſe inuerſum reſpectu eius, quae eſt in E.

Scholion.

72. Cum in hoc capite plures lentes ſim conſideraturus, pro cuiuslibet determinatione, ſpectandae ſunt primo binae diſtantiae determinatrices, altera obiecti ſeu imaginis, a qua lens radios accipit ante lentem, altera imaginis inde per lentem repraeſentatae poſt lentem: quae diſtantiae ex imaginibus principalibus aeſtimentur. Deinde cuiusque lentis craſſities in computum eſt ducenda. Tertio cum his lens nondum prorſus determinetur, inſuper pro quaque lente accedit diſtantia quaedam arbitraria hactenus per litteram k indicata. Quarto vero imprimis ratio haberi deberet, ſpatii diffuſionis, quod cuique lenti pro data apertura conueniat. Quemadmodum autem ex binis diſtantiis determinatricibus, craſſitie lentis et quantitate illa arbitraria k, cum binae lentis facies tum

etiam ſpatium diffuſionis pro apertura, cuius ſemidiameter ponitur $= x$ definiatur, in praecedente capite fuſius eſt expoſitum. Hic igitur cum plures lentes ſint conſiderandae, pro ſingulis haec elementa ſequentibus litteris indicabo.

Pro Lente	Diſtantiae determinatrices obiecti	Diſtantiae determinatrices imaginis	Craſſities lentis.	Quantitas arbitraria	Spatium diffuſionis pro aperturae ſemidiametro x
Prima	a	α	v	k	$P\alpha\alpha xx$
Secunda	b	β	v'	k'	$Q\beta\beta xx$
Tertia	c	γ	v''	k''	$R\gamma\gamma xx$
Quarta	d	δ	v'''	k'''	$S\delta\delta xx$
Quinta	e	ε	v''''	k''''	$T\varepsilon\varepsilon xx$

Sin autem ratio refractionis in ſingulis lentibus diſcrepet, pro prima lente eam ponamus $= n$; pro ſecunda $= n'$; pro tertia $= n''$ etc.

Prima autem lens hic mihi perpetuo erit ea, quae obiecto eſt proxima indeque recedendo reliquas lentes ordine numero: ex quo ſimul habebuntur diſtantiae lentium, ſcilicet ſecundae a prima $= \alpha + b$, tertiae a ſecunda $= \beta + c$; quartae a tertia $= \gamma + d$; quintae a quarta $= \delta + e$ etc. quae diſtantiae ex ſua natura debent eſſe poſitiuae etiamſi ſingulae diſtantiae determinatrices praeter primam a, quippe quae ad ipſum obiectum neceſſario ante lentem primam conſtituendum refertur, ſint quandoque negatiuae. Craſſitiem lentium hic

hic littera v defigno, quia littera d inter diftantias determinatrices, fi numerus lentium vltra ternarium affurgat, reperitur. Pro craffitie autem et quantitate arbitraria iisdem litteris vtor, commatibus infcriptis diftinguendis ob penuriam litterarum diuerfarum. Ceterum obferuandum eft me omnes lentes tanquam fuper communi axe difpofitas affumere.

Problema 2.

73. Si radii ab obiecto $E\varepsilon$ emiffi per duas lentes PP et QQ transmittantur definire fpatium diffufionis Gg, a data apertura primae lentis oriundum; vt et magnitudinem imaginis principalis in G exhibitae. Tab. I. Fig. 5.

Solutio.

Sit obiecti magnitudo $E\varepsilon = z$, eiusque imago principalis per primam lentem PP proiiciatur in $F\zeta$, at per ambas in $G\eta$ erunt diftantiae determinatrices pro lente prima PP, obiecti $EA = a$, imaginis $aF = \alpha$ pro lente fecunda QQ, obiecti $FB = b$, imaginis $bG = \beta$ Deinde fit

pro lente PP, craffities $Aa = v$, quantitas arbitr: $= k$
pro lente QQ, craffities $Bb = v'$, quantitas arbitr: $= k'$
Denique pro apertura in anteriori facie vtriusque lentis cuius femidiameter fit $= x$

fpatium diffufionis primae lentis $PP = P\alpha\alpha xx$
- - - - - fecundae lentis $QQ = Q\beta\beta xx$.

 His

His positis pro imagine per primam lentem proiecta $F\zeta$ erit eius magnitudo $F\zeta = \frac{\alpha(k+v)}{a(k-v)}z$ pro inuersa habenda, si haec expressio fuerit positiua. Deinde si lentis primae PP semidiameter aperturae in anteriori facie ponatur $= x$, erit spatium diffusionis $Ff = P\alpha\alpha xx$ et inclinatio radiorum in f ad axem $= \frac{k-v}{k+v} \cdot \frac{x}{\alpha}$. tum vero in facie lentis PP posteriori semidiameter aperturae non minor esse debet quam $\frac{k-v}{k+v}x$. Tota iam haec imago diffusa per spatium Ff respectu alterius lentis QQ tanquam obiectum spectari debet, cuius proinde repraesentatio Gg per propositionem praecedentem determinabitur. Erit autem hic $\Phi = \frac{k-v}{k+v} \cdot \frac{x}{\alpha}$, et spatium ibi expressum $Ee = P\alpha\alpha xx$; tum vero pro a, α, k, d et P hic scribi oportet b, β, k', v' et Q, vnde fiet spatium diffusionis quaesitum:

$$Gg = \frac{\beta\beta}{bb}\left(\frac{k'+v'}{k'-v'}\right)^2 P\alpha\alpha xx + \frac{bb\beta\beta}{\alpha\alpha}\left(\frac{k-v}{k+v}\right)^2 Qxx$$

siue

$$Gg = \frac{\beta\beta}{bb}\left(\frac{k'+v'}{k'-v'}\right)^2 P\alpha\alpha xx + \frac{bb}{\alpha\alpha}\left(\frac{k-v}{k+v}\right)^2 Q\beta\beta xx$$

Radiorum porro in g incidentium inclinatio ad axem est $\left(\frac{k-v}{k+v}\right)\left(\frac{k'-v'}{k'+v'}\right)\frac{bx}{\alpha\beta}$. Denique cum sit $F\zeta = \frac{\alpha(k+v)}{a(k-v)}z$ erit magnitudo imaginis in G vt erutae consideratae:

$$G\eta = \frac{\alpha\beta}{ab}\left(\frac{k+v}{k-v}\right)\left(\frac{k'+v'}{k'-v'}\right)z.$$

Coroll. 1.

74. Quod ad aperturam facierum lentis QQ attinet, ea maior esse debet spatio transitus radiorum; hinc ergo erit semidiameter aperturae

faciei

faciei anterioris $> \left(\frac{k-v}{k+v}\right)\frac{bx}{\alpha}$

faciei posterioris $> \left(\frac{k-v}{k+v}\right)\left(\frac{k'-v'}{k'+v'}\right)\frac{bx}{\alpha}$.

Coroll. 2.

75. Si ponatur pro lente prima PP radius faciei anterioris $= f$, et posterioris $= g$ erit posito $n =$ [illegible]

$$f = \frac{(n-1)a(k+v)}{k+v+2na} \text{ et } g = \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$$

Similique modo si pro lente altera QQ radius faciei anterioris ponatur $= f'$, et posterioris $= g'$ erit

$$f' = \frac{(n-1)b(k'+v')}{k'+v'+2nb} \text{ et } g' = \frac{(n-1)6(k'-v')}{k'-v'-2n6}$$

omnibus scilicet faciebus vt conuexis consideratis.

Coroll. 3.

76. Pro spatio autem diffusionis ab vtraque lente productae erit quemadmodum inuenimus:

$$P = \frac{n}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k+v}{k-v}\right)^2\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2 \\ +\left(\frac{k-v}{k+v}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2 \end{array} \right\}$$

similique modo

$$Q = \frac{n}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k'+v'}{k'-v'}\right)^2\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2 \\ +\left(\frac{k'-v'}{k'+v'}\right)^2\left(\frac{n}{6}-\frac{2}{k'-v'}\right)\left(\frac{1}{6}-\frac{2}{k'-v'}\right)^2 \end{array} \right\}$$

Non opus est vt omnibus lentibus eadem refractionis ratio $n:1$ tribuatur, sed simili modo pro pluribus lentibus poni potest n, n', n'', n''', etc. vt iam supra monuimus vnde nullum aliud discrimen nascitur, nisi

nisi quod in formulis pro f' et g' inuentis loco n scribi debeat n', et Q statui debeat.

$$=\frac{n'}{2(n'-1)^2}\left\{\begin{array}{l}+\left(\frac{k'+v'}{k'-v'}\right)^2\left(\frac{n'}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2\\+\left(\frac{k'-v'}{k'+v'}\right)^2\left(\frac{n'}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2\end{array}\right\}$$

Coroll. 4.

77. Sin autem crassities lentium euanescat, et pro quantitate arbitraria k, k' introducamus numerum λ, λ', erit

pro lente PP

$$f=\frac{a\alpha}{\varrho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}};\ g=\frac{a\alpha}{\varrho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}$$

et

$$P=\mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

at pro lente QQ

$$f'=\frac{b\beta}{\varrho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}};\ g'=\frac{b\beta}{\varrho b+\sigma\beta\mp\tau(b+\beta)\sqrt{(\lambda'-1)}}$$

$$Q=\mu\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right).$$

In expressionibus autem inuentis formulae $\left(\frac{k-v}{k+v}\right)$ et $\left(\frac{k'-v'}{k'+v'}\right)$ abeunt in vnitatem.

Numerorum vero $\mu, \nu, \varrho, \sigma, \tau$ indoles in §. 55 exposita est.

Si refractio lentium differat etiam litterae $\mu, \nu, \varrho, \sigma, \tau$ diuersos valores pro singulis lentibus sortientur, quae litte-

litterae ſi etiam commatibus a prioribus diſtinguantur, vt ſit

$$\varrho' = \frac{1}{2(n'-1)} + \frac{1}{n'+2} - 1$$
$$\sigma' = 1 + \frac{1}{2(n'-1)} - \frac{1}{n'+2}$$
$$\tau' = \tfrac{1}{3}\left(\frac{1}{2(n'-1)} + \frac{1}{n'+2}\right)\sqrt{(4n'-1)}$$

pro ſecunda lente erit

$$f' = \frac{b\mathfrak{b}}{\varrho'\mathfrak{b} + \sigma' b \pm \tau'(b+\mathfrak{b})\sqrt{(\lambda'-1)}}$$
$$g' = \frac{b\mathfrak{b}}{\varrho' b + \sigma'\mathfrak{b} \mp \tau'(b+\mathfrak{b})\sqrt{(\lambda'-1)}}$$

et ob eandem rationem erit

$$\mu' = \frac{1}{4(n'+2)} + \frac{1}{4(n'-1)} + \frac{1}{2(n'+1)^2}$$
$$\text{et } \nu' = \frac{4(n'-1)^2}{4n'-1}$$

quod et de ſequentibus lentibus eſt intelligendum, ſi forte diuerſa refractionis lege fuerint praeditae.

Scholion.

78. Quo formulas in problemate inuentas magis contrahamus, vt cum ad plures lentes proceſſerimus, ſuccinctiores euadant, ponamus

$$\frac{k-v}{k+v} = i \text{ et } \frac{k'-v'}{k'+v'} = i'$$

ita vt hi numeri i et i' abeant in vnitatem euaneſcente lentium craſſitie v et v'. Tum autem erit ſpatium diffuſionis

$$Gg = \frac{1}{rr} \cdot \frac{\mathfrak{b}\mathfrak{b}}{bb} \cdot P\alpha\alpha xx + ii \cdot \frac{bb}{\alpha\alpha} \cdot Q\mathfrak{b}\mathfrak{b}xx$$

et radiorum in g incidentium inclinatio ad axem $= ii' . \frac{bx}{\alpha\beta}$: porro magnitudo imaginis $G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z$ Ac pro apertura ſingularum facierum erit vt ſequitur

Semidiam. aperturae lentis	faciei anterioris	faciei poſterioris
primae PP	x	$i x$
ſecundae QQ	$i . \frac{bx}{\alpha}$	$ii' . \frac{bx}{\alpha}$

aperturae autem hae praeter primam maiores eſſe debent aſſignatis: valores enim aſſignati ſufficerent, ſi obiectum eſſet merum punctum in axe poſitum; quando autem habet magnitudinem, radii ab eius terminis per primam faciem ingreſſi latius diuagantur, et ad ſequentes facies maiorem aperturam exigunt.

Problema 3.

Tab. II. Fig. 6. 79. Si radii ab obiecto $E\varepsilon$ emiſſi per tres lentes PP, QQ et RR refringantur definire ſpatium diffuſionis Hh ob datam aperturam lentis primae oriundum, vt et magnitudinem imaginis principalis in H exhibitae.

Solutio.

Poſita obiecti magnitudine $E\varepsilon = z$, cadat eius imago principalis per lentem primam PP in $F\zeta$, dehinc per ſecundam QQ in $G\eta$, tum vero per tertiam RR in $H\theta$. Erunt ergo diſtantiae determinatrices,

pro lente PP, obiecti $EA = a$, imaginis $aF = \alpha$
pro lente QQ, obiecti $FB = b$, imaginis $bG = \beta$
pro lente RR, obiecti $GC = c$; imaginis $cH = \gamma$

Deinde

Deinde fit

pro lente PP, craffities $Aa = v$, quantitas arbitr: $= k$
pro lente QQ, craffities $Bb = v'$, quantitas arbitr: $= k'$
pro lente RR craffities $Cc = v''$, quantitas arbitr: $= k''$
ac ponatur breuitatis gratia:

$$\frac{k-v}{k+v} = i;\ \frac{k'-v'}{k'+v'} = i';\ \frac{k''-v''}{k''+v''} = i''$$

Denique pro quauis lente, fi effet fingularis, eiusque aperturae femidiameter effet $= x$, fit

fpatium diffufionis primae lentis $PP = P\alpha\alpha xx$
- - - - fecundae lentis $QQ = Q\beta\beta xx$
- - - - tertiae lentis $RR = R\gamma\gamma xx$

Iam in praecedente problemate inuenimus fore fpatium diffufionis per duas lentes priores productum

$$Gg = \frac{1}{i'i'} \cdot \frac{ee}{bb} P\alpha\alpha xx + ii. \frac{bb}{\alpha\alpha} \cdot Q\beta\beta xx$$

et radiorum in g concurrentium inclinationem ad axem $= ii'. \frac{bx}{\alpha\beta}$ imaginisque in G magnitudinem $G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z$.

Haec igitur fi ad problema 1. transferantur, ibi loco

Ee, $\frac{\alpha\alpha}{aa}$, $\left(\frac{k+d}{k-d}\right)^2$, $Paa\alpha\alpha$ et Φ fcribi debebit

Gg; $\frac{\gamma\gamma}{cc}$; $\frac{1}{i''i''}$, $Rcc\gamma\gamma$ et $ii'. \frac{bx}{\alpha\beta}$,

hincque fpatium diffufionis per tres lentes productum orietur.

$$Hb = \frac{1}{i'i'.i''i''} \cdot \frac{ee\gamma\gamma}{bb\,cc} P\alpha\alpha xx + \frac{ii}{i''i''} \cdot \frac{bb\gamma\gamma}{\alpha\alpha cc} \cdot Q\beta\beta xx$$
$$+ ii.\ i'i'. \frac{bbcc}{\alpha\alpha\beta\beta} \cdot R\gamma\gamma xx$$

at radiorum in b concurrentium inclinatio ad axem erit $= i i' i''. \frac{bcx}{\alpha \mathfrak{b} \gamma}$. Denique imaginis in H formatae magnitudo erit $H\theta = \frac{1}{i i' i''}. \frac{\alpha \mathfrak{b} \gamma}{abc} z$ ad ſitum inuerſum relata.

Coroll. 1.

80. Quod ad aperturas ſingularum facierum attinet, eas ſequenti modo comparatas eſſe conuenit.

Semid. aperturae Lentis	faciei anterioris	faciei poſterioris
Primae P P	x	$i x$
Secundae Q Q	$i. \frac{bx}{\alpha}$	$i i'. \frac{bx}{\alpha}$
Tertiae R R	$i i'. \frac{bcx}{\alpha \mathfrak{b}}$	$i i' i''. \frac{bcx}{\alpha \mathfrak{b}}$

his ſcilicet valoribus non debent eſſe minores.

Coroll. 2.

81. Si inclinatio radiorum in b concurrentium ad axem vocetur $= \Phi$. Cum ſit $\Phi = i i' i''. \frac{bcx}{\alpha \mathfrak{b} \gamma}$ et $H\theta = \frac{1}{i i' i''}. \frac{\alpha \mathfrak{b} \gamma}{abc} z$ erit $\Phi. H\theta = \frac{xz}{a}$: quae. proprietas pro lentium numero quantumuis magno locum habet.

Coroll. 3.

82. Quemadmodum radii ſingularum facierum determinandi ſint, ex praecedentibus facile liquet:

Erit nempe pro	Radius faciei anterioris	Radius faciei poſterioris
Lente prima PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Lente ſecunda QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)\mathfrak{b}(k'-v')}{k'-v'-2n\mathfrak{b}}$
Lente tertia RR	$\frac{(n-1)c(k''+v'')}{k''+v''+2nc}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$

exiſtente pro vitro $n = \frac{31}{20}$.

Coroll.

Coroll. 4.

83. Tum vero valores literarum P, Q, R ita ſe habebunt

$$P = \frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2 + ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2\right)$$

$$Q = \frac{n}{2(n-1)^2}\left(\frac{1}{i'i'}\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2 + i'i'\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2\right)$$

$$R = \frac{n}{2(n-1)^2}\left(\frac{1}{i''i''}\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{c}+\frac{2}{k''+v''}\right)^2 + i''i''\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{1}{\gamma}-\frac{2}{k''-v''}\right)^2\right)$$

quibus valoribus ſpatia diffuſionis definiuntur.

Si refractio differat:

litteris	tribuatur refr.
P	n
Q	n'
R	n''.

Coroll. 5.

84. Iuuabit etiam ſpatia diffuſionis, prout per vnam, duas ac tres lentes producuntur, inter ſe comparaſſe, quod ita commodiſſime fieri videtur.

$$Ff = \alpha\alpha xx.\ P$$

$$Gg = \beta\beta xx\left(\frac{1}{i'i'}.\frac{\alpha\alpha}{bb}P + ii.\frac{bb}{\alpha\alpha}Q\right)$$

$$Hh = \gamma\gamma xx\left(\frac{1}{i'i'i''i''}.\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{ii}{i''i''}.\frac{bb\beta\beta}{\alpha\alpha cc}Q + ii.i'i'.\frac{bbcc}{\alpha\alpha\beta\beta}R\right)$$

Scholion.

85. Hinc facile perſpicitur, quomodo determinatio ſpatii diffuſionis, ad plures lentes extendi debeat;

vnde problema generale ſtatim ad numerum lentium quemcunque accommodabo, idque pro quacunque lentium craſſitie.

Problema 4.

86. Si radii ab obiecto $E\varepsilon$ emiſſi per lentes quotcunque PP, QQ, RR, SS etc. ſuper communi axe diſpoſitas refringantur, definire ſpatium diffuſionis a data apertura lentis primae oriundum vt et magnitudinem imaginis repraeſentatae.

Solutio.

Poſita obiecti magnitudine $E\varepsilon = z$ imagines eius principales contemplemur: cadat igitur eius imago principalis per lentem primam PP in $F\zeta$ deinde per ſecundam QQ in $G\eta$, tum per tertiam RR in $H\theta$, porro per quartam SS in $I\iota$, per quintam TT in $K\varkappa$ etc. Hinc pro ſingulis lentibus habebimus diſtantias determinatrices quas ita litteris indicemus.

			craſſitiem
Pro lente PP	obiecti $EA = a$,	imaginis $aF = \alpha$;	$Aa = v$
Pro lente QQ	obiecti $FB = b$;	imaginis $bG = \beta$;	$Bb = v'$
Pro lente RR	obiecti $GC = c$;	imaginis $cH = \gamma$;	$Cc = v''$
Pro lente SS	obiecti $HD = d$;	imaginis $dI = \delta$;	$Dd = v'''$
Pro lente TT	obiecti $IE = e$;	imaginis $eK = \varepsilon$;	$Ee = v''''$

etc.

Deinde

Deinde cum determinatio cuiusuis lentis non solum has distantias determinatrices cum crassitie cuiusque sed etiam quantitatem quandam arbitrariam inuoluat, a qua spatium diffusionis cuiusque pendet, ponamus si quaelibet lens esset solitaria eiusque aperturae semidiameter $= x$.

Pro lente	quant: arbitr:	Spatium Diffusionis
Prima PP	k	$P\alpha\alpha xx$
Secunda QQ	k'	$Q\beta\beta xx$
Tertia RR	k''	$R\gamma\gamma xx$
Quarta SS	k'''	$S\delta\delta xx$
Quinta TT	k''''	$T\varepsilon\varepsilon xx$
	etc.	

Hinc ergo constructio cuiusque lentis ita se habebit posito $n = \frac{31}{20}$: vel si refractio differat, cuique lenti suus tribuatur valor primae n, secundae n', tertiae n'' etc.

Erit nempe pro	Radius faciei anterioris	Radius faciei posterioris
Lente prima PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Lente secunda QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Lente tertia RR	$\frac{(n-1)c(k''+v'')}{k''+v''+2nc}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$
Lente quarta SS	$\frac{(n-1)d(k'''+v''')}{k'''+v'''+2nd}$	$\frac{(n-1)\delta(k'''-v''')}{k'''-v'''-2n\delta}$
Lente quinta TT	$\frac{(n-1)e(k''''+v'''')}{k''''+v''''+2ne}$	$\frac{(n-1)\varepsilon(k''''-v'''')}{k''''-v''''-2n\varepsilon}$
	etc.	

at

at si ponamus breuitatis gratia:

$$\frac{k-v}{k+v}=i\,;\ \frac{k'-v'}{k'+v'}=i'\,;\ \frac{k''-v''}{k''+v''}=i''\,;\ \frac{k'''-v'''}{k'''+v'''}=i'''\,;\ \frac{k''''-v''''}{k''''+v''''}=i''''\ \text{etc.}$$

pro spatiis diffusionis habebimus hos valores:

$$P=\frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2+ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2\right)$$

$$Q=\frac{n}{2(n-1)^2}\left(\frac{1}{i'i'}\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2+i'i'\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2\right)$$

$$R=\frac{n}{2(n-1)^2}\left(\frac{1}{i''i''}\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{c}+\frac{2}{k''+v''}\right)^2+i''i''\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{1}{\gamma}-\frac{2}{k''-v''}\right)^2\right)$$

$$S=\frac{n}{2(n-1)^2}\left(\frac{1}{i'''i'''}\left(\frac{n}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{d}+\frac{2}{k'''+v'''}\right)^2+i'''i'''\left(\frac{n}{\delta}-\frac{2}{k'''-v'''}\right)\left(\frac{1}{\delta}-\frac{2}{k'''-v'''}\right)^2\right)$$

$$T=\frac{n}{2(n-1)^2}\left(\frac{1}{i''''i''''}\left(\frac{n}{e}+\frac{2}{k''''+v''''}\right)\left(\frac{1}{e}+\frac{2}{k''''+v''''}\right)^2+i''''i''''\left(\frac{n}{\varepsilon}-\frac{2}{k''''-v''''}\right)\left(\frac{1}{\varepsilon}-\frac{2}{k''''-v''''}\right)^2\right)$$

etc.

His constitutis pro magnitudine singularum imaginum habebimus:

ad situm

pro vna lente imaginem $F\zeta=\frac{1}{i}\cdot\frac{\alpha}{a}z$ inuersum

pro duabus lentibus .... $G\eta=\frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}z$ erectum

pro tribus lentibus .... $H\theta=\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{abc}z$ inuersum

pro quatuor lentibus .... $I\iota=\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma\delta}{abcd}z$ erectum

pro quinque lentibus .... $K\varkappa=\frac{1}{ii'i''i'''i''''}\cdot\frac{\alpha\beta\gamma\delta\varepsilon}{abcde}z$ inuersum

etc.

Deni-

Denique dum aperturae lentium non ſint minores, quam ſequentes formulae exhibent:

Semid. aperturae Lentis	faciei anterioris	faciei poſterioris
primae P P	x	$i\,x$
ſecundae Q Q	$i.\frac{bx}{\alpha}$	$ii'.\frac{bx}{\alpha}$
tertiae RR	$ii'.\frac{bcx}{\alpha\beta}$	$ii'i''.\frac{bcx}{\alpha\beta}$
quartae SS	$ii'i''.\frac{bcdx}{\alpha\beta\gamma}$	$ii'i''i'''.\frac{bcdx}{\alpha\beta\gamma}$
quintae T T	$ii'i''i'''.\frac{bcdex}{\alpha\beta\gamma\delta}$	$ii'i''i'''i''''.\frac{bcdex}{\alpha\beta\gamma\delta}$

etc.

erit vt ſequitur pro quolibet lentium numero:

I. Pro vna lente

Spatium diffuſionis $Ff = \alpha\alpha xx.\,P$

inclinatio radiorum in f concurrentium ad axem $= i.\frac{x}{\alpha}$

II. Pro duabus lentibus

Spatium diffuſionis

$$Gg = \beta\beta xx\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P + ii.\frac{bb}{\alpha\alpha}Q\right)$$

et radiorum in g concurrentium

inclinatio ad axem $= ii'.\frac{bx}{\alpha\beta}$

III. Pro tribus lentibus

Spatium diffuſionis:

$$Hh = \gamma\gamma xx\left(\frac{1}{i'i'.i''i''}\cdot\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{ii}{i''i''}\cdot\frac{bb\beta\beta}{\alpha\alpha c}\cdot Q + ii.i'i'.\frac{bbcc}{\alpha\alpha\beta\beta}R\right)$$

et radiorum in h concurrentium

inclinatio ad axem $= ii'i''.\frac{bcx}{\alpha\beta\gamma}$.

IV. Pro quatuor lentibus.

Spatium diffusionis:

$$Ii = \delta\delta xx \left\{ \begin{array}{l} \frac{i}{i'i'.i''i''.i'''i'''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd} P + \frac{ii'}{i''i''.i'''i'''} \cdot \frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd} Q \\ + \frac{ii.i'i'}{i'''i'''} \cdot \frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd} R + ii.i'i'.i''i'' \cdot \frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma} S \end{array} \right\}$$

et radiorum in i concurrentium

inclinatio ad axem $= ii'i''i''' \cdot \frac{bcdx}{\alpha\beta\gamma\delta}$.

V. Pro quinque lentibus.

Spatium diffusionis:

$$Kk = \varepsilon\varepsilon xx \left\{ \begin{array}{l} + \frac{i}{i'i'.i''i''.i'''i'''.i''''i''''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee} P \\ + \frac{ii'}{i''i''.i'''i'''.i''''i''''} \cdot \frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee} Q \\ + \frac{ii.i'i'}{i'''i'''.i''''i''''} \cdot \frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee} R \\ + \frac{ii.i'i'.i''i''}{i''''i''''} \cdot \frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee} S \\ + ii.i'i'.i''i''.i'''i''' \cdot \frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta} T \end{array} \right\}$$

et radiorum in k concurrentium

inclinatio ad axem $= ii'i''i'''i'''' \cdot \frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$.

Vnde progressio harum formularum ad plures adhuc lentes satis est manifesta. Si in lentibus ratio refractionis sit diuersa atque ad singulas lentes ordine his litteris indicetur n, n', n'', n''', etc.; haec diuersitas facile ad formulas hic inuentas accommodabitur. Primum enim haec correctio occurrit in formulis pro radiis lentium,

lentium, ita, vt quemadmodum formulae f et g pro prima lente numerum n inuolunt, ita pro secunda lente numerus n', pro tertia n'' et ita porro introducatur. Similem correctionem etiam requirunt valores litterarum P, Q, R, S etc. et loco litterae n, quae in valore P occurrit, in valoribus Q, R, S etc. scribi oportet n', n'', n''' etc.

Coroll. 1.

87. Si obiectum sit tantum punctum in axe positum, sufficit vt lentium aperturae sint tantae, quantas assignauimus sin autem obiectum habeat quandam magnitudinem, tum aperturae praeter primam eo magis mensuras assignatas superare debent, quo maior fuerit obiecti magnitudo z.

Coroll. 2.

88. In expressione spatii diffusionis quadratum semidiametri aperturae primae faciei xx primo multiplicatur per quadratum distantiae postremae imaginis ab vltima lentium: quae ergo si fuerit infinita, etiam spatium diffusionis fit infinitum.

Coroll. 3.

89. Ceteris ergo paribus, quotcunque fuerint lentes, spatium diffusionis semper est proportionale quadrato diametri aperturae primae faciei, hoc est ipsi huic aperturae. Vnde diametro aperturae primae

faciei ad femiffem redacto fpatium diffufionis quadruplo fiet minus.

Scholion.

90. Confiderauimus hic ftatim loca fingularum imaginum principalium tanquam data ex iisque ftructuram cuiusque lentis quantitatem arbitrariam introducendo determinauimus. Quod fi vero ipfae lentes fuerint datae ita vt tam radii ambarum facierum cuiusque quam craffities, vna cum earum interuallis cognofcantur tum ope formularum exhibitarum, viciffim diftantiae determinatrices innotefcunt. Sint fcilicet radii facierum anterioris et pofterioris primae lentis PP, f, g, fecundae lentis QQ, f', g'; tertiae lentis RR, f'', g'' etc. craffitie earum exiftente v, v' v'' etc. tum verodentur diftantiae $a\mathrm{B}=\mathrm{F}$; $b\mathrm{C}=\mathrm{G}$; $c\mathrm{D}=\mathrm{H}$ etc. Praeterea autem diftantia obiecti ante lentem primam fit $\mathrm{AE}=a$, ac fequenti modo omnia elementa ad problema fuperius neceffaria elicientur;

$$\begin{cases} 1.\ \frac{n-1}{f}=\frac{1}{a}+\frac{2n}{k+v}, & \text{hinc reperitur } k \\ 2.\ \frac{n-1}{g}=\frac{1}{\alpha}-\frac{2n}{k-v}, & \text{hinc vero } \alpha \\ 3.\ \mathrm{F}=\alpha+b & \text{vnde } b=\mathrm{F}-\alpha \end{cases}$$

$$\begin{cases} 4.\ \frac{n-1}{f'}=\frac{1}{b}+\frac{2n}{k'+v'}, & \text{hinc reperitur } k' \\ 5.\ \frac{n-1}{g'}=\frac{1}{\mathfrak{C}}-\frac{2n}{k'-v'}, & \text{hinc vero porro } \mathfrak{C} \\ 6.\ \mathrm{G}=\mathfrak{C}+c & \text{vnde } c=\mathrm{G}-\mathfrak{C} \end{cases}$$

(7.

$$\begin{cases} 7.\ \frac{n-1}{f''} = \frac{1}{c} + \frac{2n}{k''+\nu''}, & \text{hinc reperitur } k'' \\ 8.\ \frac{n-1}{g''} = \frac{1}{\gamma} - \frac{2n}{k''-\nu''}, & \text{hinc vero } \gamma \\ 9.\ H = \gamma + d & \text{vnde } d = H - \gamma \end{cases}$$

etc.

Vtcunque ergo lentes datae fuerint dispositae super axe communi, si ante eas constituatur obiectum in data distantia $AE = a$, inde singulae distantiae determinatrices $\alpha, b, \mathfrak{E}, c, \gamma$ etc. cum arbitrariis k, k', k'', k''' etc. facile definiuntur ex iisque porro spatium diffusionis cum reliquis phaenomenis in solutione problematis commemoratis assignabitur. Operae pretium autem erit casum, quo crassities lentium vt euanescens spectatur, accuratius euoluisse.

Problema 5.

91. Si crassities lentium euanescat, et quotcunque huiusmodi lentes super communi axe fuerint dispositae ante quas existat obiectum $E\varepsilon$, definire spatium diffusionis, per quod imago erit dissipata, vt et magnitudinem imaginis.

Solutio.

Si obiecti magnitudo $E\varepsilon = z$, cuius imagines principales successiue cadant in $F\zeta$, $G\eta$, $H\theta$, $I\iota$, $K\varkappa$ etc. hincque pro singulis lentibus sequentes habebimus distantias determinatrices, cum imago per quamuis lentem repraesentata respectu lentis sequentis vicem obiecti gerat.

		distantia	distantia
Pro lente PP	obiecti	$EA = a$;	imaginis $aF = \alpha$
Pro lente QQ	obiecti	$FB = b$;	imaginis $bG = \beta$
Pro lente RR	obiecti	$GC = c$;	imaginis $cH = \gamma$
Pro lente SS	obiecti	$HD = d$;	imaginis $dI = \delta$
Pro lente TT	obiecti	$IE = e$;	imaginis $eK = \varepsilon$

etc.

Porro autem sint numeri arbitrarii vnitate maiores cuiusque lentis figuram determinantes, λ pro lente PP, λ' pro QQ, λ'' pro RR, λ''' pro SS, λ'''' pro TT, etc. ita vt ponendo breuitatis gratia

$$\rho = 0,190781,\ \sigma = 1,627401,\ \tau = 0,905133$$

Pro lente PP radius faciei $\begin{cases} \text{anter.} = \dfrac{a\alpha}{\rho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda - 1)}} \\ \text{poster.} = \dfrac{a\alpha}{\rho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda - 1)}} \end{cases}$

Pro lente QQ radius faciei $\begin{cases} \text{anter.} = \dfrac{b\beta}{\rho\beta + \sigma b \pm \tau(b+\beta)\sqrt{(\lambda' - 1)}} \\ \text{poster.} = \dfrac{b\beta}{\rho b + \sigma\beta \mp \tau(b+\beta)\sqrt{(\lambda' - 1)}} \end{cases}$

Pro lente RR radius faciei $\begin{cases} \text{anter.} = \dfrac{c\gamma}{\rho\gamma + \sigma c \pm \tau(c+\gamma)\sqrt{(\lambda'' - 1)}} \\ \text{poster.} = \dfrac{c\gamma}{\rho c + \sigma\gamma \mp \tau(c+\gamma)\sqrt{(\lambda'' - 1)}} \end{cases}$

Pro lente SS radius faciei $\begin{cases} \text{anter.} = \dfrac{d\delta}{\rho\delta + \sigma d \pm \tau(d+\delta)\sqrt{(\lambda''' - 1)}} \\ \text{poster.} = \dfrac{d\delta}{\rho d + \sigma\delta \mp \tau(d+\delta)\sqrt{(\lambda''' - 1)}} \end{cases}$

Pro lente TT radius faciei $\begin{cases} \text{anter.} = \dfrac{e\varepsilon}{\rho\varepsilon + \sigma e \pm \tau(e+\varepsilon)\sqrt{(\lambda'''' - 1)}} \\ \text{poster.} = \dfrac{e\varepsilon}{\rho e + \sigma\varepsilon \mp \tau(e+\varepsilon)\sqrt{(\lambda'''' - 1)}} \end{cases}$

etc.

Deinde

Deinde si quaelibet lens cum binis suis distantiis determinatricibus seorsim consideretur, eiusque aperturae semidiameter foret $= x$, posito $\mu = 0,938191$ et $\nu = 0,232692$ esset spatium diffusionis

$$\text{Lentis } PP = \mu\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

$$\text{Lentis } QQ = \mu\beta\beta xx\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\left(\lambda'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu}{b\beta}\right)$$

$$\text{Lentis } RR = \mu\gamma\gamma xx\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu}{c\gamma}\right)$$

$$\text{Lentis } SS = \mu\delta\delta xx\left(\tfrac{1}{d}+\tfrac{1}{\delta}\right)\left(\lambda'''\left(\tfrac{1}{d}+\tfrac{1}{\delta}\right)^2+\tfrac{\nu}{d\delta}\right)$$

$$\text{Lentis } TT = \mu\varepsilon\varepsilon xx\left(\tfrac{1}{e}+\tfrac{1}{\varepsilon}\right)\left(\lambda''''\left(\tfrac{1}{e}+\tfrac{1}{\varepsilon}\right)^2+\tfrac{\nu}{e\varepsilon}\right)$$

etc.

His constitutis pro magnitudine singularum imaginum habebimus

Pro vna lente: $F\zeta = \frac{\alpha}{a}z$ situ inuerso

Pro duabus lentibus $G\eta = \frac{\alpha\beta}{ab}z$ situ erecto

Pro tribus lentibus $H\theta = \frac{\alpha\beta\gamma}{abc}z$ situ inuerso

Pro quatuor lentibus $I\iota = \frac{\alpha\beta\gamma\delta}{abcd}z$ situ erecto

Pro quinque lentibus $K\varkappa = \frac{\alpha\beta\gamma\delta\varepsilon}{abcde}z$ situ inuerso

etc.

At si semidiameter aperturae primae lentis PP ponatur $= x$, necesse est, vt reliquarum lentium aperturae superent sequentes valores

Semi-

Semidiameter aperturae
Lentis fecundae $QQ > \frac{b}{\alpha} x$
Lentis tertiae $RR > \frac{bc}{\alpha\beta} x$
Lentis quartae $SS > \frac{bcd}{\alpha\beta\gamma} x$
Lentis quintae $TT > \frac{bcde}{\alpha\beta\gamma\delta} x$
etc.

Hinc fpatium diffufionis pro quolibet lentium numero ita fe habebit.

I. Pro vna lente

fpatium diffufionis

$$Ff = \mu\, aaxx \left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

radiorum in f concurrentium
inclinatio ad axem $= \frac{x}{\alpha}$

II. Pro duabus lentibus

fpatium diffufionis

$$Gg = \mu\,\beta\beta xx \begin{cases} +\frac{\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\frac{bb}{\alpha\alpha}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \end{cases}$$

et radiorum in g concurrentium
inclinatio ad axem $= \frac{bx}{\alpha\beta}$

III. Pro tribus lentibus

fpatium diffufionis

$$Hh = \mu\,\gamma\gamma xx \begin{cases} +\frac{\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{\alpha a}\right) \\ +\frac{bb\beta\beta}{\alpha\alpha cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \\ +\frac{bbcc}{\alpha\alpha\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \end{cases}$$

et

et radiorum in b concurrentium
inclinatio ad axem $\frac{bcx}{\alpha\beta\gamma}$

IV. Pro quatuor lentibus

ſpatium diffuſionis

$$Ii = \mu\delta\delta xx \left\{ \begin{array}{l} +\frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \\ +\frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \\ +\frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right) \end{array} \right.$$

et radiorum in i concurrentium
inclinatio ad axem $= \frac{bcdx}{\alpha\beta\gamma\delta}$

V. Pro quinque lentibus

ſpatium diffuſionis

$$Kk = \mu\varepsilon\varepsilon xx \left\{ \begin{array}{l} +\frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \\ +\frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \\ +\frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right) \\ +\frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)\left(\lambda''''\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)^2+\frac{\nu}{e\varepsilon}\right) \end{array} \right.$$

et radiorum in k concurrentium
inclinatio ad axem $= \frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$
neque ergo caſus, quibus plures occurrunt lentes, vlla amplius laborant difficultate.

Si lentes ratione refractionis diſcrepent, ad easque referendae ſint litterae n, n', n'', n''', etc. formulae in hoc problemate inuentae sequenti modo facile ad hunc casum latius patentem adcommodabuntur. Primo scilicet in formulis pro radiis facierum inuentis litterae ϱ, σ, et τ tantum ad primam lentem pertinent, earumque loco pro secunda lente scribi oportet ϱ', σ', et τ'; pro tertia autem ϱ'', σ'' et τ'' et ita porro. Praeterea vero spatia diffusionis hinc aliquam mutationem requirunt, eritque spatium diffusionis.

I. Pro vna lente

$$\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\mu\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right).$$

II. Pro duabus lentibus

$$\beta\beta xx\left\{\begin{array}{l}\frac{\mu\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb}{\alpha\alpha}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right).\end{array}\right.$$

III. Pro tribus lentibus

$$\gamma\gamma xx\left\{\begin{array}{l}+\frac{\mu\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb\beta\beta}{\alpha\alpha cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right)\\ +\frac{\mu'' bbcc}{\alpha\alpha\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu''}{c\gamma}\right).\end{array}\right.$$

IV. Pro quatuor lentibus

$$\delta\delta xx\left\{\begin{array}{l}+\frac{\mu\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right)\\ +\frac{\mu'' bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu''}{c\gamma}\right)\\ +\frac{\mu''' bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu'''}{d\delta}\right).\end{array}\right.$$

valores autem harum litterarum μ', ν', μ'', ν'' etc. iam supra definiuimus §. 77.

Coroll.

Coroll. 1.

92. Si lentis primae PP ponatur radius faciei anterioris $=f$ et posterioris $=g$ erit

$$\frac{1}{f}=\frac{\varrho}{a}+\frac{\sigma}{\alpha}\pm\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}$$

$$\frac{1}{g}=\frac{\varrho}{\alpha}+\frac{\sigma}{a}\mp\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}$$

vnde si detur distantia obiecti $EA=a$, primo inuenitur α ex hac aequatione

$$\frac{1}{f}+\frac{1}{g}=(\varrho+\sigma)\left(\frac{1}{a}+\frac{1}{\alpha}\right)=1,818182\left(\frac{1}{a}+\frac{1}{\alpha}\right)=\frac{20}{11}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

Inuenta autem distantia α numerus λ reperitur ex hac aequatione.

$$\frac{1}{f}-\frac{1}{g}+(\sigma-\varrho)\left(\frac{1}{a}-\frac{1}{\alpha}\right)=2\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}.$$

Coroll. 2.

93. Deinde si distantia secundae lentis a prima sit $=F$, ob $F=\alpha+b$ habetur $b=F-\alpha$; qua distantia b cognita, si pro lente secunda datus sit radius faciei anterioris $=f'$, et posterioris $=g'$ habebuntur iterum duae aequationes

$$\frac{1}{f'}=\frac{\varrho}{b}+\frac{\sigma}{\beta}\pm\tau\left(\frac{1}{b}+\frac{1}{\beta}\right)\sqrt{(\lambda'-1)}$$

$$\frac{1}{g'}=\frac{\varrho}{\beta}+\frac{\sigma}{b}\mp\tau\left(\frac{1}{b}+\frac{1}{\beta}\right)\sqrt{(\lambda'-1)}$$

ex quibus cum distantiam β tum numerum λ' definire licet. Similique modo ex forma sequentium lentium earumque distantia reliqua elementa innotescent.

Coroll. 3.

94. Si singulae lentes ad minimum spatium diffusionis fuerint accommodatae, erit $\lambda=1, \lambda'=1, \lambda''=1, \lambda'''=1$ etc. sin autem hae lentes alia forma fuerint praeditae, isti numeri erunt vnitate maiores.

Scholion.

95. Quo plures fuerint lentes eo pluribus constabit membris spatium diffusionis ab iis productum; neque tamen propterea aucto lentium numero spatium diffusionis necessario augetur. Cum enim quantitates α, b, β, c, γ, d, δ, etc. valores quoque negatiuos recipere queant, dummodo binarum summae $\alpha+b$; $\beta+c$; $\gamma+d$; $\delta+e$ etc. vtpote lentium distantiae maneant positiuae, fieri potest, vt vnum vel aliquot membra fiant negatiua, hincque spatium diffusionis diminuatur, quin etiam interdum prorsus euanescat, quo casu repraesentatio sine dubio erit perfectissima. Verum in instrumentis dioptricis ad visionem instructis veluti Telescopiis ac Microscopiis non tam hoc, quod definiuimus, spatium diffusionis quam confusio in ipsa visione orta spectari debet; quae autem etsi a spatio diffusionis plurimum differt, tamen ex eo definiri potest vti mox explicabimus. Ante autem conueniet lentes compositas seu multiplicatas considerare, cuiusmodi oriuntur, si duae pluresue lentes, quarum crassities tam est parua vt negligi queat, immediate iungantur quo quidem pacto instar lentium simplicium spectari possunt; verum tali coniunctione effici potest, vt spatium diffusionis multo fiat minus quam si lens simplex adhiberetur, atque adeo euanescat valorque numeri λ istiusmodi lenti compositae conueniens vnitate minor sit prodirurus vnde maxima commoda ad confusionem diminuendam obtinebuntur.

CAPVT III.

CAPVT III.

DE LENTIBVS COMPOSITIS SEV MVLTIPLICATIS.

Definitio I.

96.

Lens duplicata oritur, si duae lentes super communi axe sibi immediate iungantur.

Crassitiem hic vtriusque lentis tanquam nullam assumo et quia distantia inter lentes nulla ponitur, crassities etiam lentis duplicatae pro nulla haberi poterit.

Coroll. I.

97. Binae ergo lentes PP et QQ sibi ad contactum fere coniunctae lentem duplicatam constituunt; de qua tamen notandum est, eius crassitiem minus tuto negligi posse quam vtriusque lentis simplicis seorsim sumtae. Si enim lentes immediate se in puncto contingerent; phaenomena colorum a *Newtono* obseruata essent metuenda tum vero etiam ostendemus, quomodo ratio distantiae inter binas lentes haberi possit. Tab. II. Fig. 7.

Coroll. 2.

98. Si lentis anterioris PP diſtantiae determinatrices ſint a et α, lentis poſterioris vero QQ, b et $\mathfrak{b}$, neceſſe eſt vt ſit $\alpha + b = 0$ ſeu $\alpha = -b$. Tum vero obiecti ante lentem ad diſtantiam $AE = a$ poſiti imago principalis repraeſentabitur poſt lentem ad diſtantiam $G = \mathfrak{b}$.

Coroll. 3.

99. Erunt ergo a et $\mathfrak{b}$ quaſi diſtantiae determinatrices lentis duplicatae; ac ſumendo α vel b ad libitum infinita paria lentium pro his diſtantiis exhiberi poſſunt. Cum deinde vtraque lens praeterea numerum indefinitum λ recipiat inſuper infinita varietas locum habet.

Coroll. 4.

100. Quia craſſities pro nihilo reputatur aperturae in ſingulis faciebus eadem eſt ratio; ſcilicet ſi in prima facie ſemidiameter aperturae ſit $= x$, in reliquis quoque faciebus apertura eadem vel ſaltem non minor eſſe debet.

Scholion.

101. Non opus eſt, vt lentes plane ad contactum coniungantur quoniam forte refractionis lex turbari poſſet: hoc autem vel minima interpoſita diſtantia euitabitur id quod ad inſtitutum noſtrum ſufficit, cum etiam craſſities vtriusque non omnino ſit nulla.

Proble-

Problema 1.

102. Omnes lentes duplicatas describere, quibus obiectum $E\varepsilon$ in data ante lentem distantia AE propositum post lentem in data distantia bG repraesentetur, simulque diffusionem imaginis Gg pro data lentis apertura definire.

Solutio.

Sit distantia obiecti $AE = a$, imaginis principalis $bG = \beta$, tum vero lentis primae PP distantiae determinatrices a, et α; lentis posterioris vero QQ, b et β; iam quia distantia lentium est nulla, erit $\alpha + b = 0$, seu $\alpha = -b$; Hinc si obiecti magnitudo sit $E\varepsilon = z$, erit imaginis principalis magnitudo $G\eta = \frac{\beta}{a} z$ pro situ inuerso. Cum porro vtraque lens infinitis modis formari possit, sit λ numerus arbitrarius pro prima PP et λ' pro secunda QQ, quarum ergo constructio ita se habebit:

Pro Lente

$$PP \text{ radius faciei} \begin{cases} \text{anterioris} = \dfrac{a\alpha}{\rho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}} \\ \text{posterioris} = \dfrac{a\alpha}{\rho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}} \end{cases}$$

$$QQ \text{ radius faciei} \begin{cases} \text{anterioris} = \dfrac{b\beta}{\rho\beta + \sigma b \pm \tau(b+\beta)\sqrt{(\lambda'-1)}} \\ \text{posterioris} = \dfrac{b\beta}{\rho b + \sigma\beta \mp \tau(b+\beta)\sqrt{(\lambda'-1)}} \end{cases}$$

Si ratio refractionis sit diuersa; pro secunda lente scribi debet ρ', σ' et τ', loco ρ, σ et τ.

Pro

Pro ſpatio autem diffuſionis Gg inueniendo, ſit ſemidiameter aperturae lentis duplicatae $= x$, eritque

$$Gg = \mu \mathfrak{b}\mathfrak{b}xx. \begin{cases} +\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)^2+\frac{\nu}{b\mathfrak{b}}\right) \end{cases}$$

et radiorum in g concurrentium inclinatio ad axem $= \frac{x}{\mathfrak{b}}$. Si ratio refractionis diſcrepet; in parte ex lente ſecunda orta ſcribatur μ', ν', loco μ, ν.

Coroll. 1.

103. Huius ergo lentis duplicatae diſtantiae determinatrices ſunt a et $\mathfrak{b}$, praeterea vero duo numeri arbitrarii λ et λ', vna cum diſtantia α vel b eius perfectam determinationem conſtituunt, vnde in huiusmodi lentibus multo maior varietas locum habet, quam in lentibus ſimplicibus.

Coroll. 2.

104. Si ex eisdem diſtantiis determinatricibus a et $\mathfrak{b}$ adiungendo numero arbitrario λ° lens ſimplex conſtruatur, ea imaginem eadem magnitudine $G\eta = \frac{\mathfrak{b}}{a}z$ referet; ſed pro eadem apertura, cuius ſemidiameter $= x$ habebitur ſpatium diffuſionis

$$Gg = \mu \mathfrak{b}\mathfrak{b}xx\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)\left(\lambda^\circ\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^2+\frac{\nu}{a\mathfrak{b}}\right)$$

Coroll. 3.

105. Fieri ergo poterit vt lens duplicata modo maiorem modo minorem diffuſionem gignat. Lens autem

autem ſimplex minimam parit diffuſionem, ſi $\lambda^{\circ} = 1$; ergo tum demum lentes duplicatae ſimplicibus erunt praeferendae, cum adhuc minorem diffuſionem producent.

Scholion.

106. Concipi quidem ſemper poterit lens ſimplex eandem diffuſionem gignens ac lens duplicata, ſi pro numero λ° omnes valores admittamus; quomodocunque enim lens duplicata fuerit comparata, ſi ſpatium diffuſionis inde productum huic ex lente ſimplici nato aequale ſtatuatur, determinatus valor pro numero λ° elicitur: qui ſi fuerit poſitiuus et vnitate maior, realis lens ſimplex aequiualens exhiberi poterit, ſin autem prodeat vnitate minor, vel adeo negatiuus, lens ſimplex inter imaginaria erit referenda. Quando autem ſit $\lambda^{\circ} > 1$, euidens eſt lentem ſimplicem eundem plane effectum eſſe edituram ac duplicatam ideoque ſemper expediet lente ſimplici potius vti quam duplicata; quodſi vero prodierit $\lambda^{\circ} < 1$, quo caſu lens ſimplex ſit imaginaria, tum lentes duplicatae effectum praeſtabunt a ſimplicibus non expectandum, qui adeo cum inſigni hoc commodo, quod ſpatium diffuſionis futurum ſit minus, erit coniunctus. Talibus ergo lentibus duplicatis maximo cum ſucceſſu vti poterimus, eoque magis eae ſimplicibus erunt anteponendae, quo minor fuerit valor numeri λ° iis reſpondens.

Problema 2.

107. Data lente duplicata ad binas distantias determinatrices $AE = \mathfrak{a}$ et $bG = \mathfrak{b}$ relata pro iisdem distantiis definire lentem simplicem, quae pro eadem apertura, eandem diffusionem imaginis producat.

Solutio.

Totum ergo negotium huc redit, vt spatium diffusionis Gg a lente simplici productum (104) aequale ponatur spatio diffusionis a lente duplicata orto, cuius expressio in problemate praecedente (101) est inuenta, indeque valor numeri λ° pro constructione lentis simplicis eliciatur. Quae inuestigatio quo commodius institui possit, ponamus:

$$\frac{1}{a} = \frac{f-1}{\alpha} + \frac{f}{\mathfrak{b}} \text{ eritque } \frac{1}{b} = \frac{-f+1}{\alpha} - \frac{f}{\mathfrak{b}}$$

ita vt loco quantitatis α vel b numerum f introduamus eritque

$$\frac{1}{a} + \frac{1}{\alpha} = f\left(\frac{1}{\alpha} + \frac{1}{\mathfrak{b}}\right) \text{ et } \frac{1}{b} + \frac{1}{b} = (1-f)\left(\frac{1}{\alpha} + \frac{1}{\mathfrak{b}}\right)$$

vnde spatium diffusionis a lente duplicata ortum prodit:

$$Gg = \mu \mathfrak{b}\mathfrak{b}xx \begin{cases} +f\left(\frac{1}{\alpha}+\frac{1}{\mathfrak{b}}\right)\left(\lambda ff\left(\frac{1}{\alpha}+\frac{1}{\mathfrak{b}}\right)^2 + \nu\left(\frac{f-1}{\alpha\alpha}+\frac{f}{\alpha\mathfrak{b}}\right)\right) \\ +(1-f)\left(\frac{1}{\alpha}+\frac{1}{\mathfrak{b}}\right)\left(\lambda'(1-f)^2\left(\frac{1}{\alpha}+\frac{1}{\mathfrak{b}}\right)^2 + \nu\left(\frac{1-f}{\alpha\mathfrak{b}}-\frac{f}{\mathfrak{b}\mathfrak{b}}\right)\right) \end{cases}$$

quae expressio reducitur ad hanc formam:

$$Gg = \mu\mathfrak{b}\mathfrak{b}xx\left(\frac{1}{\alpha}+\frac{1}{\mathfrak{b}}\right)\left((\lambda f^3 + \lambda'(1-f)^3)\left(\frac{1}{\alpha}+\frac{1}{\mathfrak{b}}\right)^2 + \nu\left(\frac{f(f-1)}{\alpha\alpha} + \frac{1-2f+2ff}{\alpha\mathfrak{b}} + \frac{f(f-1)}{\mathfrak{b}\mathfrak{b}}\right)\right)$$

Verum postremum membrum $\frac{f(f-1)}{\alpha\alpha} + \frac{1-2f+2ff}{\alpha\mathfrak{b}} + \frac{f(f-1)}{\mathfrak{b}\mathfrak{b}}$ mutatur

mutatur in $f(f-1)(\frac{1}{a}+\frac{1}{b})^2+\frac{1}{ab}$, ficque habebimus pro lente duplicata:

$$Gg = \mu bbxx(\tfrac{1}{a}+\tfrac{1}{b})((\lambda f^3+\lambda'(1-f)^3-\nu f(1-f))(\tfrac{1}{a}+\tfrac{1}{b})^2+\tfrac{\nu}{ab})$$

quae forma iam facillime cum fpatio diffufionis lentis fimplicis comparatur indeque manifefto colligitur:

$$\lambda^{\circ} = \lambda f^3+\lambda'(1-f)^3-\nu f(1-f)$$

Cum autem loco quantitatum a et b numerum f introduxerimus conftructio lentis duplicatae ita fe habebit

Pro lente radius faciei

$$\text{prima PP}\begin{cases}\text{anterioris} = \dfrac{ab}{(\rho-\sigma(1-f))b+\sigma fa\pm\tau f(a+b)\sqrt{(\lambda-1)}}\\[2ex]\text{pofterioris} = \dfrac{ab}{(\sigma-\rho(1-f))b+\rho fa\mp\tau f(a+b)\sqrt{(\lambda-1)}}\end{cases}$$

$$\text{fecunda QQ}\begin{cases}\text{anterioris} = \dfrac{ab}{(\sigma-\rho f)a+\rho(1-f)b\pm\tau(1-f)(a+b)\sqrt{(\lambda'-1)}}\\[2ex]\text{pofterioris} = \dfrac{ab}{(\rho-\sigma f)a+\sigma(1-f)b\mp\tau(1-f)(a+b)\sqrt{(\lambda'-1)}}\end{cases}$$

Tum vero inuento numero λ° conftructio lentis fimplicis aequiualentis erit

$$\text{Radius faciei}\begin{cases}\text{anterioris} = \dfrac{ab}{\rho b+\sigma a\pm\tau(a+b)\sqrt{(\lambda^{\circ}-1)}}\\[2ex]\text{pofterioris} = \dfrac{ab}{\rho a+\sigma b\mp\tau(a+b)\sqrt{(\lambda^{\circ}-1)}}\end{cases}$$

Coroll. I.

108. Quoties ergo fuerit

$$\lambda f^3+\lambda'(1-f)^3-\nu f(1-f) > 1$$

femper lens fimplex parari poteft duplicatae aequiualens

lens

lens iisque ergo casibus praestabit lente simplici vel potius quam lente duplicata.

Coroll. 2.

109. Verum si fuerit

$$\lambda f^3 + \lambda'(1-f)^3 - \nu f(1-f) < 1$$

ob $\lambda^\circ < 1$, constructio lentis simplicis fit impossibilis, ac lens duplicata minorem pariet diffusionem quam per vllam lentem simplicem obtineri potest.

Coroll. 3

110. Si esset $f = 0$, prodiret $\lambda^\circ = \lambda'$, et in lente duplicata anteriori nullam refractionem produceret ob facies parallelas, resque eodem rediret, ac si posterior sola adesset. Sin autem sumatur $f = 1$, fit $\lambda^\circ = \lambda$, et lens posterior superflua; vtroque ergo casu nullum lucrum impetratur.

Coroll. 4.

111. Cum autem numeri λ et λ' vnitate nequeant esse minores, sitque $\nu = 0,232692$, patet pro f nullum valorem inter limites 0 et 1 assumi posse, vnde fiat $\lambda^\circ = 0$. At si f extra hos limites capiatur, vtique pro λ et λ' eiusmodi numeri vnitate maiores assignari poterunt, vt fiat $\lambda^\circ = 0$.

Scholion 1.

112. Ratione ergo numeri f tres casus lentium duplicatarum considerari conueniet, prout vel f intra limi-

limites o et 1 continetur, vel fuerit $f > 1$ vel $f < 0$. Primo casu euenire non poteſt, vt fiat $\lambda^{\circ} = 0$; ſed plurimum intererit eam determinaſſe lentem duplicatam, pro qua λ° minimum obtineat valorem qui quo magis infra vnitatem cadat, eo perfectior lens erit cenſenda, maioreque iure ſimplicibus anteferenda. Binis reliquis vero caſibus $f > 1$ et $f < 0$ eiusmodi adeo lentes duplicatae parari poterunt, quae praebeant $\lambda^{\circ} = 0$, quae ergo pro perfectiſſimis eſſent habendae. Verum hic quoque ad praxin eſt reſpiciendum, quae cum ſemper a praeceptis Theoriae aberrare ſoleat, euenire poteſt, vt leui errore commiſſo numerus λ° non ſolum non euaneſcat, ſed adeo vnitatem excedat quo caſu vtique expediret lente vti ſimplici.

Scholion 2.

113. Cum tanti ſit momenti rationem aberrationis a qua praxis vix liberari poteſt habere, eam etiam in lentibus ſimplicibus perpendi conueniet. Poſtulauimus autem pro datis diſtantiis determinatricibus lentem conſtrui poſſe in qua numerus λ datum obtineat valorem dum ne ſit vnitate minor; hic igitur obſeruari oportet, quo maior fuerit λ, eo difficilius fore errorem euitare; ſi enim in conſtructione leuiſſimus error committatur alius eo magis diuerſus valor pro λ orietur, quo maior fuerit λ. Verum e contrario cum vnitas ſit minimus valor, quem λ recipere poteſt ex natura minimi liquet etiamſi in praxi a praeſcripta regula notabiliter recedatur tamen inde vix

 ſenſi-

ſenſibile diſcrimen in valorem λ eſſe redundaturum. Ex quo concludimus feliciſſimo cum ſucceſſu eiusmodi lentes ſimplices parari poſſe, pro quibus futurum ſit $\lambda=1$, neque hic errores praxeos, niſi fuerint enormes, admodum eſſe pertimeſcendos. Deinde quo minus numerus λ vnitatem ſuperare debeat, eo certiores eſſe poterimus de ſucceſſu, ſed non eo gradu, quo caſu, $\lambda=1$; at ſi opus ſit eiusmodi lente, pro qua valor ipſius λ debeat eſſe numerus ſatis magnus, difficillime per praxin ſatisfiet ac fortaſſe ingentem lentium numerum parare oportebit, antequam vna obtineatur ſcopo ſatisfaciens. Quamobrem ſi praxi conſulere velimus, vix alias lentes exigere debemus, niſi pro quibus numerus λ vel ſit vnitas ipſa, vel parumper maior. Sin autem ad inſigne aliquod commodum aliae lentes requirantur, labori non erit parcendum, cum fortaſſe non niſi poſt plurimos conatus irritos voti tandem compotes reddi queamus.

Problema 3.

114. Definire eam lentem duplicatam, pro qua, ſi numerus f intra limites 0 et 1 accipiatur numerus λ° minimum adipiscatur valorem.

Solutio.

Poſitis iisdem, quae in praecedentibus problematibus ſunt conſtituta, inuenimus eſſe

$$\lambda^\circ=\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)$$

vbi cum f intra limites 0 et 1 aſſumi debeat, ambo termini

termini λf^3 et $\lambda'(1-f)^3$ erunt poſitiui. Quare vt λ° omnium minimum valorem nanciſcatur, neceſſe eſt vtrique numero λ et λ' minimum valorem cuius eſt capax tribui.

Sit ergo $\lambda = 1$ et $\lambda' = 1$, ac habebimus

$$\lambda^\circ = 1 - 3f + 3ff - \nu f + \nu ff = 1 - (3+\nu)f(1-f)$$

quae expreſſio vt minima reddatur, oportet fieri $f(1-f)$ maximum, id quod fit ſumendo $f = \frac{1}{2}$; hincque oritur

$$\lambda^\circ = 1 - \tfrac{1}{4}(3+\nu) = \tfrac{1-\nu}{4} = 0,191827$$

Quare conſtructio huius lentis duplicatae ita ſe habebit:

Pro lente P P radius faciei $\begin{cases} \text{anterioris} = \frac{2\alpha\beta}{(2\rho-\sigma)\beta+\sigma\alpha} \\ \text{poſterioris} = \frac{2\alpha\beta}{(2\sigma-\rho)\beta+\rho\alpha} \end{cases}$

Pro lente QQ radius faciei $\begin{cases} \text{anterioris} = \frac{2\alpha\beta}{(2\sigma-\rho)\alpha+\rho\beta} \\ \text{poſterioris} = \frac{2\alpha\beta}{(2\rho-\sigma)\alpha+\sigma\beta} \end{cases}$

et ſi aperturae ſemidiameter ſit $= x$, erit ſpatium diffuſionis

$$Gg = \mu\beta\beta xx\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)\left(0,191827\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)^2 + \tfrac{\nu}{\alpha\beta}\right)$$

Si eiusmodi vitro vtamur, pro quo eſt $n = 1,60 = \frac{8}{5}$ tum ob $\nu = \frac{4}{11}$, prodiret $\lambda^\circ = 0,183333$ ideoque haec vitri ſpecies adhuc minorem confuſionem pareret.

Coroll.

Coroll. 1.

115. Si pro iisdem diſtantiis determinatricibus a et $\mathfrak{b}$ lens ſimplex minimam diffuſionem pariens conſtruatur quod fit ſumendo

$$\text{radium faciei} \begin{cases} \text{anterioris} = \dfrac{a\mathfrak{b}}{\varrho\mathfrak{b}+\sigma a} \\ \text{poſterioris} = \dfrac{a\mathfrak{b}}{\varrho a+\sigma\mathfrak{b}} \end{cases}$$

ſpatium diffuſionis foret

$$\mu\mathfrak{b}\mathfrak{b}xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\left(\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2+\tfrac{\nu}{a\mathfrak{b}}\right).$$

Coroll. 2.

116. Apparet ergo a lente duplicata deſcripta multo minorem oriri diffuſionem, quam a lente ſimplici, etiamſi haec iam ad minimam diffuſionem ſit inſtructa: Cum enim ceterae partes ſint pares coefficiens membri $\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^3$ plus quam quintuplo minor eſt in duplicata quam in ſimplici.

Coroll. 3.

117. Si ponamus $\lambda = 1$ et $\lambda' = 1$, vel ſaltem $\lambda' = \lambda$, minor valor pro λ° obtineri nequit, quam inuenimus, etiamſi pro f alios valores admittere velimus. Vnde ſi vtraque lens per ſe iam minimam diffuſionem pariat, pro lente duplicata valor ipſius λ° minor quam $0,191827$ fieri nequit.

Scholion.

118. Huiusmodi ergo lentes duplicatae maxime ſunt notatu dignae cum loco ſimplicium adhibitae multo

multo minorem diffuſionem pariant, ex quo in conſtructione Teleſcopiorum et Microſcopiorum earum ampliſſimus erit vſus. Neque vero hac inſigni proprietate ſunt praeditae, ſed etiam earum conſtructio in praxi minimis difficultatibus eſt obnoxia: propterea quod etſi a praeſcriptis regulis parumper aberretur, effectus tamen inde vix vllam mutationem patiatur. Siue enim in conſtructione vtriusque ſeorſim leuis error committatur, valores numerorum λ et λ' vnitatem haud ſenſibiliter excedent, ſiue in quantitate f valor iuſtus $f = \frac{1}{2}$ non exacte obſeruetur, error vix ſentietur, quoniam hi numeri ex natura minimi ſunt eruti. Quomodocunque autem a regulis praeſcriptis aberretur, valor ipſius λ° inde pauliſper maior prodibit. Veluti ſi eiusmodi errores committantur, vt ſit $\lambda = 1 + \frac{1}{10}$; $\lambda' = 1 + \frac{1}{10}$ et $f = \frac{1}{2} \pm \frac{1}{20}$, prodibit $\lambda^{\circ} = 0,191827 + 0,02558$ ſeu $\lambda^{\circ} = 0,2174$, ita vt diſcrimen partem tantum quadrageſimam vnitatis conficiat. Facile autem intelligitur, dummodo λ° prodeat minus quam $\frac{1}{4}$, quod nunquam non, facile praeſtari poſſe videtur, ab his lentibus duplicatis inſignem vtilitatem expectandam eſſe. Etſi ergo eiusmodi lentes duplicatae confici poſſunt, pro quibus numerus λ° plane euaneſcat, ob harum lubricam conſtructionem illae iſtis anteferendae videntur, vt mox clarius patebit.

Problema 4.

119. Pro datis diſtantiis determinatricibus $AE = a$ et $bG = \mathfrak{b}$ eas lentes duplicatas inuenire, in quibus ſit $\lambda^{\circ} = 0$.

Solutio.

Cum fieri nequeat $\lambda^{\circ}=0$, nisi numerus f extra limites 0 et 1 accipiatur, simulque numeri λ et λ' fuerint inaequales, ita vt alterutra saltem lens non debeat seorsim minimam diffusionem parere; ponamus esse vel $f>1$ vel $f<0$. Sit ergo primo $f=1+\xi$, et cum sit

$$\lambda^{\circ}=\lambda(1+\xi)^3-\lambda'\xi^3+\nu\xi(1+\xi)$$

vt fiat $\lambda^{\circ}=0$ oportet esse:

$$\lambda'=\lambda(1+\tfrac{1}{\xi})^3+\tfrac{\nu(1+\xi)}{\xi^2}$$

vnde λ' necessario vnitatem superabit, cuius valor ne prodeat nimis magnus, sumi conueniet $\lambda=1$, ita vt sit

$$\lambda'=(1+\tfrac{1}{\xi})^3+\tfrac{0,232692(1+\xi)}{\xi^2} \text{ et } \lambda=1.$$

Quicunque ergo valor ipsi ξ tribuatur, lens duplicata habetur, pro qua sit $\lambda^{\circ}=0$, ac propterea spatium diffusionis $=\mu\mathfrak{B}\mathfrak{B}xx(\frac{1}{a}+\frac{1}{\xi})\frac{\nu}{a\xi}$. Videamus nonnullos casus speciales.

$f=\frac{3}{2}$; $\xi=\frac{1}{2}$; $\lambda=1$ et $\lambda'=28,396152$

$f=2$; $\xi=1$; $\lambda=1$ et $\lambda'=8,465384$

$f=3$; $\xi=2$; $\lambda=1$ et $\lambda'=3,549519$

$f=4$; $\xi=3$; $\lambda=1$ et $\lambda'=2,473789$

$f=5$; $\xi=4$; $\lambda=1$ et $\lambda'=2,025841$

$f=6$; $\xi=5$; $\lambda=1$ et $\lambda'=1,783846$

etc.

Pro

Pro altero cafu fit $f = -\xi$, ideoque

$$\lambda^\circ = -\lambda\xi^3 + \lambda'(1+\xi)^3 + \nu\xi(1+\xi)$$

vnde facto $\lambda^\circ = 0$ prodit

$$\lambda = \lambda'(1+\tfrac{1}{\xi})^3 + \tfrac{\nu(1+\xi)}{\xi^2}.$$

Statui ergo conueniet $\lambda' = 1$, ac pro λ notentur cafus fequentes.

$f = -\frac{1}{2}$; $\xi = \frac{1}{2}$; $\lambda = 28,396152$ et $\lambda' = 1$

$f = -1$; $\xi = 1$; $\lambda = 8,465384$ et $\lambda' = 1$

$f = -2$; $\xi = 2$; $\lambda = 3,549519$ et $\lambda' = 1$

$f = -3$; $\xi = 3$; $\lambda = 2,473789$ et $\lambda' = 1$

$f = -4$; $\xi = 4$; $\lambda = 2,025841$ et $\lambda' = 1$

$f = -5$; $\xi = 5$; $\lambda = 1,783846$ et $\lambda' = 1$

ficque patet infinitis modis huiusmodi lentes duplicatas parari poffe, pro quibus fit $\lambda^\circ = 0$, et fpatium diffufionis

$$Gg = \mu \mathfrak{C}\mathfrak{C} xx \left(\tfrac{1}{a} + \tfrac{1}{\xi}\right)\tfrac{\nu}{a\xi}.$$

Scholion I.

120. Si huiusmodi lentes accuratiffime parari poffent, nullum eft dubium, quin praecedentibus effent anteferendae propterea quod diffufio iis adhuc magis diminuitur. Verum dolendum eft, quod minimus error in earum conftructione commiffus omnem

fere vſum deſtruat. Quo hoc facilius diiudicare queamus, examinemus eum caſum, quo eſt

$$f = 5, \quad \lambda = 1 \quad \text{et} \quad \lambda' = 2,025841$$

hincque

$$\lambda^\circ = \lambda f^3 - \lambda'(f-1)^3 + \nu f(f+1) = 0.$$

Ponamus autem in conſtructione errorem eſſe commiſſum vt reuera non ſit $f = 5$ ſed $f = 5\frac{1}{10}$, dum numeri λ et λ' ſuos iuſtos valores obtineant: ob hunc autem vix vitandum errorem non fit $\lambda^\circ = 0$, ſed adeo $\lambda^\circ = -2,011$; ſicque haec lens duplicata ſimplicibus longe eſt poſtponenda, ſimili modo ſi f eſſet $= 5$, ſed vel λ vel λ' tantillum a praeſcripto valore aberraret, enorme ſtatim diſcrimen in valorem ipſius λ° redundaret. Minus quidem error metuendus videtur in ea ſpecie, qua $\lambda = 1$, $\lambda' = 28,396152$ et $f = \frac{1}{2}$; ſed praeterquam, quod lens ſimplex poſterior difficillime parari queat, pro qua λ' praeciſe valorem aſſignatum conſequatur, talis lens ad vſum dioptricum plane eſt inepta, ob ingentem alterius faciei curvaturam. Quae cum ita ſint, quia tam exiguus error in paratione huiusmodi lentium commiſſus facit, vt λ° adeo ſupra vnitatem excreſcat, vix ſperare poterimus, vt vnquam talis lens duplicata perficiatur, pro qua λ° vsque ad $\frac{1}{2}$ diminuatur. In lentibus autem praecedentis generis ſucceſſus vix fallere poterit, niſi in praxi enormiter a praeſcripta regula aberretur: ex quo his ſolis lentibus duplicatis cum fructu vti licebit, dum

contra

contra eae, quas in praeſente problemate deſcripſimus, penitus profligandae videntur.

Scholion 2.

121. Pro datis ergo binis diſtantiis determinatricibus a et $\mathfrak{b}$ ſemper eiusmodi lens duplicata parari poteſt, ex qua naſcatur ſpatium diffuſionis

$$Gg = \mu \mathfrak{b} \mathfrak{b} xx \left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)\left(\lambda^{\circ}\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^{2}+\frac{\nu}{a\mathfrak{b}}\right)$$

ita vt λ° numerum quemcunque denotare poſſit. Ad hoc enim ſatisfieri oportet huic aequationi:

$$\lambda^{\circ} = \lambda f^{3} + \lambda'(1-f)^{3} - \nu f(1-f)$$

id quod ſemper fieri poteſt, cum f plane ab arbitrio noſtro pendeat, et numeri λ et λ' tantum non vnitate minores accipi debeant. Definitis autem his tribus numeris λ, λ' et f ita, vt λ° datum valorem obtineat, binae lentes ſimplices, ex quibus duplicata eſt componenda, ſecundum formulas §. 107 datas confici debent. Vbi quidem tenenda ſunt ea, quae modo obſeruauimus, in praxi eas lentes facillime obtineri, quando numeri λ et λ' vnitatem parum ſuperant, f vero propemodum $\frac{1}{2}$ denotat, cum contra quo magis hi numeri ab iſtis terminis recedant, eo maius ſit periculum ne effectus enormiter fallat. Ceterum cum diffuſionem a lente duplicata oriundam ad eandem formam reduxerimus, qua diffuſio lentis ſimplicis exprimitur, inde id commodi conſequimur, vt ſimili modo diffuſionem a lentibus magis multiplicatis ortum definire valeamus.

Supplementum I.

De lentibus duplicatis.

Si pro lente anteriori ratio refractionis fit $n:1$, pro lente posteriori vero alia ratio $n':1$ locum habeat; problemata hic tractata sequenti modo resolui poterunt

Pro Problemate I.

Si pro numeris ρ, σ, τ, qui ex n oriuntur, quaerantur simili modo ex n' valores ρ', σ', et τ' erunt

Pro Lente

$$\text{P P radius faciei} \begin{cases} \text{anter.} = \dfrac{a\alpha}{\rho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}} \\ \text{poster.} = \dfrac{a\alpha}{\rho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}} \end{cases}$$

$$\text{QQ radius faciei} \begin{cases} \text{anter.} = \dfrac{b\mathfrak{b}}{\rho'\mathfrak{b} + \sigma' b \pm \tau'(b+\mathfrak{b})\sqrt{(\lambda'-1)}} \\ \text{poster.} = \dfrac{b\mathfrak{b}}{\rho' b + \sigma'\mathfrak{b} \mp \tau'(b+\mathfrak{b})\sqrt{(\lambda'-1)}} \end{cases}$$

et definitis simili modo valoribus μ', ν' ex ratione $n':1$ reperietur spatium diffusionis

$$\mathfrak{b}\mathfrak{b}xx \begin{cases} +\mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\mu'\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)^2+\frac{\nu'}{b\mathfrak{b}}\right) \end{cases}$$

reliqua manent vt in problemate.

Pro Problemate II.

Quoniam hic duae vitri species occurrunt, ponamus lentem simplicem quaesitam ex alio quocunque vitri genere parari, cuius ratio refractionis fit $n^\circ:1$, vnde prodeant numeri μ° et ν°; superfluum autem foret, numeros ρ°, σ° et τ° computari, quoniam radii

dii

dii facierum fiunt imaginarii, ita, vt eos exprimere non fit opus, et quoniam pro hac lente fimplici aequiualente numerus λ° eft introductus, erit huius lentis fpatium diffufionis

$$\beta\beta xx.\mu^\circ\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)\left(\lambda^\circ\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu^\circ}{a\beta}\right)$$

quod vt cum fpatio diffufionis lentis duplicatae aequale fiat; ponatur, vti in problemate eft factum,

$$\tfrac{1}{\alpha}=\tfrac{f-1}{a}+\tfrac{f}{\beta} \text{ et } \tfrac{1}{b}=\tfrac{1-f}{a}-\tfrac{f}{\beta}$$

vt prodeat

$$\tfrac{1}{a}+\tfrac{1}{\alpha}=f\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right) \text{ et } \tfrac{1}{b}+\tfrac{1}{\beta}=(1-f)\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)$$

vnde peruenietur ad hanc aequationem

$$\mu f\left(\lambda f^2\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)^2+\nu\left(\tfrac{f-1}{a^2}+\tfrac{f}{a\beta}\right)\right)$$
$$+\mu'(1-f)\left(\lambda'.(1-f)^2\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)^2+\nu'\left(\tfrac{1-f}{a\beta}-\tfrac{f}{\beta^2}\right)\right)$$
$$=\mu^\circ\left(\lambda^\circ\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu^\circ}{a\beta}\right)$$

quae ita repraefentari poteft

$$\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)^2\left(\mu\lambda f^3+\mu'\lambda'(1-f)^3\right)$$
$$+\left(\tfrac{f-1}{a}+\tfrac{f}{\beta}\right)\left(\tfrac{\mu\nu f}{a}-\tfrac{\mu'\nu'(1-f)}{\beta}\right)$$
$$=\mu^\circ\lambda^\circ\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu^\circ\mu^\circ}{a\beta})$$

Vnde

$$\lambda^\circ=\tfrac{\mu\lambda f^3+\mu'\lambda'(1-f)^3}{\mu^\circ}+$$
$$\tfrac{aa\beta\beta}{\mu^\circ(a+\beta)^2}\left(\left(\tfrac{f-1}{a}+\tfrac{f}{\beta}\right)\left(\tfrac{\mu\nu f}{a}-\tfrac{\mu'\nu'(1-f)}{\beta}\right)-\tfrac{\nu^\circ\mu^\circ}{a\beta}\right)$$

Deni-

Denique si ex ratione refractionis $n' : 1$ computentur numeri ϱ', σ' et τ', radii facierum lentis duplicatae erunt

Pro Lente

$$\text{P P radius faciei} \begin{cases} \text{anterioris} = \dfrac{a\mathfrak{b}}{(\varrho-\sigma(1-f))\mathfrak{b}+\sigma f a+\tau f(a+\mathfrak{b})\surd(\lambda-1)} \\ \text{posterioris} = \dfrac{a\mathfrak{b}}{(\sigma-\varrho(1-f))\mathfrak{b}+\varrho f a+\tau f(a+\mathfrak{b})\surd(\lambda-1)} \end{cases}$$

$$\text{QQ radius faciei} \begin{cases} \text{anter.} = \dfrac{a\mathfrak{b}}{(\sigma'-\varrho' f)a+\varrho'(1-f)\mathfrak{b}\pm\tau'(1-f)(a+\mathfrak{b})\surd(\lambda'-1)} \\ \text{poster.} = \dfrac{a\mathfrak{b}}{(\varrho'-\sigma' f)a+\sigma'(1-f)\mathfrak{b}+\tau'(1-f)(a+\mathfrak{b})\surd(\lambda'-1)} \end{cases}$$

Ad Problema III.

Hoc problema non solum pro diuersa refractione n et n' hic generalius pertractabo, sed etiam rationem distantiae inter binas lentes habebo. Primum igitur vtramque lentem ad distantias determinatrices lentis duplicatae, a et $\mathfrak{b}$, reuocabo, ponendo

$$\tfrac{1}{a}+\tfrac{1}{\alpha}=f\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right) \text{ et } \tfrac{1}{b}+\tfrac{1}{\mathfrak{b}}=g\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)$$

vt sit

$$\tfrac{1}{\alpha}=\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{b}} \text{ et } \tfrac{1}{b}=\tfrac{g}{a}+\tfrac{g-1}{\mathfrak{b}}$$

hincque

$$\alpha=\frac{a\mathfrak{b}}{fa+(f-1)\mathfrak{b}} \text{ et } b=\frac{a\mathfrak{b}}{(g-1)a+g\mathfrak{b}}$$

ideoque distantia lentium

$$\alpha+b=\frac{a\mathfrak{b}(f+g-1)(a+\mathfrak{b})}{(fa+(f-1)\mathfrak{b})((g-1)a+g\mathfrak{b})}$$

quae si deberet esse $=0$, capi oporteret $g=1-f$ sed si distantiam aliquam inter lentes admittamus, statuamus

tuamus $f+g-1=\omega$, denotante ω fractionem quandam minimam, siue positiuam siue negatiuam vt distantia lentium prodeat positiua et valde parua. Cum hinc igitur sit $g=1+\omega-f$; erit lentium distantia

$$a+b=\frac{a\mathfrak{b}(a+\mathfrak{b})\omega}{(fa+(f-1)\mathfrak{b})((\omega-f)a+(1+\omega-f)\mathfrak{b})}$$

Spatium autem diffusionis nunc ita exprimetur:

$$\mathfrak{b}\mathfrak{b}xx.\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\left\{\begin{array}{l}+\lambda\mu f^3\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2+\lambda'\mu' g^3\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2\\ +\frac{\mu\nu f}{a}\left(\frac{f-1}{a}+\frac{f}{\mathfrak{b}}\right)\\ +\frac{\mu'\nu' g}{\mathfrak{b}}\left(\frac{g}{a}+\frac{g-1}{\mathfrak{b}}\right)\end{array}\right.$$

quae cum in hoc problemate ita tractari debeat, vt tam f, quam g positiue sumantur, et haec formula minima reddatur; euidens est, litteris λ et λ' minimos valores tribui debere, scilicet $\lambda=1$ et $\lambda'=1$; deinde pro hoc casu minimi f et g conuenienter definiantur, vbi quia $f+g-1=\omega$ ideoque constans in differentiatione, habebimus $dg=-df$; vndeobtinebimus hanc aequationem

$$\left(3\mu f^2-3\mu' g^2\right)\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2+\frac{\mu\nu}{a}\left(\frac{2f-1}{a}+\frac{2f}{\mathfrak{b}}\right)$$
$$-\frac{\mu'\nu'}{\mathfrak{b}}\left(\frac{2g}{a}+\frac{2g-1}{\mathfrak{b}}\right)=0$$

cui proxime satisfit, ponendo $f=g=\frac{1+\omega}{2}$ quibus valoribus substitutis spatium diffusionis ipsum minimum, erit proxime

$$\frac{\mathfrak{b}^2x^2(1+\omega)}{8}\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\left\{\begin{array}{l}\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2(1+\omega)^2(\mu+\mu')\\ +\frac{2\mu\nu}{a}\left(\frac{\omega-1}{a}+\frac{1+\omega}{\mathfrak{b}}\right)\\ +\frac{2\mu'\nu'}{\mathfrak{b}}\left(\frac{1+\omega}{a}+\frac{\omega-1}{\mathfrak{b}}\right)\end{array}\right.$$

Tum vero distantia lentium erit

$$\alpha + b = \frac{4\alpha\beta(\alpha+\beta)\omega}{((1+\omega)\alpha+(\omega-1)\beta)((\omega-1)\alpha+(1+\omega)\beta)}$$

$$= \frac{4\alpha\beta(\alpha+\beta)\omega}{-(1-\omega^2)(\alpha^2+\beta^2)+2(1+\omega)^2\alpha\beta}$$

cuius denominator cum sit negatiuus ob ω minimum; necesse est, fractionem ω sumi debere negatiuam; hincque adeo spatium diffusionis minus reddetur.

Coroll.

Si ergo distantia obiecti α fuerit infinita seu $\alpha = \infty$, habebitur primo distantia lentium $= \frac{-4\beta\omega}{1-\omega^2}$; et secundo spatium diffusionis

$$\frac{x^2(1+\omega)}{4\beta}\left\{\begin{array}{l}(1+\omega)^2(\mu+\mu')\\ -2\mu'\nu'(1-\omega)\end{array}\right.$$

quod cum ω debeat esse negatiuum, non mediocriter minus erit, quam si distantia lentium esset nulla.

Ad Problema IV.

In hoc problemate etiam distantiam lentium non negligamus; factaque reductione, vt ante, statuamus spatium diffusionis plane euanescens; id quod fieri nequit, nisi altera litterarum f et g sit negatiua, quod cum etiam fiat, quando confusio a diuersa radiorum refrangibilitate oriunda ad nihilum redigi debet, vt infra videbimus; hic casus multo magis euolutionem meretur. Ponatur igitur $g = -\zeta f$, vbi ζ ex illa

illa conditione determinatur, vt primo pro diſtantia lentium ſit

$$\alpha+b=\frac{-\alpha\beta(\alpha+\beta)(f-\zeta f-1)^2}{(f\alpha+(f-1)\beta)((\zeta f+1)\alpha+\zeta f\beta)}$$

et poſito $f+g-1=\omega$, prodeat $f-\zeta f=1+\omega$ ideoque

$$f=\frac{1+\omega}{1-\zeta} \text{ et } g=\frac{-\zeta(1+\omega)}{1-\zeta} \text{ ſicque}$$

$$\alpha+b=\frac{-\alpha\beta(\alpha+\beta)\omega(1-\zeta)^2}{((1+\omega)\alpha+(\omega+\zeta)\beta)((1+\omega\zeta)\alpha+\zeta(1+\omega)\beta)}$$

ac ſi in denominatore ω reiiciatur, erit

$$\alpha+b=\frac{-\alpha\beta(\alpha+\beta)(1-\zeta)^2\omega}{(\alpha+\zeta\beta)}$$

ſicque patet ω negatiue capi debere.

Poſito autem $g=-\zeta f$, fit ſpatium diffuſionis

$$\beta^2x^2\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)\begin{cases}(\lambda\mu f^3-\lambda'\mu'\zeta^3f^3)\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)^2\\ +\frac{\mu\nu f}{\alpha}\left(\frac{f-1}{\alpha}+\frac{f}{\beta}\right)\\ -\frac{\mu'\nu\zeta f}{\beta}\left(\frac{-\zeta f}{\alpha}-\frac{(\zeta f+1)}{\beta}\right)\end{cases}$$

ad nihilum redigendum; vnde ſequitur fore

$$\lambda'=\frac{\lambda\mu}{\mu'\zeta^3}+\frac{\mu\,\nu.\,\alpha.\,\beta^2}{\mu'\zeta^3f^2(\alpha+\beta)^2}\left(\frac{f-1}{\alpha}+\frac{f}{\beta}\right)$$
$$+\frac{\nu.\,\alpha^2\beta}{\zeta^2f^2.(\alpha+\beta)^2}\left(+\frac{\zeta f}{\alpha}+\frac{\zeta f+1}{\beta}\right)$$

qui valor cum debeat eſſe maior vnitate, ſi forte eueniat, vt minor prodeat, tunc non λ' ſed λ definiri conueniet, vbi notandum eſt eſſe $f=\frac{1+\omega}{1-\zeta}$. Hincque per formulas ante datas facile eruuntur radii facierum vtriusque lentis.

Etſi formula ſuperius data pro ſpatio diffuſionis iam ad caſum, quo diſtantia lentium eſt nulla, eſt

 ad-

adcommodata; tamen quia hic diſtantiam minimam aſſumimus, nullus inde error eſt metuendus.

Definitio 2.

122. *Lens triplicata eſt, quae conſiſtit ex tribus lentibus ſimplicibus ſibi immediate iunctis ad communem axem.*

Hic quidem etiam craſſitiem negligo, etiamſi neceſſario maior ſit quam in lentibus duplicatis In ſuplemento autem oſtendetur, quomodo etiam diſtantiarum inter lentes ratio ſit habenda.

Corollarium.

123. Poteſt ergo lens triplicata conſiderari, quaſi compoſita ex lente duplicata et lente ſimplici, hocque duplici modo, prout vel binae anteriores, vel binae poſteriores lentem duplicatam conſtituere concipiuntur.

Scholion.

124. Lentibus triplicatis tum demum vſum in praxi concedi conueniet, cum eadem commoda per lentes ſimplices vel duplicatas conſequi non licet. Haec autem commoda in paruitate numeri λ conſiſtunt, qui quamdiu vnitate fuerit maior, ſemper lente ſimplici vti praeſtat, cum vero circumſtantiae in expreſſione diffuſionis minorem numerum λ requirunt, ad lentes multiplicatas erit confugiendum. Quoniam igitur eiusmodi lentes duplicatas conficere docuimus, in

in quibus valor ipsius λ non solum ad nihilum vsque, sed adeo ad negatiua diminui queat, vsus lentium triplicatarum superfluus videtur. Verum iam obseruauimus in praxi aegre eiusmodi duplicatas lentes parari posse, pro quibus valor ipsius λ minor sit quam $0,191827$, propterea quod si leuissimus error committatur, omnis labor irritus reddatur. His igitur casibus imprimis, quando minori valore numeri λ opus est, lentes triplicatae in vsum erunt vocandae: et quia respectu ad praxin habito non omnes aequo successu construere licet, errores ineuitabiles hic quoque imprimis spectari oportet, vt pateat quousque numerus λ cum successu diminui queat, ac si adhuc minore valore opus fuerit lentes adeo quadruplicate erunt inducendae.

Problema 5.

125. Datis binis distantiis determinatricibus omnes lentes triplicatas definire, simulque spatium diffusionis ab iis ortum. Tab. II. Fig. 8.

Solutio.

Sit Ee obiectum, cuius magnitudo $=z$, et distantia a lente $AE=a$: tum primae lentis PP distantiae determinatrices sint a et α, secundae lentis QQ b et β; ac tertiae RR, c et γ. His positis quia lentes immediate iunctae sumuntur, erit $\alpha+b=0$, $\beta+c=0$, et a et γ distantiae determinatrices lentis triplicatae, ita vt α et β arbitrio nostro relinquantur.

 Si

Si igitur lens prima ſola PP adeſſet, imago repraeſentaretur in $F\zeta$, vt eſſet $AF=\alpha$ et $F\zeta=\frac{\alpha}{a}z$: ſi binae priores PP et QQ ſolae adeſſent imago exhiberetur in $G\eta$, vt eſſet $AG=\beta$ et $G\eta=\frac{\beta}{a}z$; at per lentem triplicatam referetur in $H\theta$ vt ſit $cH=\gamma$ et $H\theta=\frac{\gamma}{a}z$, pro ſitu inuerſo ob $\frac{\alpha\beta}{bc}=1$. Hactenus ſcilicet res perinde ſe habet, ac ſi in A haberetur lens ſimplex ad diſtantias determinatrices a et γ accommodata.

At ſi ad ſpatium diffuſionis Hb reſpiciamus figuram ſingularum lentium in computum ducere debemus, quatenus praeter diſtantias determinatrices numeri arbitrarii $\lambda, \lambda', \lambda''$ innoluuntur; ex quibus facies ſingularum lentium ſupra §. 91 ſunt definitae. Indidem autem colligitur pro apertura cuius ſemidiameter $=x$, ſpatium diffuſionis ob $\frac{\alpha}{b}=-1$ et $\frac{\beta}{c}=-1$ fore:

$$Hb=\mu\gamma\gamma xx\begin{cases}+\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{\alpha a}\right)\\+\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right)\\+\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right)\end{cases},$$

quae expreſſio, vt ad formam vni lenti reſpondentem reducatur ſtatuamus:

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{b}+\frac{1}{\beta}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{c}+\frac{1}{\gamma}=h\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

et

et quia $\frac{1}{\alpha}+\frac{1}{b}=0$, et $\frac{1}{\mathfrak{b}}+\frac{1}{c}=0$, his aequationibus addendis adipiscimur: $f+g+b=1$. Porro vero erit

$$\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)-\frac{1}{a};\quad \frac{1}{b}=\frac{1}{a}-f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{\mathfrak{b}}=(f+g)\left(\frac{1}{a}+\frac{1}{\gamma}\right)-\frac{1}{a};\quad \frac{1}{c}=\frac{1}{a}-(f+g)\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

siue ob $1=f+g+b$

$$\frac{1}{a}=\frac{f+g+b}{a};\quad \frac{1}{\alpha}=\frac{-g-b}{a}+\frac{f}{\gamma}$$

$$\frac{1}{b}=\frac{g+b}{a}-\frac{f}{\gamma};\quad \frac{1}{\mathfrak{b}}=-\frac{b}{a}+\frac{f+g}{\gamma}$$

$$\frac{1}{c}=\frac{b}{a}-\frac{f-g}{\gamma};\quad \frac{1}{\gamma}=\frac{f+g+b}{\gamma}$$

Vnde cum spatium diffusionis fiat.

$$Hb=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left((\lambda f^3+\lambda' g^3+\lambda'' b^3)\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\nu\left(\frac{f}{a\alpha}+\frac{g}{b\mathfrak{b}}+\frac{b}{c\gamma}\right)\right)$$

reducetur id ad hanc formam

$$Hb=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left((\lambda f^3+\lambda' g^3+\lambda'' b^3-\nu(1-f)(1-g)(1-b))\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$$

Radiorum autem in b concurrentium inclinatio ad axem erit $=\frac{x}{\gamma}$.

Coroll. 1.

126. Haec igitur lens triplicata idem producit spatium diffusionis quod produceret lens simplex ad easdem distantias determinatrices instructa, numero eius arbitrario (per litteram λ indicato) existente

$$\lambda f^3+\lambda' g^3+\lambda'' b^3-\nu(1-f)(1-g)(1-b)$$

vbi quidem est $f+g+b=1$.

Coroll.

Coroll. 2.

127. Quatenus ergo haec quantitas reddi poteſt minor non ſolum vnitate; ſed etiam fractione 0,191827 ita ſcilicet, vt praxis non enormi aberrationi ſit expoſita, eatenus lentibus triplicatis vſus erit concedendus.

Coroll. 3.

128. Si ſit vel $f=0$, vel $g=0$, vel $h=0$, vna lentium habebit facies parallelas, et lens triplicata aequiualebit duplicatae ac ſi duae litterarum f, g, h ſimul euaneſcant, tertia in vnitatem abeunte, caſus habebitur lentis ſimplicis.

Coroll. 4.

129. Si ſit $f=1$, ideoque $h=-g$, valor numeri λ pro lente triplicata erit $\lambda+(\lambda'-\lambda'')g^3$, ideoque ſi $\lambda''=\lambda'$ lens triplicata ſimplici aequiualebit, quod idem euenit ſi fuerit vel $g=1$ vel $h=1$.

Coroll. 5.

130. Sumtis autem pro f, g, h numeris idoneis ob $f+g+h=1$, conſtructio lentis triplicatae ex formulis §. 91 exhibitis eſt petenda ſumendo.

$$\frac{1}{\mathfrak{a}}=\frac{-1+f}{a}+\frac{f}{\gamma};\quad \frac{1}{b}=\frac{1-f}{a}-\frac{f}{\gamma}$$

$$\frac{1}{\mathfrak{c}}=\frac{-h}{a}+\frac{1-h}{\gamma};\quad \frac{1}{c}=\frac{h}{a}-\frac{1+h}{\gamma}$$

Scholion

Scholion 1.

131. Quemadmodum hinc expressio inuenta prodeat, notandum est fore $\frac{f}{a\alpha}+\frac{g}{b\beta}+\frac{h}{c\gamma}=$

$$+\frac{1}{\alpha\alpha}(-f(1-f)-gh(1-f))$$
$$+\frac{1}{\alpha\gamma}(+ff+g(1-f)(1-h)+fgh+hh)$$
$$+\frac{1}{\gamma\gamma}(-fg(1-h)-h(1-h))$$

sed $-f(1-f)-gh(1-f)=-(1-f)(f+gh)=-(1-f)(1-g)(1-h)$ ob $f=1-g-h$ ideoque $f+gh=(1-g)(1-h)$. Simili modo pro $\frac{1}{\gamma\gamma}$ est $-fg(1-h)-h(1-h)=-(1-h)(h+fg)=-(1-f)(1-g)(1-h)$ ob $h=1-f-g$. Denique pro $\frac{1}{\alpha\gamma}$, quia est $ff+hh=(f+h)^2-2fh=(1-g)^2-2fh=1-2g+gg-2fh$, hoc valore substituto coefficiens ipsius $\frac{1}{\alpha\gamma}$ erit

$$1-2g-2fh+gg+fgh+g(1-f)(1-h)=$$
$$1+(g+fh)(g-2)-g(1-f)(1-h)=1-2(1-f)(1-g)(1-h)$$

ob $g+fh=(1-f)(1-h)$. Consequenter colligitur

$$\frac{f}{a\alpha}+\frac{g}{b\beta}+\frac{h}{c\gamma}=-(1-f)(1-g)(1-h)\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)^2+\frac{1}{\alpha\gamma}.$$

Scholion 2.

132. Hoc problema etiam ope praecedentium facilius sequenti modo resolui potest. Considerentur scilicet binae lentes PP et QQ iunctim sumtae tanquam lens duplicata ad distantias determinatrices a et β per numeros arbitrarios λ, λ' et f instructa, ac posito $\lambda f^2+\lambda'(1-f)^2-\nu f(1-f)=\lambda^{(2)}$, spatium diffusionis ex ea sola ortum erit

$$\mu\beta\beta xx\left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\lambda^{(2)}\left(\frac{1}{a}+\frac{1}{\beta}\right)^2+\frac{\nu}{a\beta}\right)$$

quae lens ſi iam in compoſitione cum tertia R R tanquam ſimplex tractetur exinde elicitur ſpatium diffuſionis perinde atque ex coniunctione duarum ſimplicium

$$Hb = \mu\gamma\gamma xx \begin{cases} +(\frac{1}{a}+\frac{1}{\beta})(\lambda^{(2)}(\frac{1}{a}+\frac{1}{\beta})^2+\frac{\nu}{a\beta}) \\ +(\frac{1}{c}+\frac{1}{\gamma})(\lambda''(\frac{1}{c}+\frac{1}{\gamma})^2+\frac{\nu}{c\gamma}) \end{cases}$$

vbi notandum eſt eſſe $\beta + c = 0$, et conſtructio lentis duplicatae ex §. 107 erit petenda lentis vero ſimplicis RR ex diſtantiis determinatricibus $c = -\beta$ et γ vna cum numero arbitrario λ''. Ponatur iam $\frac{1}{\beta} = \frac{g-1}{a} + \frac{g}{\gamma}$ vt ſit $\frac{1}{c} = \frac{1-g}{a} - \frac{g}{\gamma}$, eritque ſpatium diffuſionis huius lentis triplicatae,

$$Hb = \mu\gamma\gamma xx(\tfrac{1}{a}+\tfrac{1}{\gamma})((\lambda^{(\,)}g^3+\lambda''(1-g)^3-\nu g(1-g))(\tfrac{1}{a}+\tfrac{1}{\gamma})^2+\tfrac{\nu}{a\gamma})$$

Quare ſi pro lente triplicata ponatur

$$\lambda^{(\,)}g^3+\lambda''(1-g)^3-\nu g(1-g) = \lambda^{(\,)}$$

ita vt iam numerus g inſuper arbitrio noſtro relinquatur, habebitur ſpatium diffuſionis more hactenus recepto expreſſum

$$Hb = \mu\gamma\gamma xx(\tfrac{1}{a}+\tfrac{1}{\gamma})(\lambda^{(3)}(\tfrac{1}{a}+\tfrac{1}{\gamma})^2+\tfrac{\nu}{a\gamma})$$

Hinc iam id intelligitur, quod ex praecedente ſolutione minus patet; ſi numerus $\lambda^{(\,)}$ fuerit vnitate maior loco binarum priorum lentium PP et QQ commodius vnicam ſimplicem adhiberi; ex quo eatenus tantum lentes triplicatae reſultare cenſendae ſunt, quatenus nume-

numerus $\lambda^{(2)}$ vnitate eſt minor. Vidimus autem ſucceſſum tuto ſperari non poſſe, niſi $\lambda^{(\,)}$ aequalis ſit fractioni 0, 191827, vel ea non multo maior; vnde ſi praxi conſulere velimus, ipſi $\lambda^{(\,)}$ minorem valorem tribui non conuenit atque ob eandem rationem numerus g intra terminos 0 et 1 accipi debebit; cuius valor imprimis ad praxin erit accommodatus ſi reddat numerum $\lambda^{(\,)}$ minimum, quia tum leues errores negotium minime turbant. Quodſi vero pro $\lambda^{(2)}$ valorem aſſignatum ſubſtituamus, obtinebimus

$$\lambda^{(\,)}=\lambda f^3g^3+\lambda' g^3(1-f)^3+\lambda''(1-g)^3-\nu g^3 f(1-f)-\nu g(1-g)$$

Coniucet autem vtramque expreſſionem pro numero $\lambda^{(\,)}$, quo ſpatium diffuſionis a lente triplicata ortum definitur, hic expoſuiſſe cum aliae concluſiones ex altera facilius deducantur. Etſi autem hinc omnes valores pro $\lambda^{(\,)}$ obtineri poſſunt tamen eos tantum, qui prope minimum ſubſiſtunt ad praxin adhiberi conueniet.

Problema. 6.

133. Datis diſtantiis determinatricibus $AE=a$ et $cH=\gamma$ definire eam lentem triplicatam, quae minimum ſpatium diffuſionis producat.

Solutio 1.

Duplici modo hoc problema ſolui poteſt, prout ſpatium diffuſionis vel ita exprimitur, vti in ſolutione problematis praecedentis, vel in Scholio 2. Priori modo numeros f, g, h ita determinari oportet, vt minima reddatur haec expressio:

$$\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h)$$

vbi notandum est esse $f+g+h=1$. Eius ergo differentiali nihilo aequali posito habebimus:

$$3\lambda ff df+3\lambda' gg dg+3\lambda'' hh dh$$

$$+\nu df(1-g)(1-h)+\nu dg(1-f)(1-h)+\nu dh(1-f)(1-g)=0.$$

Cum autem sit $dh=-df-dg$ erit

$$\left.\begin{array}{l} +3\lambda ff df-3\lambda'' hh df+\nu df(1-g)(f-h) \\ +3\lambda' gg dg-3\lambda'' hh dg+\nu dg(1-f)(g-h) \end{array}\right\}=0$$

quia vero bina differentialia df et dg a se inuicem non pendent ambo membra huius aequationis seorsim euanescere debent, vnde ob $1-g=f+h$ et $1-f=g+h$ has duas nanciscimur aequationes:

$$3\lambda ff-3\lambda'' hh+\nu ff-\nu hh=0$$

$$3\lambda' gg-3\lambda'' hh+\nu gg-\nu hh=0$$

ex quibus elicimus:

$$f=h\sqrt{\tfrac{3\lambda''+\nu}{3\lambda+\nu}} \text{ et } g=h\sqrt{\tfrac{3\lambda''+\nu}{3\lambda'+\nu}}.$$

Cum autem sit

$$f+g+h=1 \text{ seu } \tfrac{1}{h}=1+\tfrac{f}{h}+\tfrac{g}{h} \text{ erit}$$

$$\tfrac{1}{h}=1+\sqrt{\tfrac{3\lambda''+\nu}{3\lambda+\nu}}+\sqrt{\tfrac{3\lambda''+\nu}{3\lambda'+\nu}}$$

$$\tfrac{1}{g}=1+\sqrt{\tfrac{3\lambda'+\nu}{3\lambda+\nu}}+\sqrt{\tfrac{3\lambda'+\nu}{3\lambda''+\nu}}$$

$$\tfrac{1}{f}=1+\sqrt{\tfrac{3\lambda+\nu}{3\lambda'+\nu}}+\sqrt{\tfrac{3\lambda+\nu}{3\lambda''+\nu}}$$

Quicun-

Quicunque ergo numeri λ, λ', λ'' in conſtructione ſingularum lentium fuerint vſurpati, hinc numeri f, g, et b determinantur, ex quibus ſpatium diffuſionis minimum reſultet. Pro lentium autem conſtructione hinc diſtantiae α, b, $\mathfrak{b}$, c ita definiuntur, vt ſit

$$\frac{1}{\alpha} = \frac{-1+f}{a} + \frac{f}{\gamma};\quad \frac{1}{b} = \frac{1-f}{a} - \frac{f}{\gamma}$$

$$\frac{1}{\mathfrak{b}} = \frac{-b}{a} + \frac{1-b}{\gamma};\quad \frac{1}{c} = \frac{b}{a} - \frac{1+b}{\gamma}$$

eſt vero:

$$f = \frac{\sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)} + \sqrt{(3\lambda+\nu)(3\lambda''+\nu)} + \sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

$$g = \frac{\sqrt{(3\lambda+\nu)(3\lambda''+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)} + \sqrt{(3\lambda+\nu)(3\lambda''+\nu)} + \sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

$$b = \frac{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)} + \sqrt{(3\lambda+\nu)(3\lambda''+\nu)} + \sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

Ex diſtantiis autem a, α, b, $\mathfrak{b}$, c, γ cum numeris λ, λ', λ'' lentes ipſae per formulas §. 91 exhibitas conſtruuntur.

Coroll. I.

134. Si pro hac lente triplicata ponatur

$$\lambda f^3 + \lambda' g^3 + \lambda'' b^3 - \nu(1-f)(1-g)(1-b) = \lambda^{(3)}$$

ſubſtituendis his valoribus pro f, g, et b reperietur

$$\lambda^{(3)} = \frac{1}{3\left(\frac{1}{\sqrt{(3\lambda+\nu)}} + \frac{1}{\sqrt{(3\lambda'+\nu)}} + \frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2} - \frac{1}{4}\nu$$

vnde ſpatium diffuſionis fit

$$\mathrm{H}b = \mu\gamma\gamma x x\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)^2 + \frac{\nu}{\alpha\gamma}\right)$$

Coroll. 2.

135. Hoc autem ſpatium diffuſionis omnium fiet minimum ſi numeris λ, λ', λ'' minimi valores, quos accipere poſſunt, tribuantur. Sit ergo $\lambda = 1$, $\lambda' = 1$, $\lambda'' = 1$, eritque

$$f = \tfrac{1}{3};\quad g = \tfrac{1}{3},\quad h = \tfrac{1}{3}$$

$$\text{et } \lambda^{(\prime)} = \frac{\nu+1}{27} - \frac{\nu}{3} = \frac{1-8\nu}{27} = 0,042165$$

ob 0, 232692; qui ergo valor multo eſt minor, quam caſu lentium duplicatarum.

Coroll. 3.

136. Hoc porro caſu erit $\frac{1}{a} = -\frac{2}{3\alpha} + \frac{1}{3\gamma}$; $\frac{1}{b} = \frac{2}{3\alpha} - \frac{1}{3\gamma}$; $\frac{1}{\mathfrak{b}} = \frac{-1}{3\alpha} + \frac{2}{3\gamma}$ et $\frac{1}{c} = \frac{1}{3\alpha} - \frac{2}{3\gamma}$; vnde conſtructio lentium teruarum ſimplicium ita ſe habebit.

Pro Lente

Prima radius faciei $\begin{cases} \text{anterioris} = \dfrac{3\alpha\gamma}{(3\varrho-2\sigma)\gamma+\sigma\alpha} \\ \text{poſterioris} = \dfrac{3\alpha\gamma}{(3\sigma-2\varrho)\gamma+\varrho\alpha} \end{cases}$

Secunda radius faciei $\begin{cases} \text{anterioris} = \dfrac{3\alpha\gamma}{(2\varrho-\sigma)\gamma+(2\sigma-\varrho)\alpha} \\ \text{poſterioris} = \dfrac{3\alpha\gamma}{(2\sigma-\varrho)\gamma+(2\varrho-\sigma)\alpha} \end{cases}$

Tertia radius faciei $\begin{cases} \text{anterioris} = \dfrac{3\alpha\gamma}{\varrho\gamma+(3\sigma-2\varrho)\alpha} \\ \text{poſterioris} = \dfrac{3\alpha\gamma}{\sigma\gamma+(3\varrho-2\sigma)\alpha} \end{cases}$

Solutio altera Problematis.

137. Conſideremus binas lentes priores PP et QQ vt lentem duplicatam ad diſtantias determinatrices a et $\mathfrak{b}$ ita

ita instructam vt posito $\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)=\lambda^{(2)}$ spatium diffusionis inde oriundum sit

$$=\mu\,\mathfrak{b}\mathfrak{b}xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\left(\lambda^{(2)}\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2+\tfrac{\nu}{a\mathfrak{b}}\right).$$

sed pro constructione binarum lentium simplicium, ex quibus haec lens est composita, recordandum est esse

$$\tfrac{1}{\alpha}=\tfrac{-1+f}{a}+\tfrac{f}{\mathfrak{b}}\ \text{ et }\ \tfrac{1}{b}=\tfrac{1-f}{a}-\tfrac{f}{\mathfrak{b}}$$

Adiuncta iam tertia lente RR ad distantias determinatrices $c=-\mathfrak{b}$ et γ per numerum arbitrarium λ'' instructa si ponamus $\frac{1}{\mathfrak{g}}=\frac{-1+g}{a}+\frac{g}{\gamma}=-\frac{1}{c}$, et

$$\lambda^{(2)}g^3+\lambda''(1-g)^3-\nu g(1-g)=\lambda^{(3)}$$

erit spatium diffusionis

$$Hb=\mu\,\gamma\gamma xx\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu}{a\gamma}\right)$$

quod vt fiat minimum, valor ipsius $\lambda^{(3)}$ minimus reddi debet, quaeratur ergo primo g dum $\lambda^{(2)}$ vt numerus datus spectatur et habebimus

$$3\lambda^{(2)}gg-3\lambda''(1-g)^2-\nu+2\nu g=0$$

vnde elicitur

$$g=\frac{-3\lambda''-\nu+\sqrt{(3\lambda^{(2)}+\nu)(3\lambda''+\nu)}}{3\lambda^{(2)}-3\lambda''}\quad\text{siue}$$

$$g=\frac{\sqrt{(3\lambda''+\nu)}}{\sqrt{(3\lambda^{(2)}+\nu)}+\sqrt{(3\lambda''+\nu)}}$$

et hinc valor ipsius $\lambda^{(3)}$ erit

$$\lambda^{(3)}=\frac{1}{3\left(\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right)}-\frac{1}{3}\nu$$

vel

vel

$$\frac{1}{3\lambda^{(3)}+\nu}=\left(\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2$$

Sicque fit:

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

Simili modo si f ita definiatur, vt $\lambda^{(2)}$ fiat minimum reperietur:

$$f=\frac{\sqrt{(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)}+\sqrt{(3\lambda'+\nu)}}$$

hocque valore substituto

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}$$

Quare si numeris $\lambda, \lambda', \lambda''$, arbitrio nostro relictis bini numeri f et g ita definiantur, vt $\lambda^{(3)}$ consequatur valorem minimum erit

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

vnde idem valor pro $\lambda^{(3)}$ reperitur, quem ante inuenimus.

Coroll. 1.

138. Ex hac ergo solutione numeri f et g ita definiuntur vt sit

$$\frac{1}{f\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}} \quad \text{et}$$

$$\frac{1}{g\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

siue

fiue hoc modo

$$\left(\tfrac{1}{f}-1\right)\tfrac{1}{\sqrt{(3\lambda+\nu)}}=\tfrac{1}{\sqrt{(3\lambda'+\nu)}} \text{ et}$$

$$\left(\tfrac{1}{g}-1\right)\left(\tfrac{1}{\sqrt{(3\lambda+\nu)}}+\tfrac{1}{\sqrt{(3\lambda'+\nu)}}\right)=\tfrac{1}{\sqrt{(3\lambda''+\nu)}}$$

Coroll. 2.

139. Ex inuentis minimis valoribus numerorum $\lambda^{(1)}$ et $\lambda^{(2)}$ numeri f et g etiam ita definiuntur vt fit

$$\frac{1}{f\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(1)}+\nu)}} \text{ feu } f=\sqrt{\left(\frac{3\lambda^{(1)}+\nu}{3\lambda+\nu}\right)} \text{ et}$$

$$\frac{1}{g\sqrt{(3\lambda^{(1)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}} \text{ feu } g=\sqrt{\left(\frac{3\lambda^{(2)}+\nu}{3\lambda^{(1)}+\nu}\right)}$$

Coroll. 3

140. Diftantiae autem determinatrices fingularum lentium ita per a et γ prodibunt expreffae:

$$\tfrac{1}{\alpha}=-\tfrac{1}{b}=\tfrac{-1+fg}{a}+\tfrac{fg}{\gamma}$$

$$\tfrac{1}{\beta}=-\tfrac{1}{c}=\tfrac{-1+g}{a}+\tfrac{g}{\gamma}$$

vbi cum fit

$$g=\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda^{(1)}+\nu}} \text{ notandum eft effe } fg=\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}}.$$

Coroll. 4.

141. Eliminando autem numero $\lambda^{(1)}$ erit

$$\frac{1}{\alpha}=-\frac{1}{b}=-\frac{1}{a}\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda'+\nu}}-\frac{1}{a}\sqrt{\frac{3\lambda^{(1)}+\nu}{3\lambda''+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(1)}+\nu}{3\lambda+\nu}}$$

$$\frac{1}{\beta}=-\frac{1}{c}=-\frac{1}{a}\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda''+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda'+\nu}}$$

ex quibus formulis, inuento iam valore minimo $\lambda^{(7)}$ ſingulae lentes commodiſſime determinantur.

Coroll. 5.

142. Si praeterea ſingulae lentes ita fuerint comparatae vt per ſe minimam confuſionem pariant quod fit ſi $\lambda = \lambda' = \lambda'' = 1$, erit

$$\frac{1}{\surd(3\lambda^{(2)}+\nu)} = \frac{2}{\surd(3+\nu)} \text{ hincque } 3\lambda^{(2)}+\nu = \frac{3+\nu}{4},$$

vnde fit $\lambda^{(2)} = \frac{1-\nu}{4}$. Deinde vero habebitur:

$$\frac{1}{\surd(3\lambda^{(3)}+\nu)} = \frac{3}{\surd(3+\nu)} \text{ hincque } 3\lambda^{(3)}+\nu = \frac{3+\nu}{9}$$

ac propterea

$\lambda^{(3)} = \frac{3-8\nu}{27}$. Sin autem fuerit tantum $\lambda = \lambda' = \lambda''$ reperietur:

$$\lambda^{(2)} = \frac{3\lambda-3\nu}{12} \text{ et } \lambda^{(3)} = \frac{3\lambda-8\nu}{27}.$$

Coroll. 6.

143. Eodem autem caſu, quo $\lambda = \lambda' = \lambda''$, ob $\surd(3\lambda^{(3)}+\nu) = \frac{1}{3}\surd(3\lambda+\nu)$, diſtantiae determinatrices pro lentibus ſimplicibus erunt:

$$\frac{1}{\alpha} = \frac{-2}{3a} + \frac{1}{3\gamma}; \quad \frac{1}{b} = \frac{2}{3a} - \frac{1}{3\gamma}$$

$$\frac{1}{\beta} = \frac{-1}{3a} + \frac{2}{3\gamma}; \quad \frac{1}{c} = \frac{1}{3a} - \frac{2}{3\gamma}$$

vnde eaedem formulae pro earum conſtructione naſcuntur quae ſupra (136) ſunt allatae niſi quod iam in denominatoribus membra $\pm \tau(a+\gamma)\surd(\lambda-1)$ adiungi debeant. Scholi-

Scholion.

144. Ternarum ergo lentium simplicium idonea coniunctione effici potest, vt in expressione spatii diffusionis $Hb = \mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$ numerus $\lambda^{(3)}$ fiat $= 0,042165$. In lentibus autem duplicatis vidimus minimum valorem numeri $\lambda^{(2)}$ esse $= 0,191827$: sicque in triplicatis hic numerus fere quinquies minor reddi potest at is fere vicies quater est minor, quam per lentes simplices obtineri potest. Loquor hic autem de lentibus ex principio minimi petitis, quippe quae ad praxin maxime sunt accommodatae, dum constructio leuibus erroribus non admodum turbatur. Quanquam enim lentes triplicatae perinde ac duplicatae parari possent, pro quibus numerus conueniens λ non solum nihilo aequalis, sed etiam negatiuus resultaret tamen earum constructio tam est lubrica, vt minimus error totum laborem irritum reddat. Interim tamen periculum in triplicatis non tantum est quam duplicatis, vnde sequens problema soluisse operae erit pretium.

Problema 7.

145. Pro datis distantiis determinatricibus a et γ eas definire lentes triplicatas, pro quibus valor ipsius $\lambda^{(3)}$ prorsus in nihilum abeat.

Solutio.

Consideretur numerus $\lambda^{(2)}$ ex duabus prioribus lentibus natus vt datus, et cum sit secundum solutionem posteriorem praecedentis problematis

$$\lambda^{(1)} = \lambda^{(2)} g^3 + \lambda''(1-g)^3 - \nu g(1-g)$$

definiri debet g ita vt ista quantitas euanescat. Verum cum $\lambda^{(2)}$ commode nequeat minor effici quam $\frac{1-\nu}{4}$, statuamus $\lambda^{(2)} = \frac{1-\nu}{4}$, fierique oportet:

$$o = \tfrac{1}{4}g^3 + \lambda''(1-g)^3 - \nu g(1-\tfrac{1}{2}g)^2$$

sed quia λ'' vnitate minor esse nequit, necesse est vt g capiatur vnitate maior euoluantur ergo quidam casus.

I. $g = \frac{5}{4}$; $o = \frac{125}{4 \cdot 64} - \frac{\lambda''}{64} - \frac{5 \cdot 9}{4 \cdot 64}\nu$ et $\lambda'' = \frac{125 - 45\nu}{4}$

II. $g = \frac{6}{4}$; $o = \frac{216}{4 \cdot 64} - \frac{8\lambda''}{64} - \frac{6 \cdot 4}{4 \cdot 64}\nu$ et $\lambda'' = \frac{216 - 24\nu}{4 \cdot 8} = \frac{27 - 3\nu}{4}$

III. $g = \frac{7}{4}$; $o = \frac{343}{4 \cdot 64} - \frac{27\lambda''}{64} - \frac{7 \cdot 1}{4 \cdot 64}\nu$ et $\lambda'' = \frac{343 - 7\nu}{4 \cdot 27}$

IV. $g = \frac{8}{4}$; $o = \frac{512}{4 \cdot 64} - \frac{64\lambda''}{64}$ et $\lambda'' = 2$

V. $g = \frac{9}{4}$; $o = \frac{729}{4 \cdot 64} - \frac{125\lambda''}{64} - \frac{9 \cdot 1}{4 \cdot 64}\nu$ et $\lambda'' = \frac{729 - 9\nu}{4 \cdot 125}$

VI. $g = \frac{10}{4}$; $o = \frac{1000}{4 \cdot 64} - \frac{216\lambda''}{64} - \frac{10 \cdot 4}{4 \cdot 64}\nu$ et $\lambda'' = \frac{1000 - 40\nu}{4 \cdot 216} = \frac{125 - 5\nu}{4 \cdot 27}$

In primo et secundo casu fit valor ipsius λ'' nimis magnus, quam vt ista lens commode in praxin recipi queat; ac si ipsi g multo maior tribuatur valor, leuis error ingentem effectum producit. Ponamus enim loco g per errorem sumi $g + \omega$, et cum λ'' ex g rite fuerit definitum; fiet

$$\lambda^{(1)} = \omega\left(\tfrac{3}{4}gg - 3\lambda''(1-g)^2 - \nu(1-\tfrac{1}{2}g)(1-\tfrac{3}{2}g)\right)$$

et

et pro λ'' ſubſtituto valore:

$$\lambda^{(3)} = \frac{\omega}{4(1-g)}(3gg - \nu(4-gg))$$

vnde patet, quo minus g vnitatem excedat, ac ſimul quo maius fuerit g, valorem ipſius $\lambda^{(3)}$ ob errorem ω eo fieri maiorem Intelligitur autem hunc errorem fieri minimum ſi capiatur

$g = 1 + \sqrt{\frac{2-\nu\nu}{\nu+2}}$, ex hoc autem valore elicitur,

$$\lambda'' = \frac{(2+\nu)\sqrt{2(1-\nu)(2+\nu)} + 2(2+\nu\nu)}{g(1-\nu)}$$

Capi ergo debebit $g = 1,84384$ vnde colligitur:

$$\lambda'' = \frac{g^3 - \nu g(2-g)^2}{4(g-1)^2} = 2,60372$$

et ſi hae menſurae exacte obſeruentur, fiet $\lambda^{(3)} = 0$. At ſi in valore ipſius g particula ω aberretur, vt ſit $g = 1,84384 + \omega$, prodibit ob hunc errorem:

$$\lambda^{(3)} = -2,981\ \omega,$$

ita ſi eſſet error $\omega = \pm\frac{1}{10}$, loco $\lambda^{(3)} = 0$, prodiret:

$\lambda^{(3)} = \mp 0,2981$ ideoque lens triplicata poſtponenda duplicatae, longe tamen praeferenda foret ſimplici.

In genere igitur pro quouis valore alio ipſius $\lambda^{(3)}$ idem commodum inueſtigemus: ac primo cum ſit

$$\lambda'' = \frac{\lambda^{(3)}g^3 + \nu g(g-1)}{(g-1)^3}$$

ſi loco iuſti valoris g capiatur $g+\omega$ fiet

$$\lambda^{(3)} = \omega(3\lambda^{(3)}g^2 - 3\lambda''(g-1)^2 - \nu + 2\nu g)$$

vbi ſi pro λ'' valor ſubſtituatur, erit

$$\lambda^{(3)} = \frac{\omega(\nu - (3\lambda^{(3)} + \nu)gg)}{g-1}$$

 qui

qui error vt minimus reddatur capi debet

$$g = 1 + \sqrt{\frac{3\lambda^{(2)}}{3\lambda^{(2)}+\nu}}$$

qui est valor maxime idoneus pro g sumendus, ex quo elicitur

$$\lambda'' = \frac{1}{3\lambda^{(2)}}\left(\sqrt{(3\lambda^{(2)}+\nu)}+\sqrt{3\lambda^{(2)}}\right)\left(\tfrac{1}{3}(6\lambda^{(2)}+\nu)\sqrt{3\lambda^{(2)}}+(2\lambda^{(2)}+\nu)\sqrt{(3\lambda^{(2)}+\nu)}\right)$$

et sumto per errorem $g+\omega$ pro g erit

$$\lambda^{(3)} = -2\omega\left(3\lambda^{(2)}+\nu+\sqrt{3\lambda^{(2)}(3\lambda^{(2)}+\nu)}\right) \text{ seu}$$

$$\lambda^{(3)} = -2\omega\left(\sqrt{3\lambda^{(2)}}+\sqrt{(3\lambda^{(2)}+\nu)}\right)\sqrt{(3\lambda^{(2)}+\nu)}$$

Quo minor igitur iam fuerit valor ipsius $\lambda^{(2)}$ eo minus erit error metuendus, vnde solutio ante ex valore $\lambda^{(2)} = \frac{1-\nu}{4}$ eruta prae ceteris est commendanda. Tum autem erit $f = \frac{1}{2}$; $\lambda = 1$, $\lambda' = 1$, $\lambda'' = 2,60372$ et $g = 1,84384$, vnde pro lentium constructione habemus,

$$\frac{1}{\alpha} = -\frac{1}{b} = \frac{-1+g}{2a} + \frac{g}{2\gamma}$$

$$\frac{1}{\beta} = -\frac{1}{c} = \frac{-1+g}{a} + \frac{g}{\gamma}$$

vnde singulae lentes per formulas §. 91 construentur.

Coroll. 1.

146. Lens ergo prima PP construi debet ex distantiis determinatricibus a et $\frac{2a\gamma}{ga-(1-g)\gamma}$ cum numero $\lambda = 1$

Lens vero secunda QQ ex distantiis determinatricibus $\frac{-2a\gamma}{ga-(1-g)\gamma}$ cum numero $\lambda' = 1$

at

at lens tertia RR ex diſtantiis determinatricibus

$$\frac{-a\gamma}{ga-(1-g)\gamma} \text{ et } \gamma \text{ cum numero } \lambda'' = 2,60372$$

exiſtente $g = 1,84384$.

Coroll. 2.

147. Quoniam in formula ſpatium diffuſionis exprimente quae ob $\lambda^{(3)} = 0$ eſt $\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\frac{\nu}{a\gamma}$, reiecto factore primo $\mu\gamma\gamma xx$, cuius ratio in his inueſtigationibus non eſt habita, diſtantiae a et γ inter ſe permutari poſſunt; hinc etiam alia lens triplicata quaeſito aeque ſatisfaciens exhiberi poterit.

Coroll. 3.

148. Nempe pro hac altera lente triplicata lens prima PP conſtrui debet ex diſtantiis determinatricibus

$$a \text{ et } \frac{+a\gamma}{(1-g)a-g\gamma} \text{ cum numero } \lambda = 2,60372$$

Lens ſecunda QQ ex diſtantiis determinatricibus

$$\frac{-a\gamma}{(1-g)a-g\gamma} \text{ et } \frac{+2a\gamma}{(2-g)a-g\gamma} \text{ cum numero } \lambda' = 1$$

at lens tertia ex diſtantiis determinatricibus

$$\frac{-2a\gamma}{(2-g)a-g\gamma} \text{ et } \gamma \text{ cum numero } \lambda'' = 1$$

exiſtente vt ante $g = 1,84384$.

Coroll. 4.

149. Hinc ergo duas nacti ſumus lentes triplicatas pro diſtantiis a et γ, quae producunt ſpatium diffu-

diffusionis $Hb = \mu\gamma\gamma xx(\frac{1}{a}+\frac{1}{\gamma})\frac{v}{a\gamma}$. Atque hae inter infinitas alias eundem effectum praestantes hac gaudent praerogatiua, vt leuis error in constructione commissus scopum minime perturbet.

Coroll. 5.

150. Si in constructione harum lentium per errorem numerus g parumper maior accipiatur, quam 1, 84384, tum pro lente triplicata numerus $\lambda^{(3)}$ prodit nihilo minor seu negatiuus. Sin autem numerus g in praxi aliquantillum maior sumatur numerus $\lambda^{(3)}$ fit nihilo maior, sicque lens triplicata ad naturam duplicatarum accedet.

Coroll. 6.

151. Si ergo opus fuerit lente, pro qua numerus λ valorem habeat negatiuum, huic scopo satisfieri commode poterit per lentes descriptas triplicatas, dummodo pro g numerus aliquanto maior quam 1, 84384 assumatur. Scilicet si sumatur

$$g = 1,\ 84384 + \omega \text{ fiet } \lambda^{(3)} = -2,\ 981\ \omega.$$

Supplementum II.

De lentibus triplicatis

Si pro singulis lentibus refractio sit diuersa, pro prima $n : 1$ pro secunda $n' : 1$, pro tertia n''

n'' : 1, radiique facierum lentium ſequenti modo definiantur

	Diſt. determinatrices	Refractio et litterae inde pendentes	
I.	a et α	n : 1, μ, ν, ρ, σ, τ.	λ
II.	b et β	n' : 1, μ', ν', ρ', σ', τ'.	λ'
III.	c et γ	n'' : 1, μ'', ν'', ρ'', σ'', τ''.	λ''

ſcilicet ſi pro lente prima vocetur radius faciei anterioris $= F$; poſterioris $= G$; erit

$$\frac{1}{F} = \frac{\rho}{a} + \frac{\sigma}{\alpha} \mp \tau\left(\frac{1}{a} + \frac{1}{\alpha}\right)\sqrt{\lambda - 1}$$

$$\text{et } \frac{1}{G} = \frac{\rho}{\alpha} + \frac{\sigma}{a} \pm \tau\left(\frac{1}{a} + \frac{1}{\alpha}\right)\sqrt{\lambda - 1}$$

ſimilique modo pro reliquis lentibus, nempe pro ſecunda

$$\frac{1}{F'} = \frac{\rho'}{b} + \frac{\sigma'}{\beta} \mp \tau'\left(\frac{1}{b} + \frac{1}{\beta}\right)\sqrt{\lambda' - 1}$$

$$\frac{1}{G'} = \frac{\rho'}{\beta} + \frac{\sigma'}{b} \pm \tau'\left(\frac{1}{b} + \frac{1}{\beta}\right)\sqrt{\lambda' - 1}$$

tum vero pro tertia

$$\frac{1}{F''} = \frac{\rho''}{c} + \frac{{}''}{\gamma} \mp \tau''\left(\frac{1}{c} + \frac{1}{\gamma}\right)\sqrt{\lambda'' - 1}$$

$$\frac{1}{G''} = \frac{\rho''}{\gamma} + \frac{\sigma''}{c} \pm \tau''\left(\frac{1}{c} + \frac{1}{\gamma}\right)\sqrt{\lambda'' - 1}$$

Deinde quia diſtantiae lentium pro nihilo habentur ſcilicet $a + b = 0$ et $\beta + c = 0$, erit ſpatium diffuſionis

$$= \gamma\gamma xx \left\{ \begin{array}{l} + \mu\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{\nu}{a\alpha}\right) \\ + \mu'\left(\frac{1}{b} + \frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b} + \frac{1}{\beta}\right)^2 + \frac{\nu'}{b\beta}\right) \\ + \mu''\left(\frac{1}{c} + \frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c} + \frac{1}{\gamma}\right)^2 + \frac{\nu''}{c\gamma}\right) \end{array} \right.$$

ſtatuatur nunc

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)$$
$$\frac{1}{b}+\frac{1}{\beta}=g\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)$$
$$\frac{1}{c}+\frac{1}{\gamma}=h\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)$$

vt fiat

$$\frac{1}{a}=\frac{f-1}{\alpha}+\frac{f}{\gamma}=\frac{-1}{b};\quad \text{et}$$
$$\frac{1}{c}=\frac{h}{\alpha}+\frac{h-1}{\gamma}=\frac{-1}{\beta}$$

vnde fit

$$\frac{1}{b}+\frac{1}{\beta}=\frac{1-f-h}{\alpha}+\frac{1-f-h}{\gamma}=g\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)$$

hincque $f+g+h=1$; quibus valoribus ſubſtitutis erit primo pro radiis facierum

$$\frac{1}{F}=\frac{\rho+(f-1)\sigma}{\alpha}+\frac{\sigma f}{\gamma}\mp\tau f\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)\sqrt{\lambda-1}$$

$$\frac{1}{G}=\frac{\sigma+(f-1)\rho}{\alpha}+\frac{\rho f}{\gamma}\pm\tau f\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)\sqrt{\lambda-1}$$

$$\frac{1}{F'}=\frac{(1-f)\rho'-h\sigma'}{\alpha}+\frac{(1-h)\sigma'-f\rho'}{\gamma}\mp\tau' g\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)\sqrt{\lambda'-1}$$

$$\frac{1}{G'}=\frac{(1-f)\sigma'-h\rho'}{\alpha}+\frac{(1-h)\rho'-f\sigma'}{\gamma}\pm\tau' g\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)\sqrt{\lambda'-1}$$

et pro lente tertia

$$\frac{1}{F''}=\frac{h.\rho''}{\alpha}+\frac{(h-1)\rho''+\sigma''}{\gamma}\mp\tau''.h\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$

$$\frac{1}{G''}=\frac{h.\sigma''}{\alpha}+\frac{(h-1)\sigma''+\rho''}{\gamma}\pm\tau''.h\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$

Spatium

Spatium vero diffuſionis iam ita exprimetur

$$\gamma\gamma\, xx\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)\left\{\begin{array}{l}\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)^2(\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda''.b^3)\\ +\tfrac{\nu\mu f}{a}\left(\tfrac{f-1}{a}+\tfrac{f}{\gamma}\right)\\ +\nu'\mu' g\left(\tfrac{1-f}{a}-\tfrac{f}{\gamma}\right)\left(\tfrac{1-b}{\gamma}-\tfrac{b}{a}\right)\\ +\tfrac{\nu'\mu''.b}{\gamma}\left(\tfrac{b}{a}+\tfrac{b-1}{\gamma}\right)\end{array}\right.$$

Nunc igitur circa has lentes triplicatas ſequentia ſunt obſeruanda

I. Diuerſa media refringentia eum tantum in finem adhiberi ſolent, vt non ſolum ſpatium diffuſionis hic determinatum ad nihilum redigatur, ſed etiam confuſio a diuerſa radiorum refrangibilitate oriunda tollatur, quippe quod per lentes eiusdem refractionis obtineri nequit. Infra autem videbimus ad hanc conditionem implendam requiri, vt ſit $\zeta f+\eta g+\vartheta b=0$, exiſtente $\zeta=\frac{dn}{n-1}$; $\eta=\frac{dn'}{n'-1}$ et $\vartheta=\frac{dn''}{n''-1}$; ex quibus formis iam perſpicitur, ſi haec differentialia dn, dn' et dn'' eſſent ipſis $n-1$; $n'-1$; $n''-1$ proportionalia, vti *Newtonus* ſtatuerat; tum proditura eſſe $\zeta=\eta=\vartheta$ ſiue $f+g+b=0$; at iam vidimus, eſſe debere $f+g+b=1$; quare ſi *Newtoni* ſententia eſſet vera; tum ne quidem diuerſis refractionibus adhibendis diffuſioni a diuerſa refrangibilitate oriundae remedium adferri poſſet. Eatenus igitur tantum hoc incommodum vitari poterit, quatenus litterae ζ, η et ϑ ſunt diuerſae, ita, vt ſimul eſſe poſſit, et $f+g+b=1$ et $\zeta f+\eta g+\vartheta b=0$ ex quo perſpicuum eſt, quantitatum f, g et b vnam vel adeo duas esse debere negatiuas sicque hac conditione superaddita casus ille principalis, quo omnes tres litterae f, g et b positiuae sunt assumtae hic locum inuenire nequit.

II. Vt ergo nostrum spatium diffusionis euanescat, satisfaciendum est huic aequationi

$$\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)^2 (\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' b^3)$$
$$+\tfrac{\nu\mu f}{a}\left(\tfrac{f-1}{a}+\tfrac{f}{\gamma}\right)$$
$$+\nu'\mu' g\left(\tfrac{1-f}{a}-\tfrac{f}{\gamma}\right)\left(\tfrac{1-b}{\gamma}-\tfrac{b}{a}\right)$$
$$+\tfrac{\nu''\mu'' b}{\gamma}\left(\tfrac{b}{a}+\tfrac{b-1}{\gamma}\right)=0$$

Vnde vel λ vel λ' vel λ'' quaeri potest, dummodo caueatur, ne valor vel vnitate minor vel nimis magnus prodeat, quia prius naturae repugnat; alterum autem, quia constructio lentium fieret nimis lubrica. Hoc autem praestito circa binas reliquas litteras λ nihil amplius definitur neque etiam circa litteras f, g et b, praeterquam quod supra allatis conditionibus $f+g+b=1$ et $\zeta f+\eta g+\vartheta b=0$ continetur, vnde ex data vna harum litterarum duae reliquae sponte definiuntur sequenti scilicet modo:

$$g=\frac{(\vartheta-\zeta)f-\vartheta}{\eta-\vartheta}; \text{ et}$$
$$b=\frac{(\zeta-\eta)f+\eta}{\eta-\vartheta}$$

III. In praxi autem huiusmodi lentes triplicatae ideo potissimum quaeruntur, vt loco lentis obiectiuae

ctiuae in telescopiis substitui queant, pro quibus est $a=\infty$. Statuamus ergo statim $a=\infty$ et pro radiis facierum singularum lentium habebimus

$$\frac{1}{F}=\frac{\sigma f}{\gamma}\mp\frac{\tau f}{\gamma}\sqrt{\lambda-1}\,;\quad \frac{1}{G}=\frac{\rho f}{\gamma}\pm\frac{\tau f}{\gamma}\sqrt{\lambda-1}$$

pro secunda lente

$$\frac{1}{F'}=\frac{(1-b)\sigma'-f\rho'}{\gamma}\mp\frac{\tau' g}{\gamma}\sqrt{\lambda'-1}\,;\quad \frac{1}{G'}=\frac{(1-b)\rho'-f\sigma'}{\gamma}\pm\frac{\tau' g}{\gamma}\sqrt{\lambda'-1}$$

pro tertia lente

$$\frac{1}{F''}=\frac{(b-1)\rho''+\sigma''}{\gamma}\mp\frac{\tau''.b}{\gamma}\sqrt{\lambda''-1}\,;\quad \frac{1}{G''}=\frac{(b-1)\sigma''+\rho''}{\gamma}\pm\frac{\tau''.b}{\gamma}\sqrt{\lambda''-1}$$

Spatium autem diffusionis tum ita exprimetur

$$\frac{x^2}{\gamma}(\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' b^3-\nu'\mu'.fg(1-b)-\nu''\mu''.b(1-b)$$

Ita vt satisfieri oporteat huic aequationi

$$\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' b^3-\nu'\mu'.fg(1-b)-\nu''\mu''.b(1-b)=0.$$

Vnde vnus valorum λ, λ', λ'', qui ad vsum commodissimus videtur, determinari debet.

IV. Si tantum lentibus vitreis vti velimus, sufficiet duas tantum vitri species adhiberi; si igitur statuamus, lentem tertiam et primam ex eadem vitri specie parari, vt $\mu''=\mu$; $\nu''=\nu$; $\rho''=\rho$; $\sigma''=\sigma$; $\tau''=\tau$ et $\zeta=\vartheta$ ob $n''=n$; pro litteris autem f, g et b hae determinationes habebuntur, 1°. ex aequatione

$$\zeta f+\eta g+\zeta b=0 \text{ fit } f+b=\frac{-\eta}{\zeta}.g,$$

quo valore substituto fiet

$$g\frac{(\zeta-\eta)}{\zeta}=1\,;\quad g=\frac{\zeta}{\zeta-\eta} \text{ ideoque } f+b=\frac{-\eta}{\zeta-\eta}\,;$$

vnde

vnde patet, prouti littera g fuerit vel positiua vel negatiua, fore vicissim summam $f+b$ vel negatiuam vel positiuam et aequatio resoluenda iam erit

$$\mu(\lambda f^3+\lambda'' b^3)+\mu'\lambda' g^3-\tfrac{\mu'\nu'.\zeta}{\zeta-\eta}.f(1-b)-\mu\nu.b(1-b)=0$$

Hinc igitur elicitur

$$\mu'\lambda' g^3=-\mu(\lambda f^3+\lambda'' b^3)+\tfrac{\mu'\nu'.\zeta}{\zeta-\eta}.f(1-b)+\mu\nu.b(1-b)$$

cui facile erit pro casu quouis proposito satisfacere.

IV. Dum lens prima et tertia ex eadem vitri specie parantur, media constet ex aqua vel alia materia fluida, vt lens triplicata intra duas lentes vitreas contineat fluidum, et quia fluidum plerumque minorem refractionem patitur, quam vitrum; erit $n' < n$ indeque porro $\eta < \zeta$. Quia igitur pro hoc casu fit g positiuum seu lens aquea conuexa, lentes vitreae vel ambae vel vna saltem debent esse concauae.

Inprimis autem praeter determinationes iam inventas necesse est, vt radius faciei anterioris pro lente media aequalis et contrarius sit radio faciei posterioris lentis primae, eodemque modo radius faciei posterioris aequalis et contrarius radio faciei anterioris lentis tertiae vnde hae aequalitates nascentur

$$\tfrac{1}{F'}=\tfrac{-1}{G}, \text{ seu } F'+G=0; \text{ et } \tfrac{1}{G'}=\tfrac{-1}{F''} \text{ seu } F''+G'=0.$$

Ideoque satisfieri oportet istis aequationibus:

$$(1-b)\sigma'-f\varrho' \mp \tau' g.\sqrt{\lambda'-1}=-\varrho f \mp \tau f\sqrt{\lambda-1} \text{ et}$$

$$(1-b)\varrho'-f\sigma' \pm \tau'.g.\sqrt{\lambda'-1}=(1-b)\varrho-\sigma \mp \tau'' b\sqrt{\lambda''-1}$$

En

En igitur duas conditiones, quibus satisfieri oportet, vnde vel numeri λ et λ'' vel alter eorum cum alterutra litterarum f et h definiri debent; quem in finem probe obseruandum est, formulas $\sqrt{\lambda - 1}$ et $\sqrt{\lambda'' - 1}$ pro lubitu siue positiuas siue negatiuas assumi posse, neque a se inuicem pendere. Pro formula autem $\sqrt{\lambda' - 1}$ notandum est, si ea in priore aequatione positiue ponatur, in posteriore necessario negatiue sumi debere et viciſſim.

Problema 8.

152. Determinare eas lentes quadruplicatas ad datas distantias determinatrices $AE = a$ et $d\,I = \delta$ accommodatas, quae minimum spatium diffusionis Ii producant. Tab. II. Fig. 9.

Solutio.

Prima lens PP ad distantias determinatrices $AE = a$ et $AF = \alpha$ cum numero λ construatur, secunda QQ ad distantias $b = -\alpha$ et $AG = \mathfrak{G}$ cum numero λ', tertia RR ad distantias $c = -\mathfrak{G}$ et $AH = \gamma$ cum numero λ'': et quarta SS ad distantias $d = -\gamma$ et $dI = \delta$ cum numero λ''' construatur: vbi scilicet crassitiem lentium vt euanescentem spectamus. Posito iam semidiametro aperturae lentis x, sola prima lens PP produceret spatium diffusionis

$$Ff = \mu\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

Adiun-

Adiuncta autem secunda lente QQ, positoque

$$\frac{1}{\alpha}=\frac{-1+f}{a}+\frac{f}{\beta} \text{ et } \frac{1}{b}=\frac{1-f}{a}-\frac{f}{\beta}$$

si breuitatis gratia statuamus

$$\lambda^{(2)}=\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)$$

vidimus fore spatium diffusionis:

$$Gg=\mu\beta\beta xx\left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\lambda^{(2)}\left(\frac{1}{a}+\frac{1}{\beta}\right)^2+\frac{\nu}{a\beta}\right)$$

Adiungatur insuper tertia lens RR, numerusque g ita sumatur vt sit

$$\frac{1}{\beta}=\frac{-1+g}{a}+\frac{g}{\gamma} \text{ et } \frac{1}{c}=\frac{1-g}{a}-\frac{g}{\gamma}$$

ac si breuitatis ergo ponamus:

$$\lambda^{(3)}=\lambda^{(2)}g^3+\lambda''(1-g)^3-\nu g(1-g)$$

erit spatium diffusionis:

$$Hh=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$$

Nunc denique adiungatur lens quarta SS, et numero h ita in calculum introducto, vt sit

$$\frac{1}{\gamma}=\frac{-1+h}{a}+\frac{h}{\delta} \text{ et } \frac{1}{d}=\frac{1-h}{a}-\frac{h}{\delta}$$

si simili modo ponamus

$$\lambda^{(4)}=\lambda^{(3)}h^3+\lambda'''(1-h)^3-\nu h(1-h)$$

erit spatium diffusionis a lente quadruplicata productum:

$$Ii=\mu\delta\delta xx\left(\frac{1}{a}+\frac{1}{\delta}\right)\left(\lambda^{(4)}\left(\frac{1}{a}+\frac{1}{\delta}\right)^2+\frac{\nu}{a\delta}\right)$$

quod igitur minimum reddi debet. Hunc in finem considerentur numeri λ, λ', λ'' et λ''' vt dati, et quaerantur mantur idonei valores pro numeris f, g, et h: atque vt valor $\lambda^{(4)}$ minimus euadat, necesse est quoque valores $\lambda^{(3)}$ et $\lambda^{(2)}$ minimos fieri. Incipiamus ergo a valore $\lambda^{(2)}$ qui minimus redditur sumendo

$$f=\frac{\sqrt{(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)}+\sqrt{(3\lambda'+\nu)}} \text{ seu } \frac{1}{f}=1+\sqrt{\frac{3\lambda+\nu}{3\lambda'+\nu}}$$

vnde fit

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}.$$

Deinde numerus $\lambda^{(3)}$ minimum induet valorem capiendo

$$g=\frac{\sqrt{(3\lambda''+\nu)}}{\sqrt{(3\lambda^{(2)}+\nu)}+\sqrt{(3\lambda''+\nu)}} \text{ seu } \frac{1}{g}=1+\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda''+\nu}}$$

hincque colligitur

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

Denique numerus $\lambda^{(4)}$ ideoque et spatium diffusionis Ii minimum efficietur sumendo

$$h=\frac{\sqrt{(3\lambda'''+\nu)}}{\sqrt{(3\lambda^{(3)}+\nu)}+\sqrt{(3\lambda'''+\nu)}} \text{ seu } \frac{1}{h}=1+\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda'''+\nu}}$$

vnde obtinetur

$$\frac{1}{\sqrt{(3\lambda^{(4)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}+\frac{1}{\sqrt{(3\lambda'''+\nu)}}.$$

Quod si hic valores ante inuentos substituamus, nanciscemur

$$\frac{1}{\sqrt{(3\lambda^{(4)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}+\frac{1}{\sqrt{(3\lambda'''+\nu)}}.$$

Pro praecedentibus vero erit

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}};$$

et

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}.$$

Tum vero ex his porro consequimur:

$$f=\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}},\ g=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}},\ h=\sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(3)}+\nu}}$$

Superest ergo vt constructionem singularum lentium luculentius exponamus: et earum distantias determinatrices per solas propositas a et δ exprimamus: erit igitur

$$\frac{1}{\gamma}=\frac{-1+h}{a}+\frac{h}{\delta};\quad \frac{1}{d}=\frac{1-h}{a}-\frac{h}{\delta}$$

$$\frac{1}{\varepsilon}=\frac{-1+gh}{a}+\frac{gh}{\delta};\quad \frac{1}{c}=\frac{1-gh}{a}-\frac{gh}{\delta}$$

$$\frac{1}{\alpha}=\frac{-1+fgh}{a}+\frac{fgh}{\delta};\quad \frac{1}{b}=\frac{1-fgh}{a}-\frac{fgh}{\delta}$$

Ex superioribus vero formulis colligitur:

$$h=\sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(3)}+\nu}};\ g=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}};\ f=\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}}$$

vnde

vnde fit

$$g\,h = \surd\frac{3\lambda^{(4)}+\nu}{3\lambda^{(3)}+\nu} \text{ et } fg\,h = \surd\frac{3\lambda^{(4)}+\nu}{3\lambda+\nu}.$$

et superiores valores ita exprimi poterunt

$$\frac{1}{\alpha} = -\frac{1}{b} = -\frac{\surd(3\lambda^{(4)}+\nu)}{a}\left(\frac{1}{\surd(3\lambda'+\nu)} + \frac{1}{\surd(3\lambda''+\nu)} + \frac{1}{\surd(3\lambda'''+\nu)}\right) + \frac{\surd(3\lambda^{(4)}+\nu)}{\delta}\cdot\frac{1}{\surd(3\lambda+\nu)}.$$

$$\frac{1}{6} = -\frac{1}{c} = -\frac{\surd(3\lambda^{(4)}+\nu)}{a}\left(\frac{1}{\surd(3\lambda''+\nu)} + \frac{1}{\surd(3\lambda'''+\nu)}\right) + \frac{\surd(3\lambda^{(4)}+\nu)}{\delta}\left(\frac{1}{\surd(3\lambda+\nu)} + \frac{1}{\surd(3\lambda'+\nu)}\right)$$

$$\frac{1}{\gamma} = -\frac{1}{d} = -\frac{\surd(3\lambda^{(4)}+\nu)}{a}\cdot\frac{1}{\surd(3\lambda'''+\nu)} + \frac{\surd(3\lambda^{(4)}+\nu)}{\delta}\left(\frac{1}{\surd(3\lambda+\nu)} + \frac{1}{\surd(3\lambda'+\nu)} + \frac{1}{\surd(3\lambda''+\nu)}\right).$$

Coroll. I.

153. Si pro lentibus simplicibus numeri λ, λ' λ'' et λ''' sumantur inter se aequales, fiet pro minimo spatio diffusionis:

$$\surd(3\lambda^{(2)}+\nu) = \tfrac{1}{2}\surd(3\lambda+\nu);\quad \lambda^{(2)} = \frac{3\lambda - 1.3\nu}{3.4}$$

$$\surd(3\lambda^{(3)}+\nu) = \tfrac{2}{3}\surd(3\lambda+\nu);\quad \lambda^{(3)} = \frac{3\lambda - 2.4\nu}{3.9}$$

$$\surd(3\lambda^{(4)}+\nu) = \tfrac{3}{4}\surd(3\lambda+\nu);\quad \lambda^{(4)} = \frac{3\lambda - 3.5\nu}{3.16}$$

hinc $f = \frac{1}{2}$; $g = \frac{2}{3}$; $h = \frac{3}{4}$, et pro constructione lentium simplicium:

$$\frac{1}{\alpha} = -\frac{3}{4a} + \frac{1}{4\delta};\quad \frac{1}{b} = +\frac{3}{4a} - \frac{1}{4\delta}$$

$$\frac{1}{6} = -\frac{2}{4a} + \frac{2}{4\delta};\quad \frac{1}{c} = +\frac{2}{4a} - \frac{2}{4\delta}$$

$$\frac{1}{\gamma} = -\frac{1}{4a} + \frac{3}{4\delta};\quad \frac{1}{d} = +\frac{1}{4a} - \frac{3}{4\delta}$$

Coroll.

Coroll. 2.

154. Hinc ex §. 91 sequens quatuor lentium simplicium constructio obtinetur:

Pro lente radius faciei

Prima PP $\begin{cases} \text{anterioris} = \dfrac{4\alpha\delta}{(4\varrho - 3\sigma)\delta + \sigma\alpha \pm \tau(\alpha+\delta)\surd(\lambda-1)} \\ \text{posterioris} = \dfrac{4\alpha\delta}{(4\sigma - 3\varrho)\delta + \varrho\alpha \mp \tau(\alpha+\delta)\surd(\lambda-1)} \end{cases}$

Secunda QQ $\begin{cases} \text{anterioris} = \dfrac{4\alpha\delta}{(3\varrho - 2\sigma)\delta + (2\sigma - \varrho)\alpha \pm \tau(\alpha+\delta)\surd(\lambda-1)} \\ \text{posterioris} = \dfrac{4\alpha\delta}{(3\sigma - 2\varrho)\delta + (2\varrho - \sigma)\alpha \mp \tau(\alpha+\delta)\surd(\lambda-1)} \end{cases}$

Tertia RR $\begin{cases} \text{anterioris} = \dfrac{4\alpha\delta}{(2\varrho - \sigma)\delta + (3\sigma - 2\varrho)\alpha \pm \tau(\alpha+\delta)\surd(\lambda-1)} \\ \text{posterioris} = \dfrac{4\alpha\delta}{(2\sigma - \varrho)\delta + (3\varrho - 2\sigma)\alpha \pm \tau(\alpha+\delta)\surd(\lambda-1)} \end{cases}$

Quarta SS $\begin{cases} \text{anterioris} = \dfrac{4\alpha\delta}{\varrho\delta + (4\sigma - 3\varrho)\alpha \pm \tau(\alpha+\delta)\surd(\lambda-1)} \\ \text{posterioris} = \dfrac{4\alpha\delta}{\sigma\delta + (4\varrho - 3\sigma)\alpha \mp \tau(\alpha+\delta)\surd(\lambda-1)} \end{cases}$

Coroll. 3.

155. Si praeterea numeri $\lambda, \lambda', \lambda'', \lambda'''$ vnitati aequales statuantur, qui est valor minimus, quem recipere possunt erit: ob $\nu = 0,232692$

$$\lambda' = \frac{1 - 7\nu}{3 \cdot 4} = 0,191827$$

$$\lambda'' = \frac{1 - 8\nu}{3 \cdot 9} = 0,042165$$

$$\lambda''' = \frac{1 - 15\nu}{3 \cdot 16} = -0,010216$$

sicque pro lente quadruplicata valor numeri λ''' adeo infra nihilum deprimitur.

Coroll.

Coroll. 4.

156. Maiorem ergo valorem numeris $\lambda, \lambda', \lambda'', \lambda'''$ tribuendo effici poterit, vt valor ipsius $\lambda^{(*)}$ praecise nihilo aequalis prodeat; quippe hoc fiet sumendo $\lambda = 5\nu = 1, 163460$. Hinc habebitur $\lambda - 1 = 0, 163460$ et $\tau \surd (\lambda - 1) = 0, 365947$, vnde singulae lentes simplices duplici modo per formulas exhibitas construi poterunt.

Scholion 1.

157. Si inter se comparemus hos duos casus, quibus est vel $\lambda^{(*)} = -0,010216$ vel $\lambda^{(*)} = 0$, videmus, etiamsi discrimen vix partem centesimam vnitatis superet, in constructione tamen lentium simplicium satis magnum discrimen deprehendi, cum denominatores formularum, (154) siue augeri siue diminui debeant quantitate: $0, 365947\ (a + \delta)$; quae differentia maior est, quam errores, qui forte ab artifice non nimis rudi committi queant. Ex quo vicissim colligimus, etiamsi in constructione harum lentium quadruplicatarum ab artifice leues errores committantur, inde vix perceptibilem effectum in spatio diffusionis vel valore ipsius $\lambda^{(*)}$ esse metuendum, quam ob causam hae lentes imprimis ad praxin accommodatae videntur. Si scilicet opus fuerit lente, pro qua valor ipsius λ in nihilum abeat, multo magis his lentibus quadruplicatis Coroll. 4 descriptis erit vtendum, quam triplicatis, quas supra definiui. Quin etiam eas lentes quadruplicatas adhibere licebit,

in quibus eſt $\lambda^{(4)} = -0,010216$, quia hic numerus vix nihilo eſt minor; ac ſi in conſtructione a praeſcriptis menſuris aberretur, ille valor adhuc propius ad nihilum perducatur, ita vt hoc caſu adeo errores commiſſi ſcopo magis attingendo inſeruiant. Infra autem videbimus, plurimos dari caſus, quibus eiusmodi lentibus vti conueniat, pro quibus valor ipſius λ non ſolum ſit nihilo aequalis, ſed etiam tantillum infra nihilum deprimatur; tunc optimo cum ſucceſſu huiusmodi lentes quadruplicatas in vſum vocabimus. Sin autem eiusmodi lentes ſufficiant, pro quibus valor ipſius λ ſit $0,042165$ vel aliquantillum excedat, lentes triplicatae erunt commendandae, quarum conſtructionem ſupra (§. 136) dedimus; quemadmodum duplicatae negotium conficient ſi non opus fuerit minore valore ipſius λ quam $0,191827$. Tota autem inſtrumentorum dioptricorum perfectio in hoc maxime conſiſtit, vt lentes habeantur, pro quibus valor ipſius λ ſit quam minimus, cum eae ſint aptiſſimae ad confuſionem penitus tollendam; ex quo lentium quadruplicatarum hic deſcriptarum vſus erit ampliſſimus.

Scholion 2.

158. Si haec, quae de lentibus quadruplicatis hic tradidimus, attente conſiderentur, facile patebit, quomodo lentes quintuplicatae magisque multiplicatae ad vſum ſint accommodandae. Eiusmodi ſcilicet ſemper conſtructione erit opus, quae a natura minimi parumper

rumper recedat, quoniam hoc modo errores in praxi commissi effectum propositum minime perturbant. Cum autem vix vnquam vsu veniat, vt lentibus opus sit, pro quibus valor numeri λ magis vltra nihilum imminuatur, superfluum foret, constructionem lentium quintuplicatarum magisue multiplicatarum vlterius prosequi. Interim tamen iuuabit, si pro huiusmodi lentibus numeri λ lentium numero conuenienter signis $\lambda^{(5)}$ $^{(6)}$ etc. indicentur, eorum valores, quos ex natura minimi recipiunt, exposuisse. Sumamus omnes numeros λ, λ', λ'', λ''', λ'''', λ''''' etc. lentibus simplicibus respondentes vnitati aequales, et eorum qui lentibus multiplicatis conueniunt ex natura minimi, ita se habebunt.

Pro Lente

Solitaria . . . $\lambda^{(1)} = \frac{3 - 0\nu}{3 \cdot 1} = 1,000000$

Duplicata . . . $\lambda^{(2)} = \frac{3 - 3\nu}{3 \cdot 4} = 0,191827$

Triplicata . . . $\lambda^{(3)} = \frac{3 - 8\nu}{3 \cdot 9} = 0,042165$

Quadruplicata $\lambda^{(4)} = \frac{3 - 15\nu}{3 \cdot 16} = -0,010216$

Quintuplicata $\lambda^{(5)} = \frac{3 - 24\nu}{3 \cdot 25} = -0,034461$

Sextuplicata $\lambda^{(6)} = \frac{3 - 35\nu}{3 \cdot 36} = -0,047632$

Septuplicata $\lambda^{(7)} = \frac{3 - 48\nu}{3 \cdot 49} = -0,055573$

Octuplicata $\lambda^{(8)} = \frac{3 - 63\nu}{3 \cdot 64} = -0,060727$

Noncuplicata $\lambda^{(9)} = \frac{3 - 80\nu}{3 \cdot 81} = -0,064261$

Decuplicata $\lambda^{(10)} = \frac{3 - 99\nu}{3 \cdot 100} = -0,066788$

ac si huiusmodi lentes in infinitum multiplicentur; valor numeri respondentis $\lambda^{(\infty)}$ erit $= -\frac{1}{3} = -0,07564$ ita vt nunquam infra hunc numerum deprimi possit: ex quo patet vix vnquam casum existere posse, quo lente saltem quintuplicata opus esset. Ceterum etiam in genere, quicunque alii valores praeter vnitatem, numeris λ, λ', λ'', λ''' etc. tribuantur, ex formulis superioribus numeri $\lambda^{(5)}$, $\lambda^{(6)}$ etc. facile deriuabuntur; quin etiam distantiae determinatrices singularum lentium simplicium indidem sine difficultate definientur, cum lex progressionis satis sit manifesta. Verum per totum hoc caput probe tenendum est, vbique crassitiem lentium tanquam euanescentem esse consideratam; in sequente autem capite iterum lentes in genere non neglecta crassitie sumus contemplaturi.

CAPVT

CAPVT IV.

DE CONFVSIONE VISIONIS NEC NON DE MAGNITVDINE APPARENTE ET CLARITATE.

Definitio 1.

159.

Visio est distincta, si omnes radii, qui ex quolibet obiecti puncto in oculum ingrediuntur in fundo oculi super retina iterum in vnum punctum congregantur.

Scholion.

160. Ad visionem distinctam requiritur, vt obiecta in certa quadam ab oculo distantia reperiantur, quae distantia pro varia oculorum indole maxime solet esse diuersa, dum Myopes exiguam, ii qui oculis valent ingentem, ac presbytes non solum infinitam sed quandoque etiam negatiuam exigunt; cuiusmodi distantia cum in veris obiectis locum habere nequeat, ope perspicillorum sibi satisfacere solent. Quilibet ergo oculus ad certam quandam distantiam obiectorum est instructus quam eius distantiam iustam appellabo; vbi quidem insignis latitudo locum habet,

propterea quod ſtructura oculi ita eſt artificioſa, vt contractione ac elongatione quadam ſe ad diſtantias aliquanto maiores et minores accommodare poſſit. Quando ergo obiecta in diſtantia ab oculo iuſta reperiuntur, viſio eſt diſtincta, dum ſingula obiectorum puncta ſuper retina ſingulis punctis exprimuntur.

Definitio 2.

161. Viſio eſt confuſa, ſi radii ex quolibet obiecti puncto in oculum immiſſi non in vno retinae puncto congregantur, ſed per aliquod ſpatium retinam afficiunt.

Corollarium I.

162. Eo maior ergo erit confuſio, quo maius fuerit hoc ſpatium in retina, per quod radii ex eodem obiecti puncto emiſſi diſſipantur. Ex quo huius ſpatii magnitudo veram confuſionis menſuram ſuppeditabit.

Coroll. 2.

163. Viſio ergo erit confuſa, cum obiecti viſi diſtantia multum fuerit diuerſa ab oculi diſtantia iuſta. Paruum enim diſcrimen vel per ſe nullam confuſionem parit vel oculus ſe ad obiecti diſtantiam, ſua qua pollet, volubilitate accommodare valet.

Scholion.

164. Si obiecta per lentes diſtincte repraeſentarentur, viſio imaginum eadem lege teneretur atque ipſo-

ipſorum obiectorum; iuſta ſcilicet earum ab oculo diſtantia viſionem diſtinctam, admodum autem diuerſa confuſam produceret. Verum ſi imago per aliquod ſpatium fuerit diffuſa etiamſi ab oculo ad diſtantiam iuſtam ſit remota, inde tamen in viſione confuſio oriatur neceſſe eſt. Quod ſi nempe non obiecta ipſa, ſed earum imagines per vnam pluresue lentes repraeſentatas intueamur ob duplicem cauſam viſio poterit eſſe confuſa; altera, ſi diſtantia imaginis ab oculo multum fuerit diuerſa a diſtantia iuſta, altera vero ſi ipſa imago per aliquod ſpatium diffundatur. Priorem quidem cauſam tollere in noſtra eſt poteſtate ſiquidem lentes ita diſponere licet, vt imaginem in iuſta ab oculo diſtantia exhibeant; quam lentium diſpoſitionem propterea hic perpetuo aſſumamus. Quamobrem in hoc capite inueſtigare conſtitui quanta confuſio in viſione imaginum a ſpatio diffuſionis oriri debeat, eiusque quantitate determinata deinceps in hoc erit elaborandum quemadmodum lentes formatas ac diſpoſitas eſſe oporteat, vt confuſio inde in viſione nata datum limitem, quo adhuc eſt tolerabilis, non excedat. In genere quidem certum eſt, quo maius fuerit ſpatium diffuſionis, eo maiorem inde in viſionem induci debere confuſionem, verum tamen mox videbimus, confuſionem viſionis non eſſe ſpatio diffuſionis proportionalem, ſed aliam omnino legem ſequi, quam accurate determinaſſe maximi erit momenti, cum ex hoc fonte conſtructio omnium inſtrumentorum Dioptricorum ad viſionem accommodatorum ſit repetenda.

Problema I.

Tab. II. Fig. 10. 165. Si oculus aspiciat imaginem cuiuspiam obiecti ab vna pluribusue lentibus per spatium Ll diffusam, atque imago principalis L in iusta ab oculo distantia reperiatur, definire confusionem, qua visio huius imaginis afficietur.

Solutio.

In spatio diffusionis Ll, in quo punctum L a radiis per lentium medium, punctum l vero a radiis per lentium extremitates transmissis exhibeatur, considerari oportet primo eius magnitudinem Ll deinde radiorum in l concurrentium inclinationem ad axem. Supra autem vidimus has res a primae lentis apertura, cuius semidiameter sit $= x$, ita pendere vt sit ipsum spatium $Ll = Vxx$, et angulus $OlT = \mathfrak{V}x$ pro quouis scilicet lentium numero valores horum coefficientium V et $\mathfrak{V}$ determinauimus. Repraesentet iam circulus OV oculum, quem hic vt exiguam cameram obscuram spectare licet, sed ita perfectam, vt radios ex vno puncto emissos iterum in vno puncto colligat, etsi enim in oculo plures fiunt refractiones, tamen conditione illa seruata vnica lens earum loco considerari potest, quae sit in TOT, a qua retina remota sit interuallo $OV = u$. Cum nunc imago principalis L in debita ab oculo distantia, quae vocetur $OL = l$ existat, puncti L imago in oculo in ipsam retinam incidet, et in V distincte ćte depingetur: puncti vero l imago non in V, ſed ante retinam in v referetur, quod interuallum Vv, ſi lentis in TOT conceptae craſſities euaneſcat, ſecundum §. 62 ita exprimitur, vt ſit $Vv = \frac{OV^2}{OL^2}$. $Ll = \frac{uu}{ll}.\, Vxx$. Cum autem radii punctum l formantes ad axem inclinati ſint angulo $OlT = \mathfrak{B}x$, ii per lentis oculi puncta TT ita intrabunt, vt neglecto interuallo Ll prae diſtantia $OL = l$, ſit $OT = l\mathfrak{B}x$, ex quo ii in puncto v concurrentes cum axe angulum facient $OvT = \frac{OT}{Ov} = \frac{OT}{OV} = \frac{l}{u}\mathfrak{B}x$. Hinc ergo vltra ad retinam pergentes ſuper ea circulum UU effingent cuius radius erit $VU = \frac{l}{u}\mathfrak{B}x.\, Vv = \frac{l}{u}.\, \mathfrak{B}x.\, \frac{uu}{ll}.\, Vxx$ et a punctis inter L et l mediis hoc ſpatium circulare ſuper retina replebitur. Quare imago per ſpatium Ll diffuſa ſuper retina circello repraeſentabitur, cuius radius $VU = \frac{u}{l}\mathfrak{B}\, Vx^2$ qui circellus veram confuſionis menſuram ſupeditat. Quodlibet ſcilicet obiecti punctum, quod lentibus per ſpatium Ll diffuſum exhibetur, in oculo ſuper retina non puncto ſed circello exprimetur cuius radius erit $= \frac{u}{l}.\, \mathfrak{B}.\, Vx^2$.

Coroll. I.

166. Videmus ergo confuſionem, qua viſio afficitur non ſolum a quantitate ſpatii diffuſionis $Ll = Vxx$, ſed inſuper ab angulo, quo radii in l concurrentes ad axem inclinantur, qui eſt $= \mathfrak{B}x$ pendere; radiumque circelli confuſionem metientis producto illius ſpatii per hunc angulum eſſe proportionalem.

Coroll. 2.

167. Cum igitur posito semidiametro aperturae primae lentis $=x$, spatium diffusionis sit vt eius quadratum xx; radius circelli confusionem metientis est vt eius cubus x^3. Ac si confusio ipsa areae huius circelli proportionalis aestimetur erit ea vt x^6, seu vt cubus aperturae primae lentis.

Coroll. 3.

168 Hinc ergo intelligitur quanti intersit per idoneam lentium dispositionem spatium diffusionis Vxx diminuisse non solum enim in eadem ratione qua spatium diffusionis Vxx seu quantitas V diminuitur sed adeo in ratione duplicata, ipsa confusio visa minor redditur.

Scholion.

169. Assumsi hic pupillam tam late patere, vt radios ab l diuergentes recipiat; at si apertura pupillae minor esset, radios a puncto l venientes nequidem caperet: hoc ergo casu res eodem rediret ac si contracta primae lentis apertura spatium diffusionis Ll eo vsque diminueretur quoad pupilla omnes radios ab imagine emissos recipere posset hocque casu manifestum est, confusionem minorem esse prodituram. Verum in sequentibus ostendetur, tam in Telescopiis quam Microscopiis hunc casum vix vnquam locum inuenire cum plerumque conus radiosus ex puncto l emissus circa ingressum in oculum multo

to tenuior sit quam pupillae apertura; quamobrem ne opus quidem est hunc casum etsi per se facile expediretur, hic expendere. Ceterum tametsi oculus non commode cum lente simplici cuius crassities evanescat, conferri possit hincque expressio spatii V v secundum §. 84. aliquantum diuersa prodire potuisset hic ad istam circumstantiam non attendimus propterea quod spatium V v. secundum certam quandam rationem auctum vel diminutum prodiisset; Hic autem non adeo necesse est ipsam quantitatem absolutam confusionis cognouisse, dummodo rationem quam sequitur accurate definiuerimus. Cum enim nostrum calculum cum experientia contulerimus, terminum cognoscemus, quem si confusio nostro more expressa superauerit, intolerabilis euadat; hincque perpetuo sufficiet confusionem simili modo expressam infra hunc terminum reduxisse. Interim tamen notasse iuvabit, per oculi conformationem confusionem adhuc multo minorem effici posse.

Problema 2.

170. Positis iisdem, quae in praecedente problemate sunt assumta, eam definire oculi conformationem, qua minima confusio percipiatur. Tab. II. Fig. 11.

Solutio.

Oculus omnis facultate praeditus est sese aliquantillum aliter conformandi, vt etiam obiecta, quorum distantia a iusta non nimis differt, distincte videat:

videat: quod quomodocunque perficiatur, id ita fieri concipere licet, ac ſi retina ad pupillam ſeu potius eam lentem, quam oculi loco conſideramus, propius admoueretur ſeu longius ab ea detorqueretur. Cum ergo ante retinam in V ſitam ſimus contemplati, hic primo ſumamus retinam per ipſum punctum v tranſire, ac manifeſtum eſt ſuper ea punctum l diſtincte expreſſum iri. Verum quia punctum L alios radios niſi axi proximos non emittit etiam punctum L ſine vlla confuſione in v repraeſentabitur; ita vt tantum puncta media inter L et l confuſionem ſint paritura. Conſideretur ergo quoduis punctum intermedium λ, quod eſſet extremum in ſpatio confuſionis, ſi ſemidiameter aperturae primae lentis minor foret quam x, qui ergo ponatur $=z$: eritque $L\lambda = Vzz$, et inclinatio radiorum in λ concurrentium ad axem $=\mathfrak{B}z$: hinc iſtius puncti imago intra oculum refertur in $\mathfrak{v}$, vt ſit $V\mathfrak{v} = \frac{uu}{ll} Vzz$, ideoque ob $Vv = \frac{uu}{ll} Vxx$ erit interuallum $v\mathfrak{v} = \frac{uu}{ll} V(xx-zz)$: et radiorum in $\mathfrak{v}$ conuergentium inclinatio ad axem $= \frac{l}{u}\mathfrak{B}z$; ex quo circelli ſuper retina in v exiſtentis radiis puncti λ expreſſi ſemidiameter erit $= \frac{u}{l}\mathfrak{B}Vz(xx-zz)$ qui euaneſcit, vti iam monuimus, ſiue ſit $z=0$, ſiue $z=x$. Quaeratur iam ille valor ipſius z, quo ille circellus fiat maximus; id quod eueniet, ſi $xx-3zz=0$ ſeu $z=\frac{x}{\sqrt{3}}$; ſicque circelli confuſionem hoc caſu metientis ſemidiameter erit $= \frac{2u}{3l\sqrt{3}}.\mathfrak{B}Vx^3$, quod eſt multo minus quam caſu praecedente, quo retina erat in V.

At

At ſi retina aliquantillum a v verſus V remoueatur, hic circellus adhuc minor effici poterit. Ponatur enim ſpatium $VS = s$, vt ſit $vS = \frac{uu}{ll} Vxx - s$, et $S\upsilon = s - \frac{uu}{ll} Vzz$. Punctum ergo l, cuius effigies in u exhibetur, ſuper retina S circello referetur, cuius radius eſt $= \frac{l}{u} \mathfrak{B} x (\frac{uu}{ll} Vxx - s)$, punctum vero λ, cuius effigies eſt in υ, ſuper retina S circello, cuius radius erit $= \frac{l}{u} \mathfrak{B} z (s - \frac{uu}{ll} Vzz)$; nunc igitur id punctum λ inueſtigemus, vnde iſte circellus minimus euadat: quod fit ſi ſit $s = \frac{3uu}{ll} Vzz$; eritque huius circuli radius $= \frac{2u}{l} \mathfrak{B} V z^3$, qui ſimul confuſionem exhiberet, ſi modo a puncto l, non maior circulus oriretur: ſed ſubſtituto pro s valore inuento huius circuli radius erit, $\frac{u}{l} \mathfrak{B} V x (xx - 3zz)$, qui ergo illi $\frac{2u}{l} \mathfrak{B} V z^3$ aequalis ſtatuatur, vnde naſcitur haec aequatio:

$$x^3 - 3xzz = 2z^3, \text{ hincque } z = \tfrac{1}{2} x.$$

Quare circelli, quo minima confuſio menſuratur, radius erit $= \frac{u}{4l} \mathfrak{B} V x^3$, quadruplo minor, quam ſi retina eſſet in V; indeque ipſa confuſio ſedecies minor. Etſi autem illa aequatio cubica tres habet radices, praeter $z = \frac{1}{2} x$ binae reliquae ſunt aequales et $z = -x$, ſicque $zz = xx$, vnde non minima ſed quaſi maxima confuſio naſceretur.

Coroll. I.

171. Cum igitur, vt oculus minimam confuſionem ſentiat, debeat eſſe $z = \frac{1}{2} x$, ideoque $s = \frac{3uu}{4ll} Vxx$,

patet retinam in eum locum S cogi debere, vt sit $VS = \frac{3}{4} Vv$ et $vS = \frac{1}{4} Vv$.

Coroll. 2.

172. Cum ergo oculus non solum praeditus sit facultate sese parumper immutandi sed etiam hac facultate vti soleat ad confusionem euitandam, nullum est dubium quin circellorum confusionem producentium radius sit $= \frac{\mu}{4\nu} \mathfrak{B} V x^3$, ideoque quadruplo minor, quam problemate praecedente inueneramus dummodo imago spectanda Ll propemodum in distantia iusta reperiatur.

Coroll. 3.

173. Haec igitur expressio $\frac{\mu}{4\nu} \mathfrak{B} V x^3$ iustam nobis exhibet mensuram confusionis, cum exprimat radium circellorum, quibus singula obiecti puncta, quatenus id per lentes spectatur, super retina repraesentantur. Si modo imago in distantia iusta ab oculo reperiatur.

Scholion.

174. Si hi circelli euanescerent, visio plane esset distincta hoc autem fieri nequit, nisi ipsum interuallum diffusionis Vxx euanescat: vnde patet si apertura primae lentis ad nihilum reduceretur, nullam confusionem sentiri debere. Verum visio non ita est delicata, vt prorsus nullam confusionem pati possit sed dummodo confusio certum quendam terminum

num non ſuperet, quaſi eſſet nulla conſiderari poteſt. Iſte terminus, ſeu valor quem expreſſio $\frac{u}{4l}\mathfrak{B}Vx^3$ excedere non debet, ex experientia potius peti debet, quam ex Theoria isque idcirco inſigni adhuc latitudine continetur: vnde fit vt pro diuerſo ſcopo modo maiore gradu confuſionis contenti eſſe ſoleamus, modo autem minorem gradum exigamus: quas circumſtantias cum ad praxin propius accedemus, accuratius ſumus examinaturi. Ceterum notandum eſt pro interuallo u, quo quaſi profunditas oculi exhibetur, vnum circiter pollicem aſſumi poſſe, quae quidem menſura fere erit arbitraria, cum deinceps limites confuſionis per experientiam conſtituemus.

Problema. 3.

175. Si oculus per vnicam lentem obiectum aſpiciat ita vt imago viſa ab oculo in diſtantia iuſta reperiatur, definire confuſionem, qua viſio afficietur. Tab. I. Fig. 3.

Solutio.

Sit diſtantia obiecti $E\epsilon$ ante lentem $AE = a$, imaginis vero principalis $F\zeta$ poſt lentem $BF = \alpha$, lentisque craſſities $AB = v$, quantitas vero arbitraria qua cum binis diſtantiis determinatricibus a et α lentis facies determinatur, ſit $= k$, ponatur autem breuitatis gratia $\frac{k-v}{k+v} = i$. Tum autem poſita ratione refractionis $= n$, debet eſſe lentis PP

radius faciei anterioris $= \frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei poſterioris $= \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$

ac ſi ſemidiameter aperturae faciei anterioris ſit $=x$, poſterioris vero $> ix$ ſpatium diffuſionis Ff erit $= P\alpha\alpha xx$ exiſtente

$$P = \frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{1}{k+v}\right)^2 + ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2\right)$$

radiorum vero in f concurrentium inclinatio ad axem $= i. \frac{x}{\alpha}$. His, quae §. 86 ſunt ſtabilita, praemiſſis, ſit diſtantia oculi poſt lentem $BO = O$, et quia imago $F\zeta$ ante oculum in diſtantia iuſta exiſtere aſſumitur, erit $O = \alpha + l$, ideoque $\alpha = O - l$, ſi locus oculi vt datus conſideretur. Hinc ergo habebimus $V = P\alpha\alpha$ et $\mathfrak{B} = \frac{i}{\alpha}$ ideoque $\mathfrak{B}V = i\alpha P$. Conſequenter radius circellorum in oculo confuſionem metientium, ſeu vt in poſterum loquemur menſura confuſionis erit

$$= \frac{n}{4l}. i\alpha x^3. P = \frac{in}{4l}(O-l)x^3 P.$$

Si lentis craſſities v euaneſcat, et pro determinatione lentis numerus arbitrarius λ loco k introducatur, erit ex §. 91 $P = \mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{v}{a\alpha}\right)$, et $i = 1$ ex quo loco citato etiam ipſa lentis conſtructio eſt petenda.

Coroll. I.

176. Si igitur lens fuerit data, locus oculi poſt lentem ita definitur, vt debeat eſſe diſtantia $BO = O = \alpha + l$, exiſtente l diſtantia oculi iuſta: Sin autem locus oculi detur, pro lentis conſtructione diſtantia determinatrix α ita capi debet, vt ſit $\alpha = O - l$.

Coroll.

Coroll. 2.

177. Quare si oculus ita fuerit comparatus, vt exigat distantiam iustam $l = \infty$, fiet $\alpha = -\infty$, et mensura confusionis erit $= -\frac{1}{4} iux^3 . P$ seu $= \frac{1}{4} iux^3 . P$, quia signum $-$ nihil mutat in magnitudine circellorum confusionem producentium.

Coroll. 3.

178. Etsi ergo hoc casu quo $\alpha = \infty$ spatium diffusionis Ff est infinitum, tamen inde confusio in visione orta est finita, quia hoc non obstante valor ipsius P manet finitus: erit enim

$$P = \frac{n}{2(n-1)^2} \left(\frac{1}{ii} \left(\frac{n}{a} + \frac{2}{k+v} \right) \left(\frac{1}{a} + \frac{2}{k+v} \right)^2 - \frac{vii}{(k-v)^2} \right)$$

Radius autem faciei posterioris fit $= -\frac{(n-1)}{2n}(k-v)$.

Coroll. 4.

179. Quia est $i = \frac{k-v}{k+v}$, valor ipsius P etiam in genere ita exprimi potest, vt sit:

$$P = \frac{n}{2(n-1)^2} \left(\left(\frac{n}{a} + \frac{2}{k+v} \right) \left(\frac{1}{ia} + \frac{2}{k-v} \right)^2 + \left(\frac{n}{a} - \frac{2}{k-v} \right) \left(\frac{i}{a} - \frac{2}{k+v} \right)^2 \right)$$

vbi est $\frac{n}{2(n-1)^2} = \frac{310}{121} = 2,561983$ ob $n = \frac{31}{20}$. Sicque etiam valores reliquarum litterarum Q, R, S etc. in §. 86 transformari poterunt.

Problema 4.

180. Si oculus per duas lentes obiectum $E\varepsilon$ aspiciat, ita vt imago per eas repraesentata $G\gamma$ in distantia iusta $OG = l$ ob oculo sit remota, definire confusionem, qua visio afficietur. Tab. I. Fig. 5.

Solutio

Solutio.

Sint vt hactenus pro lente PP distantiae determinatrices $AE = a$, $aF = \alpha$, crassities $Aa = v$, et distantia arbitraria $= k$, ita vt sit

radius faciei anterioris $= \frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei posterioris $= \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$

tum vero posito $\frac{k-v}{k+v} = i$ ponatur

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a} + \frac{2}{k+v}\right)\left(\frac{1}{ia} + \frac{2}{k-v}\right)^2 + \left(\frac{n}{\alpha} - \frac{2}{k-v}\right)\left(\frac{i}{\alpha} - \frac{2}{k+v}\right)^2\right)$$

Deinde pro lente posteriori QQ sint distantiae determinatrices $BF = b$, $bG = \beta$, crassities lentis $Bb = v'$ et distantia arbitraria $= k'$ vt sit

radius faciei anterioris $= \frac{(n-1)b(k'+v')}{k'+v'+2nb}$

radius faciei posterioris $= \frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$

tum vero posito $\frac{k'-v'}{k'+v'} = i'$ ponatur

$$Q = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{b} + \frac{2}{k'+v'}\right)\left(\frac{1}{i'b} + \frac{2}{k'-v'}\right)^2 + \left(\frac{n}{\beta} - \frac{2}{k'-v'}\right)\left(\frac{i'}{\beta} - \frac{2}{k'+v'}\right)^2\right)$$

Atque iam erit spatium diffusionis

$$Gg = \beta\beta xx\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P + ii.\frac{bb}{\alpha\alpha}Q\right) = Vxx$$

et radiorum in g concurrentium inclinatio ad axem
$i\,i'.\frac{bx}{\alpha\beta} = \mathfrak{B}x.$

Sit iam oculus in O existente $OG = l$, ac ponatur eius post lentem QQ distantia $bO = O$ erit $O = \beta + l$, ideoque

ideoque $\beta = O - l$; quibus positis cum sit mensura confusionis $= \frac{u}{4l}\mathfrak{B}x.Vxx$ erit ea pro nostro casu:

$$\frac{ii'u}{4l} \cdot \frac{b\beta}{\alpha} x^3 \left(\frac{1}{i'i'} \cdot \frac{\alpha\alpha}{bb} P + ii \frac{bb}{\alpha\alpha} Q\right)$$

vel pro β posito valore $O - l$ et signo mutato

$$\tfrac{1}{4} ii'u\left(1 - \frac{O}{l}\right) \cdot \frac{b}{\alpha} x^3 \left(\frac{1}{i'i'} \cdot \frac{\alpha\alpha}{bb} P + ii . \frac{bb}{\alpha\alpha} Q\right).$$

Coroll. 1.

181. Si ergo pro oculo fuerit $l = \infty$, ideoque et $\beta = \infty$, etsi spatium diffusionis Gg sit infinitum tamen ob $\frac{O}{l} = 0$ confusio visionem afficiens nihilominus erit finita: neque enim P neque Q ob $\beta = \infty$ fit infinita.

Coroll. 2.

182. Si crassities lentium euanescat, vt fit $v = 0$ et $v' = 0$ erit $i = 1$ et $i' = 1$. ideoque hoc casu mensura confusionis

$$\tfrac{1}{4} u\left(1 - \frac{O}{l}\right) \cdot \frac{b}{\alpha} x^3 \left(\frac{\alpha\alpha}{bb} P + \frac{bb}{\alpha\alpha} Q\right)$$

At si loco k et k' introducantur numeri λ et λ' erit

$$P = \mu\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{\nu}{a\alpha}\right) \text{ et}$$

$$Q = \mu\left(\frac{1}{b} + \frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b} + \frac{1}{\beta}\right)^2 + \frac{\nu}{b\beta}\right)$$

existente

$$\mu = 0,938191 \text{ et } \nu = 0,232692$$

Coroll.

Coroll. 3.

183. Eodem autem hoc casu constructio binarum lentium ita se habebit:

radius faciei

$$\text{Pro lente PP}\begin{cases}\text{anterioris} = \dfrac{a\alpha}{\varrho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}\\[2ex] \text{posterioris} = \dfrac{a\alpha}{\varrho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}\end{cases}$$

$$\text{Pro lente QQ}\begin{cases}\text{anterioris} = \dfrac{b\beta}{\varrho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}}\\[2ex] \text{posterioris} = \dfrac{b\beta}{\varrho b+\sigma\beta\mp\tau(b+\beta)\sqrt{(\lambda'-1)}}\end{cases}$$

existente $\varrho = 0,190781$; $\sigma = 1,627401$ et $\tau = 0,905133$.

Problema 5.

Tab. II. Fig. 6. 184. Si oculus per tres lentes PP, QQ et RR obiectum $E\varepsilon$ aspiciat, ita vt imago per eas repraesentata $H\theta$ in distantia iusta $OH = l$ ab oculo O sit remota, definire confusionem, qua visio afficietur.

Solutio.

Positis duabus prioribus lentibus PP et QQ vt in problemate praecedente, indeque determinatis valoribus P et Q cadat imago per has duas lentes repraesentata principalis in $G\eta$, post quam tertia lens RR ita collocata sit, vt sint eius distantiae determinatrices $CG = c$, $cH = \gamma$, crassities $Cc = v''$ et quantitas arbitraria $= k''$ vt sit:

$$\text{radius faciei anterioris} = \frac{(n-1)c(k''+v'')}{k''+v''+2nc}$$

$$\text{radius faciei posterioris} = \frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$$

Tum

Tum vero posito $\frac{k''-v''}{k''+v''}=i''$ ponatur

$$R=\frac{n}{2(n-1)}\left(\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{4}{i''c}+\frac{2}{k''-v''}\right)^2+\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{i''}{\gamma}-\frac{2}{k''+v''}\right)^2\right)$$

atque iam spatium diffusionis erit

$$Hb=\gamma\gamma xx\left(\frac{1}{i'i'.i''i''}\cdot\frac{\alpha\alpha\mathfrak{b}\mathfrak{b}}{bbcc}P+\frac{ii}{i''i''}\cdot\frac{bb\mathfrak{b}\mathfrak{b}}{\alpha\alpha cc}Q+ii\cdot i'i'\cdot\frac{bbcc}{\alpha\alpha\mathfrak{b}\mathfrak{b}}R\right)$$

quod est valor ipsius Vxx. Radiorum vero in b concurrentium inclinatio ad axem est

$$ii'i''\cdot\frac{bcx}{\alpha\mathfrak{b}\gamma}=\mathfrak{B}x$$

sit iam oculus in O, ac ponatur eius distantia post lentem RR $=$ O erit $O=\gamma+l$, ideoque $\gamma=O-l$. Hinc mensura confusionis in oculo ortae colligitur

$$\tfrac{1}{4}ii'i''u\left(1-\tfrac{O}{l}\right)\frac{bc}{\alpha\mathfrak{b}}x^3\left(\frac{1}{i'i'.i''i''}\cdot\frac{\alpha\alpha\mathfrak{b}\mathfrak{b}}{bbcc}P+\frac{ii}{i''i''}\cdot\frac{bb\mathfrak{b}\mathfrak{b}}{\alpha\alpha cc}Q+ii.i'i'.\frac{bbcc}{\alpha\alpha\mathfrak{b}\mathfrak{b}}R\right)$$

Coroll. 1.

185. Hic iterum vt ante patet, si fuerit $l=\infty$ ideoque et $\gamma=\infty$, quo casu diffusionis spatium in infinitum extenditur, mensuram confusionis terminis finitis contineri, quod etiam locum habet pro quouis lentium numero.

Coroll. 2.

186. Si lentium crassities pro euanescente habeatur, ob $i=1$ $i'=1$, $i''=1$, mensura confusionis ita simplicius exprimetur, vt sit

$$=\tfrac{1}{4}u\left(1-\tfrac{O}{l}\right)\frac{bc}{\alpha\mathfrak{b}}x^3\left(\frac{\alpha\alpha\mathfrak{b}\mathfrak{b}}{bbcc}P+\frac{bb\mathfrak{b}\mathfrak{b}}{\alpha\alpha cc}Q+\frac{bbcc}{\alpha\alpha\mathfrak{b}\mathfrak{b}}R\right)$$

At hoc casu loco k, k', k'' introductis numeris $\lambda, \lambda', \lambda''$ erit

$$P = \mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$
$$Q = \mu\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\left(\lambda'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu}{b\beta}\right)$$
$$R = \mu\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu}{c\gamma}\right)$$

Coroll. 3.

187. Eodem vero casu constructio lentium per istos numeros $\lambda, \lambda', \lambda''$ ita erit dirigenda vt sit

Radius faciei

Pro lente PP $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{\rho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}} \\ \text{posterioris} = \dfrac{a\alpha}{\rho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}} \end{cases}$

Pro lente QQ $\begin{cases} \text{anterioris} = \dfrac{b\beta}{\rho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}} \\ \text{posterioris} = \dfrac{b\beta}{\rho b+\sigma\beta\mp\tau(b+\beta)\sqrt{(\lambda'-1)}} \end{cases}$

Pro lente RR $\begin{cases} \text{anterioris} = \dfrac{c\gamma}{\rho\gamma+\sigma c\pm\tau(c+\gamma)\sqrt{(\lambda''-1)}} \\ \text{posterioris} = \dfrac{c\gamma}{\rho c+\sigma\gamma\mp\tau(c+\gamma)\sqrt{(\lambda''-1)}} \end{cases}$

Scholion.

188. Hinc satis manifestum est, quemadmodum hae formulae pro pluribus lentibus progrediantur: verum antequam eas exponam, conueniet alias quoque circumstantias, quae hinc facillime deducuntur, perpendi, scilicet magnitudinem obiecti visam, et copiam radiorum a singulis eius punctis in oculum transmissorum vt hae simul cum confusione visionis dein-

deinceps coniunctim pro quouis lentium numero exhiberi queant; quo pacto plures taediosas repetitiones euitabimus. Duae autem res ex hactenus allatis facile definiri possunt, quarum altera est quantitas, qua imago obiecti per lentes repraesentata ab oculo cernitur, quae quantitas aestimanda est ex angulo, sub quo imago videtur, vt is deinceps comparari possit, cum eo angulo sub quo ipsum obiectum in data distantia a nudo oculo spectaretur, vnde qua ratione magnitudo per lentes visa augeatur, intelligetur. Altera res in copia radiorum, a singulis obiecti punctis in oculum transmissorum versatur, qua claritas visione percepta continetur, a quolibet scilicet puncto conus seu cylindrus radiosus in oculum ingreditur, qui si pupillam penitus expleat, claritas ad summum gradum erit euecta, nisi forte maiori illustratione ipsi obiecto maius lumen concilietur. At si sectio illius coni aut cylindri, qua in oculum intrat, minor fuerit pupilla, in eadem ratione claritas decrescet; quod cum in omnibus instrumentis dioptricis, quibus magnitudinem visam vehementer augere propositum est, vsu venire soleat, plurimum interfuerit amplitudinem illius coni seu cylindri, qua in oculum penetrat, accurate determinasse.

Problema 6.

189 Definire quantitatem, sub qua quaeuis obiecti portio per lentes quotcunque ab oculo in distantia iusta ab imagine vltima remoto cernetur.

 Solutio.

Solutio.

Sit z linea in obiecto concepta, quae quanta per lentes oculo sit apparitura, definiri oporteat. Ostensum autem est in praecedentibus, quotcunque fuerint lentes, imaginem principalem huius lineae iterum esse lineam, cuius longitudo ad z certam teneat rationem a distantiis determinatricibus lentium et numeris i, i', i'', i''' etc. pendentem (86). Sit ergo haec longitudo imaginis $= Mz$, quae cum ab oculo in distantia l remoto aspiciatur, apparebit sub angulo $= \frac{Mz}{l}$; vel cuius tangens potius sit $= \frac{Mz}{l}$; sed quia hic angulus rarissime vltra aliquot gradus assurgere solet, tangens tuto pro ipso arcu assumitur. Effigies autem quae ab hac linea Mz in oculo exprimitur, erit $= \frac{Muz}{l}$; hic enim cogitationes a confusione abstraho, qua vtique fit, vt effigies maior imprimatur, propterea quod singula puncta circellis exhibentur. Iam positis iisdem lentium determinationibus, quibus supra §. 86 sum vsus, pro vario lentium numero angulus, sub quo linea z in obiecto sumta cernetur ita se habebit:

	Angulus visionis	situ
Pro vnica lente	$1 \cdot \frac{\alpha}{a} \cdot \frac{z}{l}$	inuerso
Pro duabus lentibus	$\frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} \cdot \frac{z}{l}$	erecto
Pro tribus lentibus	$\frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma}{abc} \cdot \frac{z}{l}$	inuerso
Pro quatuor lentibus	$\frac{1}{ii'i''i'''} \cdot \frac{\alpha\beta\gamma\delta}{abcd} \cdot \frac{z}{l}$	erecto
	etc.	

Scilicet

Scilicet si hae formulae fiat positiuae, oculus lineam z situ siue erecto siue inuerso videbit, prout est notatum sin autem fuerint negatiuae, situm indicatum in contrarium verti oportet.

Coroll. 1.

190. Si eadem obiecti linea z in distantia $= b$ ab oculo nudo cerneretur, ea apparitura esset sub angulo $= \frac{z}{b}$, vnde perspicitur, quanto ea vel maior vel minor per lentes videatur.

Coroll. 2.

191. Si distantia oculi a postrema lente ponatur $= O$, erit casu vnius lentis $\alpha = O - l$, ideoque $\frac{\alpha}{l} = -(1 - \frac{O}{l})$ vnde angulus opticus lineae obiecti z respondens erit $= \frac{1}{l}(1 - \frac{O}{l})\frac{z}{a}$ pro situ erecto, quia signum mutauimus.

Coroll. 3.

192. Simili modo casu duarum lentium ob $\beta = O - l$ erit iste angulus $= \frac{1}{ll'}(1 - \frac{O}{l})\frac{\alpha z}{ab}$ pro situ inuerso.

Casu vero trium lentium ob $\gamma = O - l$ erit iste angulus $= \frac{1}{ll'l''}(1 - \frac{O}{l})\frac{\alpha\beta z}{abc}$ pro situ erecto

Casu quatuor lentium ob $\delta = O - l$ erit iste angulus $\frac{1}{ll'l''l'''}(1 - \frac{O}{l})\frac{\alpha\beta\gamma z}{abcd}$ pro situ inuerso, et ita porro pro pluribus lentibus.

Scholion.

193. Hinc etiam modus se offert confusionem ob lentium aperturam in oculo natam distinctius aestimandi. Scilicet cum singula obiecti puncta in oculo exprimantur circulis, quorum radius est $=\frac{u}{4l}\mathfrak{B}Vx^3$, verus autem circulus cuius radius $=z$, oculo in distantia l expositus in oculo referatur circulo cuius radius $=\frac{u}{l}z$ singula puncta illius obiecti per lentes spectati, aeque magna apparebunt, ac orbes circulares radii $z=\frac{1}{4}\mathfrak{B}Vx^3$, si in distantia ab oculo $=l$ spectarentur. Vel cum horum orbium semidiameter apparens sit $=\frac{z}{l}$, ob confusionem singula obiecti puncta instar circulorum videbuntur, quorum semidiameter apparens esset $=\frac{1}{4l}\mathfrak{B}Vx^3$. In expressionibus igitur ante pro confusione inuentis deleatur quantitas u, et habebitur semidiameter apparens circulorum confusionem exprimentium. Hinc iudicari poterit quam parua esse debeat confusio, vt non amplius sentiatur, scilicet si oculus non amplius percipere valeat spatium circulare, cuius semidiameter esset 1'', seu $\frac{1}{853}$ pars radii circiter, euidens est, si fuerit nostra expressio $\frac{1}{4l}\mathfrak{B}Vx^3=\frac{1}{853}$, confusionem fore imperceptibilem. Ac experientiam consulentes deprehendimus multo maiores angulos non amplius percipi posse, ita vt confusio non sit metuenda etiamsi expressio $\frac{1}{4l}\mathfrak{B}Vx^3$ notabiliter maior fuerit, quam $\frac{1}{853}$; ne autem hic temere quicquam statuamus, ponamus limitem, quem formula $\frac{1}{4l}\mathfrak{B}Vx^3$ excedere

non

non debeat, esse $= \frac{1}{4\varkappa^3}$, ita vt esse oporteat $\frac{1}{4}\mathfrak{B} V x^3 < \frac{1}{\varkappa^3}$. Postea igitur ad praxin descendentes poterimus pro $\varkappa$ numerum vel 40 vel minorem assumere prout experientia quouis casu postulauerit. Quod si ergo hoc modo confusionis rationem habeamus, profunditas oculi a non amplius in computum ingredietur.

Definitio 3.

194. Semidiameter confusionis, est semidiameter apparens circuli, qui ab oculo aeque magnus videtur, ac singula obiecti puncta ipsi ob confusionem apparent.

Corollarium.

195. Inueniemus igitur facile semidiametrum confusionis si formulas supra pro confusione repertas per profunditatem oculi a diuidamus quo pacto eae formulae ad numeros absolutos reducentur.

Definitio 4.

196. Multiplicatio per lentes producta ex ratione quantitatis, qua obiecta per lentes spectantur, ad quantitatem, qua eadem obiecta in data distantia ab oculo nudo cernerentur aestimatur. Exponens autem multiplicationis inuenitur, si magnitudo, qua linea quaecunque in obiecto concepta per lentes videtur, diuidatur per magnitudinem, qua eadem linea in data distantia ab oculo nudo spectata esset apparitura.

Coroll.

Coroll. 1.

197. Inuoluit ergo diiudicatio multiplicationis distantiam quandam fixam, in qua rem obiecta a nudo oculo aspici assumimus quae prout diuersa assumatur, multiplicatio alio atque alio modo exprimetur.

Coroll. 2.

198. Si haec distantia fixa, ex qua multiplicatio diiudicatur ponatur $= b$ et exponens multiplicationis $= m$, sit linea quaepiam in obiecto concepta $= z$, quae ergo nudo oculo in distantia b appareret sub angulo $= \frac{z}{b}$; eadem autem linea per lentes spectetur sub angulo $= \frac{Mz}{l}$ (88) ex quo erit exponens multiplicationis $m = \frac{Mb}{l}$.

Coroll. 3.

199. In ratione ergo $m : 1$ dimensiones lineares per lentes augeri sunt censendae; vnde superficies auctae apparebunt in ratione $mm : 1$, et ipsa corpora in ratione $m^3 : 1$. Cum exponente autem multiplicationis coniungi debet situs, quo obiecta apparent siue is sit erectus siue inuersus.

Scholion.

200. Distantia haec fixa b ad quam multiplicatio refertur non eodem modo perpetuo assumi solet, quippe quod etiam pro diuersitate obiectorum omnino fieri non posset. Nam si per lentes obiecta valde

valde remota veluti coeleſtia contuemur, quoniam ea nunquam in diſtantia modica ſpectare ſolemus, conuenit vtique magnitudinem per lentes viſam cum ea comparare, qua in ea ipſa a nobis diſtantia nudis oculis cernerentur: ideoque his caſibus diſtantia fixa b ipſi diſtantiae a, qua a lentibus obiecta ſunt remota, aequalis conſtitui ſolet. Scilicet ſi diſtantia a fuerit valde magna, qui eſt caſus Teleſcopiorum, ſtatuitur $b = a$, et magnitudo per haec inſtrumenta viſa, cum magnitudine per nudos oculos viſa in eadem diſtantia commodiſſime comparatur. Sic de Teleſcopiis dicitur, quoties diametri corporum coeleſtium multiplicentur, eoque caſu littera m exponentem huius rationis indicabit. Sin autem obiecta propiora contemplamur, qui eſt vſus Microſcopiorum ea plerumque ita prope ad inſtrumentum admouentur, vt in tam exigua diſtantia nudis oculis nunquam diſtincte cerni poſſent: neque ergo his caſibus ſtatui $b = a$ conueniret. Aliam ergo rationem ineundo pro b ſumi ſolet eiusmodi diſtantia modica, in qua obiecta commode ac diſtincte cernere liceat, quae etſi vtique pro diuerſa oculorum indole diuerſa ſumi deberet; tamen vt aliquid fixi ſtatuatur, pro b diſtantia 8 pollicum vtpote maximae oculorum parti conueniens accipi ſolet, ita vt his caſibus definiamus, quoties huiusmodi obiecta maiora *appareant*, quam ſi eadem nudis oculis in diſtantia 8 digitorum aſpicerentur. Interim tamen ſi multiplicatio ad hanc diſtantiam fuerit relata, non difficile erit eam ad quamlibet aliam referre, ita vt haec hypothesis naturam horum instrumentorum afficere non sit censenda.

Problema 7.

Tab. II. Fig. 10. 201. Si obiectum per lentes quotcunque aspiciatur, definire amplitudinem coni seu cylindri luminosi, qui a singulis obiecti punctis in oculum transmittitur.

Solutio.

Reperiatur vt supra oculus in distantia iusta l post imaginem postremam per lentes repraesentatam quae etsi est per spatium aliquod Ll diffusa, hic tamen a confusione inde nata mentem abstrahimus, quoniam confusionem iam seorsim determinauimus. Ex puncto igitur l oculum versus diffunditur conus luminosus, cuius radii extremi ad axem inclinati sunt angulo OlT quem posuimus supra $= \mathfrak{B}x$. Huius igitur coni sectio circa ingressum in oculum consideretur, cuius semidiameter erit $= \mathfrak{B}lx$, vnde pro vario lentium numero hic semidiameter sequenti modo definietur.

Pro vnica lente . . . . . . $il\frac{x}{\alpha}$

Pro duabus lentibus . . . . . $ii^{I}l\frac{bx}{\alpha\beta}$

Pro tribus lentibus . . . . $ii^{I}i^{II}l\frac{bcx}{\alpha\beta\gamma}$

Pro quatuor lentibus . . . $ii^{I}i^{II}i^{III}l\frac{bcdx}{\alpha\beta\gamma\delta}$

etc.

hicque perinde est siue hae formulae sint positiuae siue negatiuae; quia circulus siue radio positiuo siue negatiuo describatur eiusdem prodit magnitudinis.

Coroll.

Coroll. 1.

202. Si iste semidiameter $\mathfrak{V}/x$ maior fuerit semidiametro pupillae, tota pupillae apertura radiis implетur neque propterea visio clarior procurari poterit, nisi forte ipsum obiectum fortiori illustratione splendidius reddatur.

Coroll. 2.

203. Sin autem haec quantitas $\mathfrak{V}/x$ minor fuerit semidiametro pupillae claritatis mensura existet, quae eo maior erit quo maior fuerit ista quantitas: dum contra minuta hac quantitate claritas tam exigua euadere potest, vt non amplius sensui visus excitando sufficiat.

Definitio 5.

204. Gradus claritatis per lentes perceptae commodissime definietur semidiametro coni luminosi, qui a quouis obiecti puncto in oculum transmittitur.

Coroll. 1.

205. Gradus ergo claritatis definitur quantitate supra inuenta $\mathfrak{V}/x$, ita vt si gradum claritatis ponamus $= y$ habeamus $y = \mathfrak{V}/x$, quo cognito facillime iudicabimus, quonam gradu visio sit clara habenda.

Coroll. 2.

206. Scilicet si semidiametrum pupillae ponamus $= \omega$, quamdiu fuerit $y > \omega$, claritate plena

 fruemur,

fruemur, quae nullius augmenti est capax nisi forte ipsam pupillam magis dilatare valeamus.

Coroll. 3.

207. At si fuerit $y < \omega$, claritatem vtique minorem percipiemus: ac si claritatem plenam vnitate designemus, claritas ex casu $y < \omega$ resultans erit $= \frac{yy}{\omega\omega}$; propterea quod copia radiorum in oculum immissorum est vt quadratum semidiametri y.

Coroll. 4.

208. Quod si gradus claritatis y eousque decrescat, vt copia radiorum nimis sit parua, quam vt sensum visus excitare possit, nihil ob summam caliginem percipi poterit, vnde manifestum est ad visionem requiri, vt gradus claritatis certum quempiam limitem superet.

Scholion.

209. Tam in Telescopiis quam Microscopiis maxime necesse est; vt obiecta certo claritatis gradu exhibeantur ne repraesentatio nimis fiat obscura. Hic autem gradus plurimum a lumine proprio obiectorum pendet, quae quo fuerint illustriora minor claritatis gradus iis satis clare videndis sufficit; ideoque stellas illustriores contemplantes minori gradu claritatis contenti esse possumus: terrestria vero obiecta multo maiorem claritatis gradum postulant. Quo igitur haec ad

ad omnes casus accommodare valeamus gradum claritatis hic littera indefinita y contentum in computum sum ducturus. Quamobrem his de multiplicatione et claritate praemissis, haec duo elementa simul cum confusione pro quouis lentium numero exhibebo: ac primo quidem non neglecta lentium crassitie, tum vero eadem seorsim lentium crassitie neglecta exponi conueniet.

Problema 8.

210. Si oculus per quotcunque lentes PP, QQ, RR, SS etc. obiectum Ee aspiciat; ita vt imago postrema per eas repraesentata ante oculum in iusta distantia $= l$ reperiatur, determinare tam multiplicationem et claritatem, quam confusionem, qua visio perturbabitur.

Solutio

Quotcunque fuerint lentes, sint pro singulis distantiae determinatrices, vt et crassities cum quantitate arbitraria, vt sequitur:

Pro Lente	Dist: determinatr:	Crassities	quant: arb:
Prima PP	$EA = a$; $aF = \alpha$;	$Aa = v$;	k
Secunda QQ	$FB = b$; $bG = \beta$;	$Bb = v'$;	k'
Tertia RR	$GC = c$; $cH = \gamma$;	$Cc = v''$;	k''
Quarta SS	$HD = d$; $dI = \delta$;	$Dd = v'''$;	k'''

etc.

 atque

atque hinc posita ratione refractionis $\frac{31}{20} = n$ constructio lentium ita se habebit;

Pro Lente	Radius Faciei anterioris	Radius Faciei posterioris
Prima PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Secunda QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Tertia RR	$\frac{(n-1)c(k''+v'')}{k''+v''+2nc}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$
Quarta SS	$\frac{(n-1)d(k'''+v''')}{k'''+v'''+2nd}$	$\frac{(n-1)\delta(k'''-v''')}{k'''-v'''-2n\delta}$

etc.

Tum posito breuitatis gratia:

$$\frac{k-v}{k+v} = i;\ \frac{k'-v'}{k'+v'} = i';\ \frac{k''-v''}{k''+v''} = i'';\ \frac{k'''-v'''}{k'''+v'''} = i''' \text{ etc.}$$

si aperturae primae lentis in facie anteriori semidiameter fuerit $= x$, tam pro facie posteriori quam pro vtraque facie singularum lentum sequentium aperturae maiores esse debent vel saltem non minores, quam sequens tabula ostendit:

Pro Lente	Semidiameter aperturae in facie anteriori	Semidiameter aperturae in facie posteriori
Prima PP	x	ix
Secunda QQ	$i.\frac{bx}{\alpha}$	$ii'.\frac{bx}{\alpha}$
Tertia RR	$ii'.\frac{bcx}{\alpha\beta}$	$ii'i''.\frac{bcx}{\alpha\beta}$
Quarta SS	$ii'i''.\frac{bcdx}{\alpha\beta\gamma}$	$ii'i''i'''.\frac{bcdx}{\alpha\beta\gamma}$

etc.

Denique

Denique ad abbreuiandum ponatur:

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{a}-\frac{2}{k-v}\right)\left(\frac{i}{a}-\frac{2}{k+v}\right)^2\right)$$

$$Q = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{b}-\frac{2}{k'-v'}\right)\left(\frac{i'}{b}-\frac{2}{k'+v'}\right)^2\right)$$

$$R = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-v''}\right)^2+\left(\frac{n}{c}-\frac{2}{k''-v''}\right)\left(\frac{i''}{c}-\frac{2}{k''+v''}\right)^2\right)$$

$$S = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{i'''d}+\frac{2}{k'''-v'''}\right)^2+\left(\frac{n}{d}-\frac{2}{k'''-v'''}\right)\left(\frac{i'''}{d}-\frac{2}{k'''+v'''}\right)^2\right)$$

etc.

His positis ponamus oculum in diſtantia $= O$ ab vltima lente locari ita vt poſt imaginem vltimam reperiatur in diſtantia $= l$: magnitudinem autem viſam comparari cum magnitudine qua idem obiectum nudo oculo in diſtantia fixa $= b$ cerneretur; ac ponatur exponens multiplicationis $= m$.

Deinde pro claritate ſit gradus claritatis $= y$, ita vt y indicet ſemidiametrum coni luminoſi in oculum intrantis.

Confuſio autem aeſtimetur per ſemidiametrum confuſionis ſupra (194) definitum.

Iam pro quouis lentium numero hae tres res ita ſe habebunt.

I. Pro vnica lente $O = a + l$

1. Exponens multiplicationis $m = \frac{1}{i} \cdot \frac{ab}{ai}$ ſitu inuerſo
2. Gradus claritatis $y = il.\ \frac{x}{a}$
3. Semid: Confuſionis $= \frac{1}{4} i \frac{a}{l} x^3 . P$

II.

II. Pro duabus Lentibus $O = \beta + l$

1. Exp. mult: $m = \frac{1}{ii'} \cdot \frac{\alpha\beta b}{abl}$ situ erecto

2. Gradus claritatis $y = ii'l \cdot \frac{bx}{\alpha\beta}$

3. Semid. Confusionis $= \frac{1}{4} ii' \cdot \frac{\beta}{l} \cdot \frac{b}{\alpha} x^3 \left(\frac{1}{i'i'} \cdot \frac{\alpha\alpha}{bb} P + ii \cdot \frac{bb}{\alpha\alpha} Q \right)$.

III. Pro tribus Lentibus $O = \gamma + l$

1. Exp. mult: $m = \frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma b}{abcl}$ situ inverso:

2. Gradus claritatis $y = ii'i''l \cdot \frac{bcx}{\alpha\beta\gamma}$.

3. Semidiameter confusionis;

$$\frac{1}{4} ii'i'' \cdot \frac{\gamma}{l} \cdot \frac{bc}{\alpha\beta} x^3 \left(\frac{1}{i'i'.i''i''} \cdot \frac{\alpha\alpha\beta\beta}{bbcc} P + \frac{ii}{i''i''} \cdot \frac{bb\beta\beta}{\alpha\alpha cc} Q + ii.i'i' \cdot \frac{bbcc}{\alpha\alpha\beta\beta} R \right).$$

IV. Pro quatuor Lentibus $O = \delta + l$

1. Exp. multipl: $m = \frac{1}{ii'i''i'''} \cdot \frac{\alpha\beta\gamma\delta b}{abcdl}$ situ erecto

2. Gradus Claritatis $y = ii'i''i'''l \cdot \frac{bcdx}{\alpha\beta\gamma\delta}$

3. Semidiameter Confusionis:

$$\frac{1}{4} ii'i''i''' \cdot \frac{\delta}{l} \cdot \frac{bcd}{\alpha\beta\gamma} x^3 \left\{ \begin{array}{l} \frac{1}{i'i'.i''i''.i'''i'''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd} P + \frac{ii}{i''i''.i'''i'''} \cdot \frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd} Q \\ + \frac{ii.i'i'}{i'''i'''} \cdot \frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd} R + ii.i'i'.i''i'' \cdot \frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma} S \end{array} \right\}$$

V.

V. Pro quinque Lentibus $O = \varepsilon + l$

1. Exp. multipl: $m = \frac{1}{i\,i'\,i''\,i'''\,i''''} \cdot \frac{\alpha\beta\gamma\delta\varepsilon b}{abcdel}$ situ inuerso

2. Gradus claritatis $y = i\,i'\,i''\,i'''\,i''''\,l \cdot \frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$

3. Semidiameter confusionis

$$\tfrac{1}{4}\,i\,i'\,i''\,i'''\,i'''' \cdot \frac{\varepsilon}{l} \cdot \frac{bcde}{\alpha\beta\gamma\delta}x^3 \left\{ \begin{array}{l} + \frac{1}{i'i'.\,i''i''.\,i'''i'''.\,i''''i''''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee}\,P \\ + \frac{ii}{i''i''.\,i'''i'''.\,i''''i''''} \cdot \frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee}\,Q \\ + \frac{ii.\,i'i'}{i'''i'''.\,i''''i''''} \cdot \frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee}\,R \\ + \frac{ii.\,i'i'.\,i''i''}{i''''i''''} \cdot \frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee}\,S \\ + ii.\,i'i'.\,i''i''.\,i'''i''' \cdot \frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}\,T \end{array} \right.$$

atque hinc etiam progressus ad plures lentes est manifestus.

Coroll. 1.

211. His omnibus casibus euidens est fore generatim
$my = \frac{bx}{a}$.

Datis scilicet multiplicatione m cum claritate y statim definitur apertura primae lentis nempe $x = my \cdot \frac{a}{b}$. Quo maior scilicet tam multiplicatio quam claritas desideratur, eo maiorem esse oportet aperturam lentis primae.

Coroll. 2.

212. Quum autem x maius accipere non liceat, quam vt confusio infra certum limitem contineatur, dato x cum exponente multiplicationis m

definitur claritatis gradus $y = \frac{bx}{ma}$, vnde patet reliquis paribus, quo maior multiplicatio exigatur, eo minori claritate contentos nos esse oportere.

Coroll. 3.

213. Imprimis autem hic obseruandum est, has formulas aeque negotium conficere, quamcunque crassitiem lentes habuerint. *Euadent autem* tractabiliores, si lentium crassities negligatur, qui casus seorsim tractari meretur.

Problema 9.

214. Iisdem positis, quae in problemate praecedente, si lentium crassities vt euanescens consideretur, determinare tam multiplicationem et claritatem, quam confusionem, qua visio perturbabitur.

Solutio.

Haec tractatio a praecedenti in isto differt, quod lentium crassities v, v', v'' etc. euanescant, et loco quantitatum arbitrariarum k, k', k'' etc. numeri arbitrarii λ, λ', λ'' etc. in calculum introducantur. Ponatur ergo

Pro Lente		Dist. determinatr.	num. arb.
Prima	PP	$EA = a$; $aF = \alpha$	λ
Secunda	QQ	$FB = b$; $bG = \beta$	λ'
Tertia	RR	$GC = c$; $cH = \gamma$	λ''
Quarta	SS	$HD = d$; $dI = \delta$	λ'''
		etc.	

Hinc

Hinc si sit breuitatis gratia!

$\varrho = 0,190781$; $\sigma = 1,627401$; et $\tau = 0,905133$

lentium constructio ita est instituenda

Pro Lente	Radius Faciei anterioris	Radius Faciei posterioris
Prima PP	$\frac{a\alpha}{\varrho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda - 1)}}$	$\frac{a\alpha}{\varrho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda - 1)}}$
Secunda QQ	$\frac{b\beta}{\varrho\beta + \sigma b \pm \tau(b+\beta)\sqrt{(\lambda' - 1)}}$	$\frac{b\beta}{\varrho b + \sigma\beta \mp \tau(b+\beta)\sqrt{(\lambda' - 1)}}$
Tertia RR	$\frac{c\gamma}{\varrho\gamma + \sigma c \pm \tau(c+\gamma)\sqrt{(\lambda'' - 1)}}$	$\frac{c\gamma}{\varrho c + \sigma\gamma \mp \tau(c+\gamma)\sqrt{(\lambda'' - 1)}}$
Quarta SS	$\frac{d\delta}{\varrho\delta + \sigma d \pm \tau(d+\delta)\sqrt{(\lambda''' - 1)}}$	$\frac{d\delta}{\varrho d + \sigma\delta \mp \tau(d+\delta)\sqrt{(\lambda''' - 1)}}$

Cum iam lentis primae PP semidiameter aperturae sit $= x$, et in qualibet lente vtriusque faciei eadem sit ratio, vt omnes radii per primam ingressi simul per reliquas transmittantur, apertura reliquarum sequentes limites superare debet:

Semid. aperturae

Lentis secundae QQ $> \frac{b}{\alpha}x$

Lentis tertiae RR $> \frac{bc}{\alpha\beta}x$

Lentis quartae SS $> \frac{bcd}{\alpha\beta\gamma}x$

etc.

Tum vero posito breuitatis ergo $\mu = 0,938191$ et $\nu = 0,232692$ statuatur:

$$P = \mu\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)^2 + \tfrac{\nu}{a\alpha}\right)$$

$$Q = \mu\left(\tfrac{1}{b} + \tfrac{1}{\beta}\right)\left(\lambda'\left(\tfrac{1}{b} + \tfrac{1}{\beta}\right)^2 + \tfrac{\nu}{b\beta}\right)$$

$$R = \mu\left(\tfrac{1}{c} + \tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c} + \tfrac{1}{\gamma}\right)^2 + \tfrac{\nu}{c\gamma}\right)$$

$$S = \mu\left(\tfrac{1}{d} + \tfrac{1}{\delta}\right)\left(\lambda'''\left(\tfrac{1}{d} + \tfrac{1}{\delta}\right)^2 + \tfrac{\nu}{d\delta}\right)$$

etc.

Sit iam O diſtantia oculi poſt vltimam lentem, ita vt ab imagine poſtrema diſtet interuallo $= l$, comparetur magnitudo viſa cum ea, qua idem obiectum in diſtantia fixa b nudo oculo cerneretur, ſitque exponens multiplicationis $= m$; et gradus claritatis $= y$, quibus poſitis erit pro quouis lentium numero vt ſequitur.

I. Pro vnica lente $O = \alpha + l$

1. Exponens multiplicationis $m = \frac{\alpha b}{al}$ ſitu inuerſo
2. Gradus claritatis $y = l.\frac{x}{\alpha}$ hinc $my = \frac{bx}{a}$
3. Semid: confuſionis $= \frac{\alpha}{4l}. x^3 P$

II. Pro duabus lentibus $O = \beta + l$

1. Exponens multiplicationis $m = \frac{\alpha\beta b}{abl}$ ſitu erecto
2. Gradus claritatis $y = l.\frac{bx}{\alpha\beta}$, hinc $my = \frac{bx}{a}$
3. Semid: confuſionis $= \frac{\beta}{4l}.\frac{b}{\alpha}x^3 (\frac{\alpha\alpha}{bb}P + \frac{bb}{\alpha\alpha}Q)$

III. Pro tribus lentibus $O = \gamma + l$

1. Exponens multiplicationis $m = \frac{\alpha\beta\gamma b}{abcl}$ ſitu inuerſo
2. Gradus claritatis $y = l.\frac{bcx}{\alpha\beta\gamma}$ hinc $my = \frac{bx}{a}$
3. Semidiameter confuſionis

$$\frac{\gamma}{4l}.\frac{bc}{\alpha\beta}x^3(\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{bb\beta\beta}{\alpha\alpha cc}Q + \frac{bbcc}{\alpha\alpha\beta\beta}R).$$

IV. Pro quatuor lentibus $O = \delta + l$

1. Exponens multiplicationis $m = \frac{\alpha\beta\gamma\delta b}{abcdl}$ ſitu erecto
2. Gradus claritatis $y = l\frac{bcdx}{\alpha\beta\gamma\delta}$ hinc $my = \frac{bx}{a}$
3. Semidiameter confuſionis:

$$\frac{\delta}{4l}.\frac{bcd}{\alpha\beta\gamma}x^3(\frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}P + \frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}Q + \frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}R + \frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}S)$$

V.

V. Pro quinque lentibus $O = \varepsilon + l$

1. Exponens multipl. $m = \frac{\alpha\beta\gamma\delta\varepsilon b}{abcdel}$ ſitu inuerſo

2. Gradus claritatis $y = l.\frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$ hinc $my = \frac{bx}{a}$

3. Semidiameter confuſionis

$$\frac{\varepsilon}{el} \cdot \frac{bcde}{\alpha\beta\gamma\delta} x^3 \left\{ \begin{array}{l} + \frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee} P + \frac{bb\beta\beta\gamma\gamma\delta\delta}{aaccddee} Q \\ + \frac{bbcc\gamma\gamma\delta\delta}{aa\beta\beta ddee} R \\ + \frac{bbccdd\delta\delta}{aa\beta\beta\gamma\gamma ee} S + \frac{bbccddee}{aa\beta\beta\gamma\gamma\delta\delta} T \end{array} \right.$$

vnde non difficile erit has formulas ad lentes etiam plures continuare.

Coroll. 1.

215. Lentes ſimplices adhibendo numeros $\lambda, \lambda', \lambda''$, etc. vnitate minores accipi non poſſunt. Verum hic nihil obſtat quo minus loco lentium ſimplicium lentes duplicatae, triplicatae vel etiam quadruplicatae in vſum vocentur, quo pacto in his formulis numeri $\lambda, \lambda', \lambda''$, etc. non ſolum infra vnitatem diminui ſed ad nihilum vſque perduci poterunt. Tum autem conſtructio lentium harum multiplicatarum ex ſuperiori capite peti et ad diſtantias determinatrices hic poſitas accommodari debet.

Coroll. 2.

216. Veluti ſi pro lente prima PP debeat eſſe

 $\lambda =$

$\lambda = 0,191827$, hanc lentem ita ex duabus componi oportet, vt sit

Pro lente	Radius faciei anterioris	Radius faciei posterioris
Priori . .	$\frac{2a\alpha}{(2\varrho-\sigma)\alpha+\sigma a}$;	$\frac{2a\alpha}{(2\sigma-\varrho)\alpha+\varrho a}$
Posteriori	$\frac{2a\alpha}{\varrho\alpha+(2\sigma-\varrho)a}$;	$\frac{2a\alpha}{\sigma\alpha+(2\varrho-\sigma)a}$.

Coroll. 3.

217. Sin autem velimus, vt pro lente prima sit $\lambda = 0,042165$, ea erit triplicanda, hoc modo

Pro lente	Radius faciei anterioris	Radius faciei posterioris
Priori	$\frac{3a\alpha}{(3\varrho-2\sigma)\alpha+\sigma a}$;	$\frac{3a\alpha}{(3\sigma-2\varrho)\alpha+\varrho a}$
Media	$\frac{3a\alpha}{(2\varrho-\sigma)\alpha+(2\sigma-\varrho)a}$;	$\frac{3a\alpha}{(2\sigma-\varrho)\alpha+(2\varrho-\sigma)a}$
Posteriori	$\frac{3a\alpha}{\varrho\alpha+(3\sigma-2\varrho)a}$;	$\frac{3a\alpha}{\sigma\alpha+(3\varrho-2\sigma)a}$

Coroll. 4.

218. At si pro lente prima requiratur $\lambda = -0,010216$, quadruplicata erit vtendum ita construenda:

Pro lente	Radius faciei anterioris	Radius faciei posterioris
Prima	$\frac{4a\alpha}{(4\varrho-3\sigma)\alpha+\sigma a}$;	$\frac{4a\alpha}{(4\sigma-3\varrho)\alpha+\varrho a}$
Secunda	$\frac{4a\alpha}{(3\varrho-2\sigma)\alpha+(2\sigma-\varrho)a}$;	$\frac{4a\alpha}{(3\sigma-2\varrho)\alpha+(2\varrho-\sigma)a}$
Tertia	$\frac{4a\alpha}{(2\varrho-\sigma)\alpha+(3\sigma-2\varrho)a}$;	$\frac{4a\alpha}{(2\sigma-\varrho)\alpha+(3\varrho-2\sigma)a}$
Quarta	$\frac{4a\alpha}{\varrho\alpha+(4\sigma-3\varrho)a}$;	$\frac{4a\alpha}{\sigma\alpha+(4\varrho-3\sigma)a}$

Coroll.

Coroll. 5.

219. Simili autem modo lens fecunda QQ ex fuis diftantiis determinatricibus b et β per multiplicationem erit conftruenda, numerus que ei refpondens λ' debet effe vel $0,191827$, vel $0,042165$ vel $-0,010216$; quod idem de reliquis lentibus eft intelligendum.

Scholion 1.

220. Alias lentium fpecies hic nollem in praxi adhiberi, cum hae folae fine metu enormis erroris confici queant; tum vero etfi aliae ab his non admodum difcrepantes fere aequo fucceffu in praxin introduci poffent, tamen quia difcrimen non admodum eft notabile, iis facile carere poterimus. Cum igitur fit $\rho = 0,190781$ et $\sigma = 1,627401$, ideoque

$$\rho = 0,190781; \qquad \sigma = 1,627401$$
$$2\rho - \sigma = -1,245839; \quad 2\sigma - \rho = 3,064021$$
$$3\rho - 2\sigma = -2,682459; \quad 3\sigma - 2\rho = 4,500641$$
$$4\rho - 3\sigma = -4,119079; \quad 4\sigma - 3\rho = 5,937261$$

easdem conftructiones in numeris euolutis exhiberi conueniet.

I. Si igitur pro lente PP debeat effe $\lambda = 1$,

ea erit fimplex hoc modo conftruenda.

Radius faciei anterioris pofterioris

$$\frac{\alpha a}{+0,190781\,a + 1,627401\,\alpha}; \quad \frac{\alpha a}{+1,627401\,a + 0,190781\,\alpha}$$

II.

II. Si pro lente PP debeat esse $\lambda = 0,191827$ ea erit duplicata hoc modo construenda

Radius Faciei

Pro lente	anterioris	posterioris
Priori	$\frac{a\alpha}{-0,622515\alpha+0,013700a}$;	$\frac{a\alpha}{+1,332010\alpha+0,095390a}$
Posteriori	$\frac{a\alpha}{+0,095390\alpha+1,332010a}$;	$\frac{a\alpha}{+0,013700\alpha-0,622515a}$

III. Si pro lente PP debeat esse $\lambda = 0,042165$ ea erit triplicata hoc modo construenda

Radius Faciei

Pro lente	anterioris	posterioris
Priori	$\frac{a\alpha}{-0,894103\alpha+0,342467a}$;	$\frac{a\alpha}{+1,500214\alpha+0,063594a}$
Media	$\frac{a\alpha}{-0,415200\alpha+1,021340a}$;	$\frac{a\alpha}{+1,021340\alpha-0,415200a}$
Posteriori	$\frac{a\alpha}{+0,063594\alpha+1,500214a}$;	$\frac{a\alpha}{+0,342467\alpha-0,894103a}$

IV. Si pro lente PP debeat esse $\lambda = -0,010216$ ea erit quadruplicata hoc modo construenda

Radius Faciei

Pro lente	anterioris	posterioris
Prima	$\frac{a\alpha}{-1,025770\alpha+0,406850a}$;	$\frac{a\alpha}{+1,484315\alpha+0,047655a}$
Secunda	$\frac{a\alpha}{-0,670615\alpha+0,766005a}$;	$\frac{a\alpha}{+1,125160\alpha-0,311460a}$
Tertia	$\frac{a\alpha}{-0,311460\alpha+1,125160a}$;	$\frac{a\alpha}{+0,766005\alpha-0,670615a}$
Quarta	$\frac{a\alpha}{+0,047655\alpha+1,484315a}$;	$\frac{a\alpha}{+0,406850\alpha-1,025770a}$

Scholion

Scholion 2.

221. Interim tamen si lentes desideremus, in quibus valor ipsius λ maior sit, quam hic est assumtus, constructio earum leui additione ex his ipsis formulis concinnari poterit. In binis scilicet fractionibus, quibus radii facierum cuiusque lentis designantur, alter denominator augeri alter vero diminui debet eadem quantitate, quae quantitas semper $= \tau(a+\alpha)\sqrt{v}$ denotante v excessum valoris ipsius λ supra ante assumtum. Ita si esse debeat

I. $\lambda = 1+v$; II. $\lambda = 0,191827+v$; III. $\lambda = 0,042165+v$
vel IV. $\lambda = -0,010216+v$

denominatores fractionum in Scholio praecedente traditarum pro quauis lente simplici alternatim sunt augendi et minuendi quantitate $0,905133\,(a+\alpha)\sqrt{v}$. Vnde si lens quadruplicata desideretur, pro qua sit praecise $\lambda = 0$, erit $v = 0,010216$ et $\tau\sqrt{v} = 0,091487$; hincque pro quauis lente simplici denominatorum alter augeri alter vero diminui debet hac quantitate

$$0,091487\alpha + 0,091487a$$

ex quo talis nascitur constructio huiusmodi lentis quadruplicatae pro qua est $\lambda = 0$

Pro lente	Radius faciei anterioris	Radius faciei posterioris
Prima	$\frac{a\alpha}{-1,121257\alpha+0,315363a}$;	$\frac{a\alpha}{+0,575802\alpha+0,139182a}$
Secunda	$\frac{a\alpha}{-0,762102\alpha+0,674518a}$;	$\frac{a\alpha}{+1,216647\alpha-0,215973a}$
Tertia	$\frac{a\alpha}{-0,402547\alpha+1,033673a}$;	$\frac{a\alpha}{+0,857498\alpha-0,579121a}$
Quarta	$\frac{a\alpha}{-0,043792\alpha+1,392828a}$;	$\frac{a\alpha}{+0,498337\alpha-0,938283a}$

Sed hoc casu quaelibet lens adhuc alio modo construi potest: sic quarta ordine retrogrado exposita permutatis a et α dabit alteram formam lentis primae.

Supplementum III.

Ad Problem. 8.

Si lentes ratione refractionis discrepent, vt sit ratio refractionis pro prima lente $= n$, pro secunda $= n'$, pro tertia $= n''$; inde neque in multiplicatione m neque in gradu claritatis quicquam mutatur; at vero in semidiametro confusionis valores litterarum P, Q, R sequenti modo immutari debent.

$$P = \frac{n}{2(n-1)^2}\left\{\begin{aligned}&\left(\frac{n}{a}+\frac{2}{k+\nu}\right)\left(\frac{1}{ia}+\frac{2}{k-\nu}\right)^2\\&+\left(\frac{n}{\alpha}-\frac{2}{k-\nu}\right)\left(\frac{i}{\alpha}-\frac{2}{k+\nu}\right)^2\end{aligned}\right.$$

$$Q = \frac{n'}{2(n'-1)^2}\left\{\begin{aligned}&\left(\frac{n'}{b}+\frac{2}{k'+\nu'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-\nu'}\right)^2\\&+\left(\frac{n'}{\beta}-\frac{2}{k'-\nu'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+\nu'}\right)^2\end{aligned}\right.$$

$$R = \frac{n''}{2(n''-1)^2}\left\{\begin{aligned}&\left(\frac{n''}{c}+\frac{2}{k''+\nu''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-\nu''}\right)^2\\&+\left(\frac{n''}{\gamma}-\frac{2}{k''-\nu''}\right)\left(\frac{i''}{\gamma}-\frac{2}{k''+\nu''}\right)^2\end{aligned}\right.$$

$$S = \frac{n'''}{2(n'''-1)^2}\left\{\begin{aligned}&\left(\frac{n'''}{d}+\frac{2}{k'''+\nu'''}\right)\left(\frac{1}{i'''d}+\frac{2}{k'''-\nu'''}\right)^2\\&+\left(\frac{n'''}{\delta}-\frac{2}{k'''-\nu'''}\right)\left(\frac{i'''}{\delta}-\frac{2}{k'''+\nu'''}\right)^2\end{aligned}\right.$$

etc.

Ad Problem. 9.

Si lentes ratione refractionis discrepent, loco litterarum ϱ, σ, τ in radiis facierum scribi oportet pro

pro fecunda lente ϱ', σ', τ'; pro tertia ϱ'', σ'', τ'', etc. tum vero expreſſiones pro multiplicatione et gradu claritatis nulla mutatione egent; pro confuſione autem notari oportet, fore

$$P = \mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

$$Q = \mu'\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right)$$

$$R = \mu''\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu''}{c\gamma}\right)$$

etc.

CAPVT V.

DE CAMPO APPARENTE OCVLIQVE LOCO MAXIME IDONEO.

Problema I.

222.

Tab. II. Fig. 10. Si ex obiecti puncto extra axem sumto ε radius quicunque εM per lentem PP transmittatur, definire eius concursum cum axe O.

Solutio.

Sit $F\zeta$ imago principalis per lentem proiecta, a diffusione enim hic mentem abstrahimus, ac ponamus pro lente eius distantias determinatrices $AE=a$, $aF=\alpha$, eius crassitiem $Aa=v$, et quantitatem arbitrariam $=k$, sitque breuitatis ergo $\frac{k-v}{k+v}=i$. His positis si puncti ε imago cadat in ζ, voceturque $E\varepsilon=z$, erit $F\zeta=\frac{1}{i}\cdot\frac{\alpha z}{a}$ at pro puncto lentis M statuatur eius distantia ab axe $AM=x$; ac supra ostendimus si radius a puncto E per punctum M transmitteretur, eum ita per m ad punctum F progressurum esse, vt foret $am=ix$. Radius igitur εM per punctum N ad ζ feretur, ita vt, cum obliquitas radiorum

rum in facies lentis incidentium perpetuo valde parva statuatur, sit proxime angulus $EM\varepsilon$ ad NMm vt n ad 1 posito $n=\frac{31}{20}$. Hinc erit $E\varepsilon$ ad mN in ratione composita istorum angulorum, et distantiarum AE ad Aa, seu $E\varepsilon : mN = na : v$, vnde fit $mN = \frac{vz}{na}$, ideoque $aN = ix - \frac{vz}{na}$. Iam vero radius a puncto N recta ad ζ pergit, et propterea axem ita in O secabit, vt sit $aN + F\zeta : aF = aN : aO$ sicque

$$aO = \left(i\alpha x - \frac{\alpha vz}{na}\right) : \left(ix - \frac{vz}{na} + \frac{\alpha z}{ia}\right) = \frac{niia\alpha x - i\alpha vz}{niiax - ivz + n\alpha z}$$

et $FO = \frac{n\alpha\alpha z}{niiax - ivz + n\alpha z}$

Si x sit semidiameter aperturae lentis in facie anteriori, haec intersectio O respondet casui, quo radius ab ε per lentis terminum summum transeat: sin autem is per terminum imum transmittatur, sumto x negatiuo fiet

$$FO = \frac{n\alpha\alpha z}{-niiax - ivz + n\alpha z}$$

At si radius ex ε per centrum lentis A transeat, punctum intersectionis ita cadet in O vt sit

$$FO = \frac{n\alpha\alpha}{na - iv}$$

Coroll. 1.

223. Si igitur x denotet semidiametrum aperturae lentis in facie anteriori $PMAP$, vt omnes radii a puncto ε in hanc faciem incidentes per lentem transmittantur, necesse est, vt faciei posterioris

PNaP ſemidiameter ſit maior quam $\pm ix - \frac{vz}{na}$, ſumto x tam negatiuo quam poſitiuo. Vnde hic ſemidiater minor eſſe nequit, quam $ix + \frac{vz}{na}$.

Coroll. 2.

224. Si apertura in facie anteriori euaneſcat, vt ſit $x = 0$ radii a puncto ε, cuius ab axe diſtantia $E\varepsilon = z$, per lentem non tranſmittentur, niſi in facie poſteriori ſemidiameter aperturae ſit $= \frac{vz}{na}$ vel maior. Vnde patet quo maior fuerit lentis craſſities v eo maiori apertura in facie poſteriori eſſe opus.

Coroll. 3.

225. Viciſſim ergo ſi detur lentis apertura in facie poſteriori, cuius ſemidiameter ſit $= \frac{vz}{na}$; inde ſimul in obiecto extremum punctum ε determinatur, a quo radius in centrum lentis A incidens per eam tranſmittatur.

Coroll. 4.

226. Hic autem radius per A immiſſus poſt tranſitum cum axe in O occurret, vt ob $x = 0$ ſit interuallum $FO = \frac{naa}{na - iv}$. ſeu $aO = \frac{-iav}{na - iv}$. Niſi ergo oculus in hoc axis loco teneatur, radius transmiſſus non in oculum ingredietur; ſi quidem apertura pupillae vt infinite parua ſpectetur.

Coroll.

Coroll. 5.

227. At si semidiameter pupillae ponatur $= \omega$, oculus etiam in o positus illum radium excipiet, si fuerit $o\,\omega = \omega$. Cum autem casu $x = 0$ sit

$$a\mathrm{N}\left(\frac{vz}{n\alpha}\right) : a\mathrm{O}\left(\frac{i\alpha v}{n\alpha - iv}\right) = o\,\omega\,(\omega) : \mathrm{O}o \text{ erit}$$

$$\mathrm{O}o = \frac{ni\alpha\omega}{z(n\alpha - iv)}.$$

Quod interuallum cum aeque positiuum ac negatiuum accipi possit, pro loco oculi o habebimus

$$ao = -\frac{i\alpha vz \pm ni\alpha\omega}{z(n\alpha - iv)} = \frac{-i\alpha(vz \pm n\alpha\omega)}{z(n\alpha - iv)}$$

Scholion.

228. In hac tractatione, vbi campum apparentem et locum oculi idoneum inuestigamus, tam aperturam lentis obiectiuae in facie anteriori quam pupillae amplitudinem pro nihilo habebimus, vt quaestiones obtineamus determinatas. Quare horum elementorum, quae in instrumentis dioptricis ad visionem accommodatis maximi sunt momenti, sequentes definitiones constituemus.

Definitio I.

229. Campus apparens est spatium in obiecto, ex cuius singulis punctis radii in centrum lentis obiectiuae incidentes per reliquas lentes omnes transmittuntur. Quod spatium cum sit circulare, eius radius vocatur semidiameter campi apparentis.

Coroll.

Coroll. 1.

230. Si ergo $E\epsilon = z$ fuerit ſemidiameter campi apparentis, erit ϵ punctum obiecti extremum, ſed ab axe maxime remotum, ex quo adhuc radii in centrum A lentis obiectiuae incidentes per omnes lentes transmittuntur.

Coroll. 2.

231. Determinatur igitur magnitudo campi apparentis per aperturam ſequentium facierum refringentium, ac fortaſſe per aperturam vnius, ſi ſcilicet radii a puncto quodam magis remoto quam ϵ venientes nullum transitum per eam inuenirent, etiamſi per reliquas facies transmitterentur.

Scholion.

232. Si radii ex ſolo puncto E, quod eſt centrum campi apparentis, conſiderentur, ii quidem qui in A incidunt, quoniam perpetuo ſecundum axem progrediuntur, per omnes reliquas facies refringentes, quantumuis parua fuerit earum apertura, certe transmittentur; hincque campus apparens nunquam penitus euaneſcere poteſt. At quo magis punctum ϵ ab axe diſtans accipitur, vt radii ab eo per omnes facies transmittantur, eo maior earum apertura requiritur, quae cum ab earum curuatura pendeat, neque certum limitem ſuperare debeat, hinc vltimum punctum ϵ, vnde radii etiam nunc transmittuntur, ac propterea ſemidiameter campi apparentis determinatur.

natur. In sequentibus quidem propositionibus campum apparentem seu eius semidiametrum $Ee = z$ vt datum assumam, et quanta esse debeat cuiusque faciei apertura, inuestigabo: hinc enim facile vicissim, si quaeque apertura fuerit cognita, campum apparentem ipsum definire licebit. Ceterum in hac definitione assumsi aperturam primae faciei esse euanescentem: ex quo facile intelligitur ea aucta etiam campum aliquantum extendi oportere; verum hoc augmentum postea in lucrum cedet, quod cum nunquam soleat esse notabile, eius rationem hic non habendam censui; quemadmodum etiam in sequente definitione aperturae pupillae rationem non habebo.

Definitio 2.

233 Locus oculi idoneus est id punctum in axe, [illegible] quo radii ab extremitate campi apparentis per lentes transmissi axem intersecant. Oculus scilicet in hoc loco constitutus totum campum apparentem conspiciet.

Coroll. 1.

234. Hinc igitur idoneus locus oculo assignabitur, si illa intersectio radiorum extremorum cum axe post lentem vltimam cadat, sin autem haec intersectio ante lentem vltimam reperiatur, fieri nequit vt oculus in eo loco teneatur neque propterea totum campum contueri poterit.

Coroll. 2.

235. At si ista intersectio pone lentem vltimam cadat, oculus totum campum apparentem perspiciet, etiamsi pupilla maxime esset contracta: neque tamen ob maiorem pupillae amplitudinem maiorem campum percipere valet.

Coroll. 3.

236. Verum ob amplitudinem pupillae hoc commodi assequimur, vt oculus etiamsi extra locum idoneum constituatur, dummodo distantia non sit nimis magna, tamen totum campum apparentem conspicere possit: id quod egregie vsu veniet iis casibus, quibus locus idoneus oculi ante faciem refringentem extremam cadit. Tum enim fieri poterit, vt oculus huic faciei immediate applicatus tamen totum campum percipiat.

Scholion.

237. Quando hic de visione loquor, id ita in genere est interpretandum, vt a puncto viso radius in oculum ingrediatur, neque hic curo, vtrum visio sit distincta nec ne? in sequentibus enim docebitur, quomodo lentes disponi conueniat, vt oculus in loco idoneo positus etiam in iusta ab vltima imagine distantia reperiatur, quo visio distincta reddatur. Hic igitur sine vllo respectu ad visionem distinctam habito, eum oculo locum assigno, vbi ab omnibus punctis

ctis in campo apparente contentis radios recipiat, et quoniam singula momenta, quae ad visionem pertinent, seorsim expediri conuenit; hic etiam non ad spatium diffusionis respicio, quod quidem semper per se euanescit, si apertura faciei primae euanescens statuatur.

Problema 2.

238. Si obiectum per vnicam lentem aspiciatur, determinare tam campum apparentem, quam locum idoneum oculi. Tab. III. Fig. 12.

Solutio.

Sint vt in problemate superiori distantiae determinatrices huius lentis, scilicet distantia obiecti ante lentem $AE = a$, et imaginis post lentem $aF = \alpha$, tum vero lentis crassities $Aa = v$, et distantia arbitraria constructionem lentis plene determinans $= k$. Deinde ponamus semidiametrum campi apparentis $E\varepsilon = z$, ita vt posita faciei anterioris PAP apertura infinite parua etiamnum a puncto ε radii per lentem transmittantur. Sit porro breuitatis gratia $\frac{k-v}{k+v} = i$, atque supra demonstrauimus campum apparentem ad ε vsque extendi, si pro facie posteriori PaP fuerit semidiameter aperturae $aN = \frac{vz}{na}$ existente $n = \frac{31}{20}$; perinde enim est, siue hic semidiameter affirmatiue accipiatur siue negatiue. Hinc ergo viciſſim si semidiameter huius aperturae ponatur $= \alpha$, erit semidiameter campi apparentis $E\varepsilon = z = \frac{na}{v}\alpha$.

 Quod

Quod ad locum idoneum oculi attinet, qui ſit in O, quoniam inuenimus $FO = \frac{n\alpha a}{n\alpha - iv}$, erit diſtantia $aO = \frac{-i\alpha v}{n\alpha - iv}$; ideoque negatiua, niſi ſit vel i numerus negatiuus, vel $iv > n\alpha$. Sin autem haec diſtantia aO fuerit poſitiua, oculus in O poſitus totum campum apparentem perſpiciet.

Coroll. 1.

239. Si craſſities lentis v euaneſcat, ob $z = \frac{n\alpha}{v} a$, campus apparens euadet infinitus, ſeu potius indeterminatus; diſtantia vero aO euaneſcet. Oculus igitur lenti immediate applicatus tantum ſpatium conſpiciet, quantum per propriam indolem complecti valebit.

Coroll. 2.

240. Sin autem ob lentis craſſitiem $Aa = v$ diſtantia $aO = \frac{-i\alpha v}{n\alpha - iv}$ prodeat poſitiua, oculus in O poſitus totum campum apparentem aſpicere valebit, ſeu in obiecto ſpatium circulare ſpectabit, cuius ſemidiameter $E\varepsilon = z = \frac{n\alpha}{v} a = \frac{r i \alpha a}{s i v}$, quod ergo eo erit maius, quo tenuior fuerit lens.

Coroll. 3.

241. At ſi diſtantia aO reſultet negatiua, oculum in loco idoneo conſtitui non licet, quoniam is neceſſario poſt lentem teneri debet. Vbicunque autem is poſt lentem collocetur, non vniuerſum campum apparentem contuebitur, ſed tantum eius partem, et quidem

quidem eo minorem, quo magis post lentem remoueatur, propterea quod hoc modo magis a loco idoneo recedit.

Coroll. 4.

242. Hoc igitur casu conueniet oculum immediate ad faciem lentis posteriorem applicare, quo situ eatenus tantum radios accipiet, quatenus pupilla patet; sicque campus visus ab apertura pupillae pendebit; quae si esset nulla etiam campus apparens euanesceret.

Coroll. 5.

243. Hinc patet si pupilla excedit aperturam faciei PaP seu si sit $\omega > a$, denotante ω semidiametrum pupillae, quia tum oculus huic faciei applicatus omnes radios transmissos recipit, eum totum campum apparentem esse visurum. Sin autem sit $\omega < a$, partem tantum totius campi perspiciet, cuius semidiameter erit $= \frac{n a}{v} \omega = \frac{3 r a \omega}{2 v}$, scilicet non maiorem quam si apertura faciei PaP aequalis esset pupillae.

Scholion.

244. Interim tamen si hoc casu postremo apertura faciei lentis PaP, cui oculus est applicatus maior sit quam pupilla, nihil obstat quominus ea successiue totam aperturam peragret, sicque pedetentim totum campum apparentem con-

ſpicere poterit, etiamſi eum ſimul contueri non valeat. Ceterum notandum eſt, ſi etiam faciei anteriori apertura tribuatur, inde campum apparentem aliquantum augeri, ſed partes obiecti vlteriores, quia non per medium lentis radios transmittunt, obſcuriores apparebunt, vnde merito a campo apparente excluduntur. At ſi faciei anteriori tribuatur apertura, cuius ſemidiameter $= x$, vt facies poſterior omnes radios in illam incidentes transmittat, eius apertura tanto maior eſſe debet, ſecundum regulas ſupra traditas; ſcilicet eius ſemidiametrum eſſe oportet $= ix + a$ ſeu $= ix + \frac{vz}{na}$.

Problema 3.

245. Si inſtrumentum dioptricum duabus conſtet lentibus, definire campum apparentem, et locum oculi idoneum.

Solutio.

Tab. III. Fig. 13. Sit pro his lentibus vt hactenus:

pro PP: $AE = a, aF = \alpha, Aa = v$; diſt. arb: $= k$, et $\frac{k-v}{k+v} = i$

pro QQ: $BF = b, bG = \beta, Bb = v'$; diſt. arb: $= k'$ et $\frac{k'-v'}{k'+v'} = i'$

ac poſito ſemidiametro campi apparentis $E\varepsilon = z$, in imaginibus erit $F\zeta = \frac{1}{i} \cdot \frac{\alpha}{a} z$ et $G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z$. Conſideretur iam radius a puncto ε in centrum lentis primae A incidens et per lentes transmiſſus qui vtique per extremitates imaginum ζ et η tranſibit.

Iam

Iam ex problemate praecedente patet esse $a\mathrm{N} = \frac{vz}{na}$, vnde in altera lente punctum M' ita definietur, vt sit

$$\mathrm{F}\zeta - a\mathrm{N} : a\mathrm{F} = \mathrm{BM}' - \mathrm{F}\zeta : \mathrm{BF} \text{ siue}$$

$$\mathrm{BM}' = \mathrm{F}\zeta + \frac{\mathrm{BF}(\mathrm{F}\zeta - a\mathrm{N})}{a\mathrm{F}} = \frac{a\mathrm{B}.\mathrm{F}\zeta - \mathrm{BF}.a\mathrm{N}}{a\mathrm{F}}$$

vnde fit

$$\mathrm{BM}' = \tfrac{1}{i}.\frac{a+b}{a}z - \frac{bvz}{naa}.$$

Nunc punctum N' hinc perinde definietur, atque ex problemate primo ex puncto M determinabatur punctum N; erit quippe

$$b\mathrm{N}' = i'.\mathrm{BM}' - \frac{v'}{n'b}.\mathrm{F}\zeta \text{ ideoque}$$

$$b\mathrm{N}' = \frac{i'}{i}.\frac{a+b}{a}z - \frac{i'bv}{naa}z - \frac{1}{i}.\frac{av'}{n'ab}z$$

Hinc autem punctum O, vbi est locus oculi idoneus facile assignabitur erit enim $b\mathrm{N}' + \mathrm{G}\eta : b\mathrm{G} = b\mathrm{N}' : b\mathrm{O}$, indeque

$$b\mathrm{O} = \frac{b\mathrm{G}.b\mathrm{N}'}{b\mathrm{N}' + \mathrm{G}\eta} = \frac{\frac{i'}{i}.\frac{a+b}{a} - \frac{i'bv}{naa} - \frac{1}{i}.\frac{av'}{nab}}{\frac{i'}{i}.\frac{a+b}{a} - \frac{i'bv}{naa} - \frac{1}{i}.\frac{av'}{nab} + \frac{1}{ii'}.\frac{a\epsilon}{ab}}\,\mathfrak{E}$$

Quod si ergo ponamus semidiametrum aperturae

pro lente PP $\begin{cases} \text{faciei anterioris} = \mathfrak{A} = x \\ \text{faciei posterioris} = \mathfrak{a} \end{cases}$

pro lente QQ $\begin{cases} \text{faciei anterioris} = \mathfrak{B} \\ \text{faciei posterioris} = \mathfrak{b} \end{cases}$

habe-

habebimus

$$\mathfrak{A} = 0$$

$$\mathfrak{a} = \frac{v}{na} z$$

$$\mathfrak{B} = \left(\frac{1}{i} \cdot \frac{\alpha+b}{a} - \frac{b\,v}{na\alpha}\right) z$$

$$\mathfrak{b} = \left(\frac{i'}{i} \cdot \frac{\alpha+b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{nab}\right) z$$

ac ſi diſtantia oculi poſt lentem QQ ponatur $bO = O$ erit

$$O = \frac{\mathfrak{b}}{\mathfrak{b} + \frac{1}{ii'} \cdot \frac{\alpha v'}{ab} z} \cdot \mathfrak{B}$$

Coroll. 1.

246. Si ambarum lentium craſſities euaneſcat erit $v = 0$. $v' = 0$, et $i = i' = 1$; quo ergo caſu noſtrae formulae in ſequentes abibunt:

$$\mathfrak{A} = 0;\ \mathfrak{a} = 0;\ \mathfrak{B} = \frac{\alpha+b}{a} z \text{ et } \mathfrak{b} = \frac{\alpha+b}{a} z$$

Coroll. 2.

247. Datis ergo viciſſim aperturis lentium ex aequationum traditarum ea, pro qua quantitas z minimum valorem adipiſcitur, definietur campus adparens.

Coroll. 3.

248. Si igitur craſſities lentium euaneſcat, campus apparens ex apertura lentis poſterioris facillime determinatur. Erit enim $z = \frac{\alpha\,\mathfrak{B}}{\alpha+b}$; id quod intelligendum eſt, ſi diſtantia $bO = O$ fuerit poſitiua oculusque in O collocetur. Coroll.

Coroll. 4.

249. Sin autem diſtantia $bO = O$ prodeat negatiua oculusque vltimae lenti immediate applicetur, tum eius apertura plus non praeſtat quam ſi amplitudini pupillae eſſet aequalis. Quare ſi $\mathfrak{b}$ maior fuerit ſemidiametro pupillae ω loco $\mathfrak{b}$ ſcribatur ω et ex vltima aequatione verus valor ipſius z elicietur niſi forte ex aliqua reliquarum aequationum adhuc minor valor pro z eſſet proditurus.

Problema 4.

250. Si inſtrumentum dioptricum ex tribus conſtet lentibus determinare cum campum apparentem, tum locum oculi idoneum. Tab. III. Fig. 14.

Solutio.

Exiſtentibus imaginibus per has lentes ſucceſſiue repraeſentatis in $F\zeta$, $G\eta$ et $H\theta$, obiecto vero ipſo in Ee, ponamus vt hactenus:

Pro Lente	Diſtantias		craſſitiem	diſt. arb: et
Prima PP..	$AE = a$; $aF = \alpha$;		$Aa = v$;	k; $\frac{k-v}{k+v} = i$
Secunda QQ..	$BF = b$; $bG = \beta$;		$Bb = v'$;	k'; $\frac{k'-v'}{k'+v'} = i'$
Tertia RR..	$CG = c$; $cH = \gamma$;		$Cc = v''$;	k''; $\frac{k''-v''}{k''+v''} = i''$

Semidiametros vero aperturarum

Pro Lente

PP faciei $\begin{cases} \text{anterioris} = \mathfrak{A} = \mathfrak{o} \\ \text{poſterioris} = \mathfrak{a} \end{cases}$

QQ faciei $\begin{cases} \text{anterioris} = \mathfrak{B} \\ \text{posterioris} = \mathfrak{b} \end{cases}$

RR faciei $\begin{cases} \text{anterioris} = \mathfrak{C} \\ \text{posterioris} = \mathfrak{c} \end{cases}$

Tum vero sit semidiameter campi apparentis $E\epsilon = z$, supraque ostendimus fore:

$$F\zeta = \frac{1}{i}\frac{\alpha}{a}z;\ G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab}z;\ H\theta = \frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma}{abc}z.$$

His positis habebimus $aN = \mathfrak{a} = \frac{vz}{na}$; vnde si per F recta ipsi NM' parallela ducta intelligatur erit

$$F\zeta - aN : aF = BM' - F\zeta : BF \text{ siue } BM' = \frac{aB.F\zeta - BF.aN}{aF}$$

ideoque

$$BM' = \frac{\alpha+b}{\alpha} . \frac{1}{i} . \frac{\alpha}{a}z - \frac{bvz}{naa} \text{ seu}$$

$$\mathfrak{B} = \frac{1}{i} . \frac{\alpha+b}{a}z - \frac{b.v}{naa}z$$

Porro vero est ex problemate primo $bN' = i'.BM' - \frac{v'}{nb}.F\zeta$,

hincque $\mathfrak{b} = \frac{i'}{i} . \frac{\alpha+b}{a}z - \frac{i'bv}{naa}z - \frac{1}{i} . \frac{\alpha v'}{nab}z$

simili modo per G ducta intelligatur recta ipsi $N'M''$ parallela, eritque $bG : bN' + G\eta = CG : CM'' - G\eta$ siue

$$CM'' = \mathfrak{C} = G\eta + \frac{c}{\beta}(\mathfrak{b} + G\eta) = \frac{\beta+c}{\beta} . G\eta + \frac{c}{\beta}\mathfrak{b},$$

vnde fit

$$\mathfrak{C} = \frac{1}{ii'} . \frac{\alpha(\beta+c)}{ab}z + \frac{i'}{i} . \frac{c(\alpha+b)}{a\beta}z - \frac{i'bcv}{naa\beta}z - \frac{1}{i} . \frac{acv'}{nab\beta}z$$

Deinde

Deinde ex CM'' ita definitur cN'' per problema primum, vt fit

$$cN'' = i''.\, CM'' - \tfrac{v''}{n c}.\, G\eta \text{ seu } \mathfrak{c} = i''.\, \mathfrak{C} - \tfrac{1}{i i'}.\, \tfrac{\alpha \beta v''}{n a b c} z$$

hincque

$$\mathfrak{c} = \tfrac{i''}{i i'}.\, \tfrac{\alpha(\beta + c)}{a b} z + \tfrac{i' i''}{i}.\, \tfrac{\alpha(\alpha + b)}{a \beta} z - i' i''.\, \tfrac{b c v}{n a \alpha c} z - \tfrac{i''}{i'}.\, \tfrac{\alpha c v'}{n a b c} z - \tfrac{1}{i i'}.\, \tfrac{\alpha \beta v''}{n a b c} z$$

Isti ergo valores sequenti modo determinantur:

$\mathfrak{A} = 0$

$\mathfrak{a} = \frac{v}{n a}.\, E\varepsilon$

$\mathfrak{B} = \frac{\alpha + b}{\alpha}.\, F\zeta - \frac{b}{a}.\, \mathfrak{a}$

$\mathfrak{b} = i'.\, \mathfrak{B} - \frac{v'}{n b}.\, F\zeta$

$\mathfrak{C} = \frac{\beta + c}{\beta}.\, G\eta + \frac{c}{\beta}.\, \mathfrak{b}$

$\mathfrak{c} = i''.\, \mathfrak{C} - \frac{v''}{n c}.\, G\eta$

Cum iam punctum O praebeat locum oculi iustum, si ponamus $cO = O$, erit $cN'' + H\theta : cH = cN'' : cO$ vnde reperitur

$$O = \tfrac{\gamma \mathfrak{c}}{\mathfrak{c} + H\theta}, \text{ vel } \tfrac{1}{O} = \tfrac{1}{\gamma} + \tfrac{1}{i i' i''}.\, \tfrac{\alpha \beta}{a b}.\, \tfrac{z}{c \mathfrak{c}}.$$

Coroll. I.

251. Ex his aequationibus sequitur fore

$$\tfrac{\mathfrak{B}}{b} + \tfrac{\mathfrak{a}}{a} = \tfrac{\alpha + b}{\alpha b}.\, F\zeta = \tfrac{1}{i}\left(\tfrac{\alpha}{b} + 1\right) \tfrac{z}{a} \text{ et}$$

$$\tfrac{\mathfrak{C}}{c} - \tfrac{\mathfrak{b}}{\beta} = \tfrac{\beta + c}{\beta c}.\, G\eta = \tfrac{1}{i i'}\left(\tfrac{\alpha \beta}{b c} + \tfrac{\alpha}{b}\right) \tfrac{z}{a}.$$

hincque porro:

$$\tfrac{i \mathfrak{a}}{a} + \tfrac{i \mathfrak{B}}{b} + \tfrac{i i' \mathfrak{b}}{\beta} - \tfrac{i i' \mathfrak{C}}{c} = \left(1 - \tfrac{\alpha \beta}{b c}\right) \tfrac{z}{a}.$$

 Coroll.

Coroll. 2.

252. Si ergo craſſities lentium euaneſcat, cum ſit $a=0$, $b=\mathfrak{B}$ et $c=\mathfrak{C}$, definitio campi apparentis reducitur ad has duas aequationes.

$$\text{I. } \mathfrak{B}\cdot\frac{1}{b}=\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$$

$$\text{II. } \mathfrak{B}\left(\frac{1}{b}+\frac{1}{c}\right)-\mathfrak{C}\cdot\frac{1}{c}=\left(1-\frac{\alpha\zeta}{bc}\right)\frac{z}{a}$$

vnde minor valor ipſius z praebet ſemidiametrum campi apparentis.

Coroll. 3.

253. Deinde ſi diſtantia $cO=O$ prodeat negatiua, vt oculum cogamur lenti vltimae immediate applicare, pro c ſcribi oportet ſemidiametrum pupillae ω, et ex vltima aequatione definietur ſemidiameter campi z, niſi forte ex alia aequatione adhuc minor valor pro z prodeat.

Problema 5.

254. Si inſtrumentum Dioptricum ex quatuor lentibus ſuper eodem axe diſpoſitis conſtet, determinare cum campum apparentem, tum locum oculi idoneum.

Solutio.

Tab. III. Fig. 15. Exiſtente obiecto $E\varepsilon=z$, ſint imagines per lentes ſucceſſiue repraeſentatae $F\zeta$, $G\eta$, $H\theta$, et $I\iota$, ponamusque

musque pro lentium singularum determinatione vt hactenus:

Pro lente	distantias		crassitiem	dist. arb.	et
Prima PP	$AE = a$;	$aF = \alpha$;	$Aa = v$;	.. k;	$\frac{k-v}{k+v} = i$
Secunda QQ	$BF = b$;	$bG = \beta$;	$Bb = v'$;	.. k',	$\frac{k'-v'}{k'+v'} = i'$
Tertia RR	$CG = c$;	$cH = \gamma$;	$Cc = v''$;	.. k'';	$\frac{k''-v''}{k''+v''} = i''$
Quarta SS	$DH = d$;	$dI = \delta$;	$Dd = v'''$;	.. k''';	$\frac{k'''-v'''}{k'''+v'''} = i'''$

Semidiametri vero aperturarum sint:

Pro Lente

Prima PP faciei $\begin{cases} \text{anterioris} = \mathfrak{A} = o \\ \text{posterioris} = \mathfrak{a} \end{cases}$

Secunda QQ faciei $\begin{cases} \text{anterioris} = \mathfrak{B} \\ \text{posterioris} = \mathfrak{b} \end{cases}$

Tertia RR faciei $\begin{cases} \text{anterioris} = \mathfrak{C} \\ \text{posterioris} = \mathfrak{c} \end{cases}$

Quarta SS faciei $\begin{cases} \text{anterioris} = \mathfrak{D} \\ \text{posterioris} = \mathfrak{d} \end{cases}$

Si iam $F\varepsilon = z$ exhibeat semidiametrum campi apparentis erit, vt iam supra ostendimus:

$$F\zeta = \frac{1}{i} \cdot \frac{\alpha}{a} z; \; G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z; \; H\theta = \frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma}{abc} z$$

$$\text{et } I\iota = \frac{1}{ii'i''i'''} \cdot \frac{\alpha\beta\gamma\delta}{abcd} z$$

Quod fi ratiocinium nunc vt ante inftituamus, obtinebimus fequentes aequationes:

$$\mathfrak{A}=0; \qquad \mathfrak{a}=\frac{v}{na}\cdot E\epsilon$$

$$\mathfrak{B}=\left(1+\frac{b}{\alpha}\right)F\zeta-\frac{b}{\alpha}\mathfrak{a}: \qquad \mathfrak{b}=i'\mathfrak{B}-\frac{v'}{nb}\cdot F\zeta$$

$$\mathfrak{C}=\left(1+\frac{c}{\beta}\right)G\eta+\frac{c}{\beta}\cdot\mathfrak{b}: \qquad \mathfrak{c}=i''\mathfrak{C}-\frac{v''}{nc}\cdot G\eta$$

$$\mathfrak{D}=\left(1+\frac{d}{\gamma}\right)H\theta+\frac{d}{\gamma}\cdot\mathfrak{c}: \qquad \mathfrak{d}=i'''\mathfrak{D}-\frac{v'''}{nd}\cdot H\theta$$

Ex quarum ordine priori confequimur:

$$\frac{\mathfrak{B}}{b}+\frac{\mathfrak{a}}{\alpha}=\left(\frac{1}{\alpha}+\frac{1}{b}\right)F\zeta=\frac{1}{i}\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$$

$$\frac{\mathfrak{C}}{c}-\frac{\mathfrak{b}}{\beta}=\left(\frac{1}{\beta}+\frac{1}{c}\right)G\eta=\frac{1}{ii'}\left(\frac{\alpha}{b}+\frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

$$\frac{\mathfrak{D}}{d}-\frac{\mathfrak{c}}{\gamma}=\left(\frac{1}{\gamma}+\frac{1}{d}\right)H\theta=\frac{1}{ii'i''}\left(\frac{\alpha\beta}{bc}+\frac{\alpha\beta\gamma}{bcd}\right)\frac{z}{a}$$

ex ordine vero pofteriori

$$\mathfrak{a}=\frac{vz}{na}$$

$$\mathfrak{b}=i'\mathfrak{B}-\frac{1}{i}\cdot\frac{\alpha}{b}\cdot\frac{v'z}{na}$$

$$\mathfrak{c}=i''\mathfrak{C}-\frac{1}{ii'}\cdot\frac{\alpha\beta}{bc}\cdot\frac{v''z}{na}$$

$$\mathfrak{d}=i'''\mathfrak{D}-\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{bcd}\cdot\frac{v'''z}{na}$$

Si denique pro loco oculi idoneo ponamus $dO=O$ erit

$$O=\frac{\delta\mathfrak{d}}{\mathfrak{d}+\delta i} \text{ feu } \frac{1}{O}=\frac{1}{\delta}+\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma}{bcd}\cdot\frac{z}{a\mathfrak{d}}.$$

Coroll.

Coroll. 1.

255. Ex aequationibus prioribus deducimus sequentes :

$$\frac{i\mathfrak{a}}{\alpha}+\frac{i\mathfrak{B}}{b}=(1+\frac{\alpha}{b})\frac{z}{a}$$

$$\frac{i\mathfrak{a}}{\alpha}+i\frac{\mathfrak{B}}{b}+\frac{ii'\mathfrak{b}}{\beta}-\frac{ii'\mathfrak{C}}{c}=(1-\frac{\alpha\beta}{bc})\frac{z}{a}$$

$$\frac{i\mathfrak{a}}{\alpha}+\frac{i\mathfrak{B}}{b}+\frac{ii'\mathfrak{b}}{\beta}-\frac{ii'\mathfrak{C}}{c}-\frac{ii'i''\mathfrak{c}}{\gamma}+\frac{ii'i''\mathfrak{D}}{d}=(1+\frac{\alpha\beta\gamma}{bcd})\frac{z}{a}.$$

quae quomodo ad plures lentes sint continuandae, facile perspicitur.

Coroll. 2.

256. Si lentium crassities euanescat, fiet $\mathfrak{a}=0$; $\mathfrak{b}=\mathfrak{B}$; $\mathfrak{c}=\mathfrak{C}$ et $\mathfrak{d}=\mathfrak{D}$, porro $i=i'=i''=i'''=1$, vnde hae aequationes in sequentes formas abibunt:

$$\mathfrak{B}.\frac{1}{b}=(1+\frac{\alpha}{b})\frac{z}{a}$$

$$\mathfrak{B}(\frac{1}{b}+\frac{1}{\beta})-\mathfrak{C}\frac{1}{c}=(1-\frac{\alpha\beta}{bc})\frac{z}{a}$$

$$\mathfrak{B}(\frac{1}{b}+\frac{1}{\beta})-\mathfrak{C}(\frac{1}{c}+\frac{1}{\gamma})+\mathfrak{D}\frac{1}{d}=(1+\frac{\alpha\beta\gamma}{bcd})\frac{z}{a}$$

ex quibus tribus aequationibus, vti in genere, valor ipsius z, qui prodierit minimus verum semidiametrum campi apparentis praebebit.

Coroll. 3.

257. Totus iste campus apparens reuera spectabitur ab oculo in puncto O constituto, dummodo distantia $dO = O$ fuerit positiua. Sed si ea sit negatiua, oculusque lenti vltimo SS immediate applicetur, pona-

ponatur $\mathfrak{b} = \omega$, scilicet semidiametro pupillae, et ex vltima aequatione elicietur semidiameter spatii in obiecto reuera conspicui.

Scholion I.

258. Hinc igitur perspicitur, quomodo campus apparens a singularum lentium apertura pendeat; simulque patet quanta esse debeat cuiusque lentis apertura, vt campus apparens datae magnitudinis obtineatur. Si enim quantitas z cum quantitatibus ad lentium determinationem pertinentibus pro data assumatur, per nostras formulas successiue semidiametri aperturarum pro singulis lentibus definiuntur: vbi quidem deinceps est dispiciendum, num lentes tantae aperturae sint capaces. Hinc scilicet campo apparenti limites praefiniuntur, quos transgredi non liceat; vnde sequitur campum apparentem maiorem assumi non posse, quam vt aperturae inde pro singulis lentibus oriundae admitti queant. His autem definitis perinde est siue cuique lenti ea ipsa, quae fuerit inuenta apertura tribuatur, siue maior, dum ne sit minor quandoquidem hic aperturam lentis obiectiuae euanescentem assumsimus. Verum si insuper claritatis ratio habeatur, necesse est vt vera apertura cuiusque lentis eam, quam hic assignauimus, aliquantum superet, et quidem ea quantitate, quam supra pro limitibus ob claritatem requisitis exhibuimus, nisi enim hoc augmentum accesserit, extremitas in campo apparente minore lumine praedita erit quam

quam medium. Tum autem campus apparens latius patebit oramque obſcuriorem complectetur; quamobrem ſi circa extremitates minori lumine contenti eſſe velimus, ne opus quidem eſt, vt lentibus maior apertura, quam quidem per formulas noſtras definitur, tribuatur; ſuperfluumque foret aperturas vltra hos limites augere, ita vt hinc cuique lenti conueniens apertura conſtituatur.

Scholion 2.

259. Etſi pro caſu, quo lentium craſſities negligitur formulae noſtrae non multo ſimpliciores euadunt, tamen in iis percommode vſu venit, vt litterarum $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, coefficientes, ſcilicet $\frac{1}{b}+\frac{1}{\beta}$; $\frac{1}{c}+\frac{1}{\gamma}$; $\frac{1}{d}+\frac{1}{\delta}$; ipſam diſtantiam focalem, inuoluant cuiuſque lentis; in praxi autem apertura ſatis tuto ex diſtantia focali colligi ſolet. Nam ſi lentis QQ diſtantia focalis ponatur $= q$, erit $\frac{1}{b}+\frac{1}{\beta}=\frac{1}{q}$, ſicque $\mathfrak{B}(\frac{1}{b}+\frac{1}{\beta})=\frac{\mathfrak{B}}{q}$; ac ne arcus nimis magni in apertura comprehendantur, neceſſe eſt vt ſit $\mathfrak{B} < \frac{1}{4}q$; et pro varia lentis forma valor fractionis $\frac{\mathfrak{B}}{q}$ vſque ad $\frac{1}{4}$, vel $\frac{1}{5}$ diminui debet. Quare ſi ponamus

$$\mathfrak{B}(\tfrac{1}{b}+\tfrac{1}{\beta})=\pi;\ \mathfrak{C}(\tfrac{1}{c}+\tfrac{1}{\gamma})=\pi';\ \mathfrak{D}(\tfrac{1}{d}+\tfrac{1}{\delta})=\pi''$$

hae litterae π, π', π'' eiusmodi denotabunt fractiones, quarum valor vt plurimum erit vel $\frac{1}{3}$ vel $\frac{1}{4}$ vel $\frac{1}{5}$, id ſcilicet tantum cauendum eſt, ne his litteris nimis magnus valor tribuatur. Quo obſeruato cum arbitrio noſtro relinquantur, imprimis conueniet has ipſas litteras in calculum introduci, ex iisque reliquas determinari; earum enim beneficio campus apparens facillime definitur. Quin etiam ipſe campus apparens ſtatim quoque in calculum induci poterit, quippe cuius determinatio deinceps per formulam ſimpliciſſimam expedietur. Hunc in finem vt tota inueſtigatio ad meros numeros redigatur, ponam $\frac{z}{a} = \Phi$ ita vt Φ ſit angulus, ſub quo ſemidiameter campi apparentis ab oculo ad lentem obiectiuam collocato ſpectaretur. Videamus ergo quomodo per hos numeros π, π', π'', etc. et Φ reliquae quantitates definiantur.

Definitio 3.

260. Ratio aperturae cuiuſque lentis mihi vocabitur quotus, qui oritur, ſi ſemidiameter aperturae diuidatur per diſtantiam focalem lentis; eius craſſitie pro nihilo habita.

Coroll. 1.

261. Ita ſi b et $\mathfrak{b}$ ſint diſtantiae determinatrices lentis, et $\mathfrak{B}$ ſemidiameter aperturae eiuſdem, quia diſtantia focalis eſt $= \frac{b\mathfrak{b}}{b+\mathfrak{b}}$, ratio aperturae erit $= \mathfrak{B}\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)$

Coroll. 2.

262. Ratio igitur aperturae cuiuſuis lentis eſt fractio minor quam $\frac{1}{2}$, quandoquidem hanc legem ſanciuimus vt neutrius faciei arcus 60 gradibus maior in apertura contineatur.

Coroll.

Coroll. 3.

263. Si ſcilicet ambae facies fuerint aeque curuae ratio aperturae per hanc legem vsque ad $\frac{1}{2}$ augeri poterit; ſin autem altera facies fuerit plana, ratio aperturae $\frac{1}{4}$ ſuperare vix poterit: ac ſi lens ſit meniscus, ea adhuc minor ſtatui debebit.

Coroll. 4.

264. Cum autem nihil ſit, quo apertura lentium accuratius definiatur, ea fere arbitrio noſtro relinquitur, et quouis caſu commodiſſime per experientiam determinatur, ſufficietque notaſſe, eam fractioni ſiue $\frac{1}{3}$ ſiue $\frac{1}{4}$ ſiue etiam $\frac{1}{5}$ pro forma lentis aequalem ſtatui debere.

Scholion.

265. Vt ſcilicet quouis caſu ratio aperturae recte definiatur, radios vtriusque faciei lentis contemplari oportet, qui ſi fuerint f et g, erit diſtantia focalis $= \frac{fg}{(n-1)(f+g)} = \frac{10}{11} \cdot \frac{fg}{f+g}$. Iam ſemidiameter aperturae minor eſſe debet quam $\frac{1}{4}f$ vel quam $\frac{1}{4}g$, prout vel f vel g fuerit minor. Sit $g < f$, et cum ſemidiameter aperturae minor eſſe debeat quam $\frac{1}{4}g$, ratio aperturae minor accipienda eſt quam $\frac{11}{40}\left(1+\frac{g}{f}\right)$. Vnde patet, ſi lens ſit vtrinque aeque conuexa, ſeu $g = f$, rationem aperturae capi debere infra $\frac{11}{20}$; ſin autem ſit altera facies plana ſeu $f = \infty$, illum limitem eſſe $\frac{11}{40}$, qui adhuc minor fiet, ſi lens ſit

 menimen'fcus, feu $\frac{\mathfrak{E}}{f}$ numerus negatiuus. Ceterum fi ratio aperturae fit $= \pi$, eique hoc modo idoneus valor tribuatur, perinde eft fiue is negatiue fiue pofitiue accipiatur: femper autem conducet rationi aperturae minorem valorem tribui, quam fecundum hanc regulam; partim vt obliquitas radiorum incidentium diminuatur, partim vero potiffimum, vt ob claritatem aperturas lentium adhuc vltra augere liceat.

Problema 6.

266. Si inftrumentum dioptricum ex quotcunque lentibus, quarum craffitiem vt nullam fpectare liceat, fit compofitum, dataque fit ratio aperturae pro fingulis lentibus vna cum campo apparente, definire diftantias determinatrices fingularum lentium.

Solutio.

Sit diftantia obiecti ante lentem primam $AE = a$, imaginisque per eam repraefentatae $aF = \alpha$, ac pro fequentibus lentibus ponatur.

Pro Lente	Dift: determinatrices	ratio aperturae
Secunda . .	$BF = b$; $bG = \mathfrak{b}$;	π
Tertia . .	$CG = c$; $cH = \gamma$;	π'
Quarta . .	$DH = d$; $dI = \delta$;	π''
Quinta . .	$EI = e$; $eK = \varepsilon$;	π'''

etc.

Hinc

Hinc ergo ſi ſemidiametri aperturarum harum lentium vt ante indicentur literis $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, etc. erit $\pi = \mathfrak{B}(\frac{1}{b}+\frac{1}{\beta})$; $\pi' = \mathfrak{C}(\frac{1}{c}+\frac{1}{\gamma})$; $\pi'' = \mathfrak{D}(\frac{1}{d}+\frac{1}{\delta})$; $\pi''' = \mathfrak{E}(\frac{1}{e}+\frac{1}{\varepsilon})$ etc. Tum vero ſi ſit ſemidiameter campi apparentis $= z$ ponatur etiam $\frac{z}{a} = \Phi$. Cum igitur hinc ſit:

$$\mathfrak{B} = \frac{\pi b \beta}{b+\beta};\ \mathfrak{C} = \frac{\pi' c \gamma}{c+\gamma};\ \mathfrak{D} = \frac{\pi'' d \delta}{d+\delta};\ \mathfrak{E} = \frac{\pi''' e \varepsilon}{e+\varepsilon} \text{ etc.}$$

habebimus ex §. 256 ſequentes aequationes:

$$\frac{\pi \beta}{b+\beta} = (1+\frac{\alpha}{b})\Phi$$
$$\pi - \frac{\pi' \gamma}{c+\gamma} = (1-\frac{\alpha\beta}{bc})\Phi$$
$$\pi - \pi' + \frac{\pi'' \delta}{d+\delta} = (1+\frac{\alpha\beta\gamma}{bcd})\Phi$$
$$\pi - \pi' + \pi'' - \frac{\pi''' \varepsilon}{e+\varepsilon} = (1-\frac{\alpha\beta\gamma\delta}{bcde})\Phi$$
etc.

Quo iam faciliüs hinc per π, π', π'', π''' etc. et Φ diſtantiae determinatrices lentium definiri queant, ponatur $\alpha = \mathrm{A}a$; $\beta = \mathrm{B}b$; $\gamma = \mathrm{C}c$; $\delta = \mathrm{D}d$; $\varepsilon = \mathrm{E}e$ etc. Ita vt litterae A, B, C, D, E etc. denotent numeros abſolutos, ac noſtrae aequationes induent has formas:

$$\frac{\mathrm{B}\pi}{\mathrm{B}+1} = (1+\frac{\mathrm{A}a}{b})\Phi$$
$$\pi - \frac{\mathrm{C}\pi'}{\mathrm{C}+1} = (1-\frac{\mathrm{AB}a}{c})\Phi$$
$$\pi - \pi' + \frac{\mathrm{D}\pi''}{\mathrm{D}+1} = (1+\frac{\mathrm{ABC}a}{d})\Phi$$
$$\pi - \pi' + \pi'' - \frac{\mathrm{E}\pi'''}{\mathrm{E}+1} = (1-\frac{\mathrm{ABCD}a}{e})\Phi$$

etc.

vnde eliciuntur ſequentes determinationes:

$$b = \frac{A(B+1)a\Phi}{B\pi-(B+1)\Phi}$$

$$c = \frac{AB(C+1)a\Phi}{C\pi'-(C+1)(\pi-\Phi)}$$

$$d = \frac{ABC(D+1)a\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$$

$$e = \frac{ABCD(E+1)a\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$$

etc.

Datis ergo praeter numeros Φ, π, π', π'', π''' etc. numeris A, B, C, D, E etc. cum diſtantia obiecti $AE = a$, per has formulas diſtantiae b, c, d, e etc. determinantur indeque inſuper alterae α, β, γ, δ, ϵ etc hoc modo:

$$\alpha = Aa$$

$$\beta = \frac{AB(B+1)a\Phi}{E\pi-(B+1)\Phi}$$

$$\gamma = \frac{ABC(C+1)a\Phi}{C\pi'-(C+1)(\pi-\Phi)}$$

$$\delta = \frac{ABCD(D+1)a\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$$

$$\epsilon = \frac{ABCDE(E+1)a\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$$

etc.

Hinc nanciſcimur diſtantias focales lentium

Primae $PP = \frac{Aa}{A+1}$

Secundae $QQ = \frac{ABa\Phi}{B\pi-(B+1)\Phi}$

Tertiae $RR = \frac{ABCa\Phi}{C\pi'-(C+1)(\pi-\Phi)}$

Quartae $SS = \frac{ABCDa\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$

Quintae $TT = \frac{ABCDEa\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$

etc.

Coroll.

Coroll. 1.

267. Ex angulo Φ cum diſtantia obiecti ante lentem primam $AE = a$, ita definitur ſemidiameter campi apparentis z, vt ſit $z = a\Phi$: neque tamen campus apparens pro lubitu aſſumi poteſt, ſed is per multiplicationem determinabitur, vt mox videbimus.

Coroll. 2.

268. Cum omnes numeri hic in calculum introducti aeque negatiue ac poſitiue accipi queant, obſeruandum eſt eos perpetuo ita aſſumi debere, vt internalla lentium quae ſunt $\alpha+b; \beta+c; \gamma+d; \delta+e;$ etc. omnia prodeant poſitiua.

Coroll. 3.

269. Quod ad aperturam cuiusque lentis attinet eius ſemidiameter habebitur ſi eius diſtantia focalis multiplicetur per rationem aperturae littera π inſignitam.

Scholion.

270. Quo formulas hic inuentas ſimpliciores reddamus quoniam litteris $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$; etc. non amplius indigebimus, ponamus ad abbreuiandum;

$$\frac{A}{A+1} = \mathfrak{A}; \ \frac{B}{B+1} = \mathfrak{B}; \ \frac{C}{C+1} = \mathfrak{C}; \ \frac{D}{D+1} = \mathfrak{D}; \ \frac{E}{E+1} = \mathfrak{E} \text{ etc.}$$

vt ſit

$$A = \frac{\mathfrak{A}}{1-\mathfrak{A}}; \ B = \frac{\mathfrak{B}}{1-\mathfrak{B}}; \ C = \frac{\mathfrak{C}}{1-\mathfrak{C}}; \ D = \frac{\mathfrak{D}}{1-\mathfrak{D}}; \ E = \frac{\mathfrak{E}}{1-\mathfrak{E}} \text{ etc.}$$

at-

atque habebimus:

$$\alpha = \mathrm{A}a; \qquad b = \frac{\mathrm{A}a\Phi}{\mathfrak{B}\pi - \Phi}$$

$$\beta = \frac{\mathrm{AB}a\Phi}{\mathfrak{B}\pi - \Phi} \qquad c = \frac{\mathrm{AB}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$\gamma = \frac{\mathrm{ABC}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \qquad d = \frac{\mathrm{ABC}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$\delta = \frac{\mathrm{ABCD}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \qquad e = \frac{\mathrm{ABCD}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

$$\varepsilon = \frac{\mathrm{ABCDE}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \qquad \text{etc.}$$

Hincque porro definientur distantiae focales lentium:

$$\text{Primae } PP = \mathfrak{A}a$$

$$\text{Secundae } QQ = \frac{\mathrm{A}\mathfrak{B}a\Phi}{\mathfrak{B}\pi - \Phi}$$

$$\text{Tertiae } RR = \frac{\mathrm{AB}\mathfrak{C}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$\text{Quartae } SS = \frac{\mathrm{ABC}\mathfrak{D}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$\text{Quintae } TT = \frac{\mathrm{ABCD}\mathfrak{E}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

etc.

et lentium interualla:

$$\text{I et II} = \frac{\mathrm{A}\mathfrak{B}a\pi}{\mathfrak{B}\pi - \Phi}$$

$$\text{II et III} = \frac{\mathrm{AB}a\Phi(\mathfrak{C}\pi' - (1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi - \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$$

$$\text{III et IV} = \frac{\mathrm{ABC}a\Phi(\mathfrak{D}\pi'' - (1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi' - \pi + \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$$

$$\text{IV et V} = \frac{\mathrm{ABCD}a\Phi(\mathfrak{E}\pi''' - (1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$$

Quae interualla debent esse positiua.

Problema 7.

271. Positis iisdem, quae in problemate praecedente sunt assumta, definire locum idoneum oculi, vnde totus campus apparens conspici queat.

So-

Solutio.

Maneant omnes denominationes vt ante, et quia apertura lentis PP vt nulla ſpectatur, pro reliquis lentibus ex data aperturae ratione, ſemidiameter aperturae cuiusque ita ſe habebit

Lentis	ſemidiameter aperturae
Secundae QQ . .	$\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}\cdot\Phi$
Tertiae RR . .	$\frac{AB\mathfrak{C}a\pi'}{\mathfrak{C}\pi'-\pi'+\Phi}\Phi$
Quartae SS . .	$\frac{ABC\mathfrak{D}a\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\Phi$
Quintae TT . .	$\frac{ABCD\mathfrak{E}a\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\Phi$

vbi litterae $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, etc. valores in praecedente Scholio aſſignatos obtinent.

Deinde magnitudines ſingularum imaginum conſiderari conuenit, quae ob $E\varepsilon = z = a\Phi$ et

$$\alpha = Aa,\ \beta = Bb;\ \gamma = Cc;\ \delta = Dd \text{ etc.}$$

erunt

$$F\zeta = \alpha\Phi = Aa\Phi$$

$$G\eta = ABa\Phi$$

$$H\theta = ABCa\Phi$$

$$I\iota = ABCDa\Phi$$

etc.

Iam pro quolibet lentium numero locus oculi idoneus ſeorſim definiri debet; denotante ergo O diſtantiam oculi poſt vltimam lentem

I. Pro vnica lente

Quia craſſities lentis vt nulla ſpectatur, euidens eſt pro loco oculi idoneo fore $O = o$

II. Pro duabus Lentibus

Tab. III. Fig. 13. Cum hic ſit $bN' + G\eta : bG = bN' : bO$ erit $bO = O = \frac{bN'}{bN' + G\eta} \mathfrak{b}$. Sed eſt $bN' = \frac{A\mathfrak{B}\pi}{\mathfrak{B}\pi - \Phi} a\Phi$ et $G\eta = ABa\Phi$ vnde fit

$$bN' + G\eta = Aa\Phi . \frac{(B+1)\mathfrak{B}\pi - B\Phi}{\mathfrak{B}\pi - \Phi} = \frac{ABa\Phi(\pi - \Phi)}{\mathfrak{B}\pi - \Phi}$$

ob $(B+1)\mathfrak{B} = B$. Erit ergo $\frac{bN'}{bN' + G\eta} = \frac{\mathfrak{B}\pi}{B(\pi - \Phi)}$, quae fractio per $\mathfrak{b} = \frac{ABa\Phi}{(\mathfrak{B}\pi - \Phi)}$ multiplicata dat locum oculi idoneum

$$O = \frac{A\mathfrak{B}a\pi\Phi}{(\pi - \Phi)(\mathfrak{B}\pi - \Phi)}$$

III. Pro tribus Lentibus

Fig. 14. Cum hic ſit $cN'' + H\theta : cH = cN'' : cO$, erit $cO = O = \frac{cN''}{cN'' + H\theta} . \gamma$. Sed eſt

$$cN'' = \frac{AB\mathfrak{C}a\pi'\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \text{ et } H\theta = ABCa\Phi$$

hincque ob $(C+1)\mathfrak{C} = C$ fiet

$$cN'' + H\theta = \frac{ABCa\Phi(\pi' - \pi + \Phi)}{\mathfrak{C}\pi' - \pi + \Phi}$$

et $\frac{cN''}{cN'' + H\theta} = \frac{\mathfrak{C}\pi'}{C(\pi' - \pi + \Phi)}$ Nunc igitur ob $\gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ habebimus diſtantiam oculi idoneam:

$$O = \frac{AB\mathfrak{C}a\pi'\Phi}{(\pi' - \pi + \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$$

IV.

IV. Pro quatuor Lentibus

Cum ſit $dN''' + I\iota : dI = dN''' : dO$, erit $dO = O = \frac{dN'''}{dN''' + I\iota} \cdot \delta$. Sed eſt Tab. III. Fig. 15.

$$dN''' = \frac{ABCDa\pi''\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \text{ et } I\iota = ABCDa\Phi$$

hincque ob $(D + 1)\mathfrak{D} = D$ fiet

$$dN''' + I\iota = \frac{ABCDa\Phi(\pi'' - \pi' + \pi - \Phi)}{\mathfrak{D}\pi'' - \pi + \pi - \Phi} \text{ et}$$

$$\frac{dN'''}{dN''' + I\iota} = \frac{\mathfrak{D}\pi''}{D(\pi'' - \pi' + \pi - \Phi)}. \text{ Ergo ob } \delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

prodit diſtantia oculi idonea

$$O = \frac{ABCDa\pi''\Phi}{(\pi'' - \pi' + \pi - \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}.$$

V. Pro quinque Lentibus

Si ratiocinium ſimili modo ad caſum quinque lentium extendatur, reperiemus diſtantiam oculi idoneam

$$O = \frac{ABCD\mathfrak{E}a\pi'''\Phi}{(\pi''' - \pi'' + \pi' - \pi + \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}.$$

VI. Pro ſex Lentibus

Eodemque modo progrediendo colligitur fore pro caſu ſex lentium diſtantiam oculi idoneam

$$O = \frac{ABCDE\mathfrak{F}a\pi''''\Phi}{(\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)(\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \Phi)}$$

ſicque vlterius, quousque libuerit progredi licet.

Coroll. 1.

272. Quouis ergo casu necesse est, vt distantia oculi idonea prodeat positiua: si enim fieret negatiua totus campus apparens nusquam conspici posset.

Coroll. 2.

273. Iis autem casibus, quibus distantia O fit negatiua oculum immediate vltimae lenti applicari conueniet; Tum vero oculus plus non cernet, quam si vltimae lentis apertura aequalis esset amplitudini pupillae.

Coroll. 3.

274. Hoc ergo casu statuatur semidiameter aperturae vltimae lentis $= \omega$ semidiametro pupillae, ex eaque aequatione eliciatur valor ipsius Φ, quo invento erit $a\Phi$ semidiameter campi apparentis, qui in obiecto reuera conspicietur.

Problema 8.

275. Positis iisdem atque in problematibus praecedentibus, eam conditionem in lentium dispositione definire, vt oculus in loco idoneo positus obiectum simul distincte videat.

Solutio.

Quia aperturam lentis obiectiuae euanescentem assumimus, in visione alia confusio locum habere nequit nisi quatenus oculus non in distantia iusta ab vltima imagine, quam intuetur exiſtit, quae ergo tolletur, ſi lentes ita diſponantur, vt imago vltima ante oculum in O ſitum in diſtantia iuſta, quam littera l deſignauimus, verſetur. Cum igitur in figuris locus oculi ante imaginem vltimam cadat haec diſtantia negatiue ſumta ipſi l aequalis eſt ponenda; vnde pro quouis lentium numero ſequentes habebimus determinationes.

I. Pro vnica lente

Cum hic ſit $O = o$, et $OF = \alpha = Aa$, oportet eſſe $Aa = -l$, ideoque $A = -\frac{l}{a}$: et $\alpha = -l$, vnde indoles huius lentis determinatur, ita vt eius diſtantia focalis eſſe debeat $= \frac{al}{l-a}$. Tab. III. Fig. 12.

II. Pro duabus lentibus

Ex inuenta diſtantia $bO = O$, erit $OG = \frac{O}{b\mathfrak{N}'}$. Fig. 13. $G\eta$. Eſt vero $\frac{O}{b\mathfrak{N}'} = \frac{1}{\pi - \Phi}$; vnde fit $\frac{ABa\Phi}{\pi - \Phi} = -l$, hincque pro ſecunda lente $B = -\frac{(\pi - \Phi)l}{Aa\Phi}$. et $\mathfrak{B} = \frac{B}{B+1}$. Vel cum ſit $Aa\Phi = -\frac{(\pi - \Phi)l}{B}$, erit pro loco oculi

$$O = \frac{-\mathfrak{B}\pi}{B(\mathfrak{B}\pi - \Phi)}l$$

III. Pro tribus lentibus

Hic eſt $OH = \frac{O}{c\mathfrak{N}''}$. $H\theta = \frac{H\theta}{\pi' - \pi + \Phi}$, vnde obtinetur: Fig. 14.

$$OH = \frac{ABCa\Phi}{\pi' - \pi + \Phi} = -l$$

ſicque pro vltima lente habebitur:

$$C = \frac{-(\pi' - \pi + \Phi)l}{ABa\Phi}$$

At fi pro determinatione primae lentis capiatur $Aa\Phi = \frac{-(\pi' - \pi + \Phi)l}{BC}$, erit diftantia oculi

$$O = \frac{-\mathfrak{C}\pi'}{C(\mathfrak{C}\pi' - \pi + \Phi)}\, l$$

IV. Pro quatuor lentibus

Tab. III. Fig. 15.

Cum fit $OI = \frac{O}{a\pi'''}$, $Ii = \frac{Ii}{\pi'' - \pi' + \pi - \Phi}$, habebitur

$$OI = \frac{ABCDa\Phi}{\pi'' - \pi' + \pi - \Phi} = -l$$

Vnde pro vltima lente

$$D = -\frac{(\pi'' - \pi' + \pi - \Phi)l}{ABCa\Phi}$$

Cum autem fit $ABC\,a\Phi = -\frac{(\pi'' - \pi' + \pi - \Phi)l}{D}$ erit in loco oculi hoc valore furrogando

$$O = \frac{-\mathfrak{D}\pi''}{D(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}\, l.$$

V. Pro quinque Lentibus

Simili modo pro quinque lentibus vltima ita comparata effe debet, vt fit

$$E = \frac{-(\pi''' - \pi'' + \pi' - \pi + \Phi)l}{ABCDa\Phi}$$

Prima autem inde definita fit

$$O = \frac{-\mathfrak{E}\pi'''}{E(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}\, l.$$

VI. Pro fex Lentibus

Eodem modo patet. pro fex lentibus fore

$$F = \frac{-(\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)l}{ABCDEa\Phi}$$

atque

atque si hinc Aa definiatur:

$$O = \frac{-\mathfrak{E}\,\pi''''}{F(\mathfrak{E}\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)}\, l$$

quas formulas quousque libuerit, continuare licet.

Coroll. 1.

276. Si distantia oculi iusta l fuerit infinita erit vt sequitur:

I. Pro vna lente $A = \infty$ et $\mathfrak{A} = 1$

II. Pro duabus lentibus $B = \infty$ et $\mathfrak{B} = 1$

III. Pro tribus lentibus $C = \infty$ et $\mathfrak{C} = 1$

IV. Pro quatuor lentibus $D = \infty$ et $\mathfrak{D} = 1$

etc.

Coroll. 2.

277. Casu ergo, quo distantia oculi iusta l est infinita, distantia oculi post lentem vltimam erit pro quouis lentium numero:

I. Pro vnica lente $O = 0$

II. Pro duabus lentibus $O = \frac{Aa\pi\Phi}{(\pi - \Phi)^2}$

III. Pro tribus lentibus $O = \frac{ABb\pi'\Phi}{(\pi' - \pi + \Phi)^2}$

IV. Pro quatuor lentibus $O = \frac{ABCa\pi''\Phi}{(\pi'' - \pi' + \pi - \pi)^2}$

etc.

Scholion.

278. Hactenus campum apparentem Φ vt datum consideraui, ex eoque tam lentium indolem

quam

quam earum dispositionem determinaui, vt campus datae amplitudinis appareat, nihilque obstare deprehendimus, quominus huic conditioni satisfiat; cum numeri A, B, C, D, etc. penitus arbitrio relinquantur, aperturarum vero rationes π, π', π'', etc. infra $\frac{1}{3}$ vel $\frac{1}{4}$ accipi debeant. Verum hic multiplicationis ratio nondum in computum est ducta qua simul campus apparens ita adstringitur, vt certum limitem excedere nequeat. Quoniam igitur in omnibus instrumentis dioptricis multiplicatio imprimis proposita esse solet, quemadmodum per eam campus apparens definiatur in sequente problemate exponamus.

Problema 9.

279. Si instrumentum dioptricum ex quotcunque lentibus fuerit compositum, quarum quidem crassities vt nulla spectetur, simul vero multiplicationis ratio sit proposita, determinare campum apparentem.

Solutio.

Manentibus omnibus denominationibus, quibus hactenus sumus vsi, ita vt $a\Phi$ semidiametrum campi apparentis denotet, sit b distantia, ad quam multiplicationem referamus. Magnitudo igitur $a\Phi$ in distantia hac $= b$ nudo oculo cerneretur sub angulo, cuius tangens est $= \frac{a\Phi}{b}$. Quare si multiplicationis ratio statuatur $= m$, necesse est vt eadem magnitudo $a\Phi$ per lentes spectetur sub angulo, cuius tangens sit $= \frac{ma\Phi}{b}$.

Iam

Iam vero ex iis, quae in problematibus praecedentibus funt tradita iste angulus facile assignatur, sicque obtinebitur angulus Φ, indeque semidiameter campi apparentis $a\Phi$. Cum autem haec multiplicatio non ad ipsos angulos sed eorum tangentes referatur euidens est tantum partes obiecti minimas circa centrum E sitas in ratione proposita multiplicari, remotiores vero in ratione minore. Quo notato hanc multiplicationis rationem m pro quouis lentium numero contemplemur.

I. Pro vnica Lente

Tangens anguli, quo imago $F\zeta$ ab oculo in O constituto conspicitur, est $\frac{F\zeta}{OF} = \frac{F\zeta}{aF}$ ob $aO = o$, Erit ergo $\frac{ma\Phi}{b} = \Phi$, seu $ma = b$. Hoc ergo casu campus apparens non determinatur sed multiplicationis ratio est $m = \frac{b}{a}$. Verum vt visio sit distincta, per superius problema debet esse $A = -\frac{l}{a}$ et distantia oculi post lentem $O = o$. At obiectum situ erecto cernetur: Tab. III. Fig. 12.

II. Pro duabus Lentibus

Tangens anguli, quo imago $G\eta$ ab oculo in O constituto cernetur, est $= \frac{bN'}{bO} = \pi - \Phi = \frac{ma\Phi}{b}$; vnde sequitur semidiameter campi apparentis Fig. 13.

$$\Phi = \frac{\pi b}{ma + b} \text{ pro situ inuerso}$$

Quo inuento, vt visio sit distincta oportet esse

$$B = \frac{-ml}{Ab}, \text{ et } \mathfrak{B} = \frac{-ml}{Ab - ml}$$

hincque prodit diſtantia oculi poſt lentem ocularem

$$O = \frac{ABl(ma+b)}{mmal+ABb}.$$

III. Pro tribus Lentibus

Tab. III. Fig. 14. Tangens anguli, quo imago $H\theta$ ab oculo in O conſtituto cernitur eſt $\frac{c\iota''}{cO} = \pi' - \pi + \Phi = \frac{ma\Phi}{b}$, vnde fit ſemidiameter campi apparentis:

$$\Phi = \frac{(\pi'-\pi)b}{ma-b} \text{ pro ſitu erecto}$$

Deinde vt viſio ſit diſtincta, oportet eſſe

$$C = \frac{-ml}{ABb} \text{ et } \mathfrak{C} = \frac{-ml}{ABb-ml}$$

Cum igitur ſit $\pi' - \pi + \Phi = \frac{ma(\pi'-\pi)}{ma-b}$ erit

$$\mathfrak{C}\pi' - \pi + \Phi = \pi' - \pi + \Phi - \frac{ABb\pi'}{ABb-ml} = \frac{ma(\pi'-\pi)}{ma-b} - \frac{ABb\pi'}{ABb-ml}.$$

hincque pro loco oculi

$$O = \frac{-ABbl\pi'}{(ABb-ml)(\mathfrak{C}\pi'-\pi+\Phi)} = \frac{ABbl(ma-b)\pi'}{(mmal-ABbb)\pi' + ma(ABb-ml)\pi}.$$

IV. Pro quatuor lentibus

Fig. 15. Tangens anguli, quo imago $I\iota$ ab oculo in O conſtituto cernitur eſt $= \frac{d\iota'''}{dO} = \pi'' - \pi' + \pi - \Phi = \frac{ma\Phi}{b}$; vnde elicitur:

$$\Phi = \frac{(\pi''-\pi'+\pi)b}{ma+b} \text{ pro ſitu inuerſo}$$

Hinc porro pro viſione diſtincta eſſe debet

$$D = \frac{-ml}{ABCb}, \text{ et } \mathfrak{D} = \frac{-ml}{ABCb-ml}$$

vnde fit

$$\mathfrak{D}\pi'' - \pi' + \pi - \Phi = \frac{ma(\pi''-\pi'+\pi)}{ma+b} - \frac{ABCb\pi''}{ABCb-ml}$$

et

et pro loco oculi

$$O = \frac{ABCbl(ma+b)\pi''}{(mmal + ABCbb)\pi'' + ma(ABCb - ml)(\pi' - \pi)}.$$

V. Pro quinque lentibus

Eodem modo progrediendo pro campo apparente reperitur:

$$\Phi = \frac{(\pi''' - \pi'' + \pi' - \pi)b}{ma - b} \text{ pro situ erecto}$$

et vt visio euadat distincta

$$E = \frac{-ml}{ABCDb} \text{ et } \mathfrak{E} = \frac{-ml}{ABCDb - ml}$$

vnde pro loco oculi idoneo concluditur

$$O = \frac{ABCDbl(ma-b)\pi'''}{(mmal - ABCDbb)\pi''' + ma(ABCDb - ml)(\pi'' - \pi' + \pi)}.$$

VI. Pro sex lentibus

Hic campus apparens ita definitur, vt sit:

$$\Phi = \frac{(\pi'''' - \pi''' + \pi'' - \pi' + \pi)b}{ma + b} \text{ pro situ inuerso}$$

visio vero distincta exigit

$$F = \frac{-ml}{ABCDEb}; \ \mathfrak{F} = \frac{-ml}{ABCDEb - ml}$$

vnde pro loco oculi idoneo

$$O = \frac{ABCDEbl(ma+b)\pi''''}{(mmal + ABCDEbb)\pi'''' + ma(ABCDEb - ml)(\pi''' - \pi'' + \pi' - \pi)}$$

sicque progressio ad plures lentes est manifesta.

Coroll. 1.

280. Datis ergo rationibus aperturarum singularum lentium π, π', π'', etc. vna cum ratione multiplicationis m, distantia b, ad quam multiplicatio

refertur, et distantia obiecti ante instrumentum a, determinatur campus apparens.

Coroll. 2.

281. Vt ergo campus apparens pro data multiplicatione maximus obtineatur litteris π, π', π'', etc. ita valores maximos tribui conueniet, vt alternatim sint positiui et negatiui.

Coroll. 3.

282. Si igitur valores π, π', π'', etc. vsque ad $\frac{1}{3}$ augeri liceat, maximus valor ipsius Φ pro quouis lentium numero erit vt sequitur:

Pro casu duarum lentium $\Phi = \frac{b}{3(ma+b)}$

Pro casu trium lentium $\Phi = \frac{2b}{3(ma-b)}$

Pro casu quatuor lentium $\Phi = \frac{3b}{3(ma+b)}$

Pro casu quinque lentium $\Phi = \frac{4b}{3(ma-b)}$

etc.

Coroll. 4.

283. Quo plures ergo lentes adhibentur eo magis campus apparens augeri potest simul vero patet, quo maior multiplicatio desideretur, eo minorem fieri campum apparentem.

Coroll.

Coroll. 5.

284. Ratio multiplicationis m tam poſitiue quam negatiue capi poteſt. Si poſitiue accipitur pro lentium numero pari ſitum inuerſum, pro impari autem ſitum erectum declarat. Contrarium vero euenit, ſi m fuerit numerus negatiuus.

Scholion I.

285. Hic autem imprimis notandum eſt, valorem ipſius Φ tum ſolum angulum EAϵ praebere, quando fuerit tam exiguus, vt aliquot gradus non ſuperet; ſi enim valor ipſius Φ prodeat multo maior, tum tangentem huius anguli EAϵ exprimit. Plerumque autem, ſi quidem multiplicatio ſit modica, iſte valor ipſius Φ tam paruus reperitur, vt ſine errore pro ipſo angulo EAϵ accipi poſſit. Hic igitur ob aliam cauſam amplitudo campi apparentis reſtringitur, vt certum limitem ſuperare nequeat; cum enim angulus, quo radii in oculum incidentes in O ad axem inclinantur, nunquam poſſit eſſe rectus, neque fortaſſe vix $60°$ ſuperare queat, quandoquidem ne nudo quidem oculo ſpatium in coelo maius quam $120°$ conſpicere valeamus; ſi illum angulum maximum, quem oculus capere valeat, circiter $63°$ ſtatuamus, vt eius tangens ſit $= 2$ pro quouis lentium numero habebimus $\frac{ma\Phi}{b} = 2$, vnde ſit $\Phi = \frac{2b}{ma}$, et $a\Phi = \frac{2b}{m}$. Data ergo multiplicatione m et diſtantia b ad quam refertur, ſemidiameter ſpatii in obiecto

conſpicui nunquam maior exiſtere poteſt quam $\frac{2b}{m}$ quotcunque etiam adhibeantur lentes, eaeque ita diſponantur vt maximum campum patefaciant. In Teleſcopiis ergo, vbi ſumitur $b=a$, et ſemidiameter campi ex ipſo angulo Φ aeſtimatur, eius tangens nunquam maior eſſe poteſt quam $\frac{2}{m}$: vnde ſequentem tabellam adiungo, quae pro quauis multiplicatione ſemidiametrum campi apparentis maximi oſtendit, quem nunquam ſuperare liceat.

Multiplicatio m	Semidiam: campi app: maximi	Multiplicatio m	Semidiam: campi app: maximi
5	21°, 48′	60	1°, 54′, 33″
10	11, 18	70	1, 38, 13
15	7, 35	80	1, 25, 57
20	5, 42	90	1, 16, 24
25	4, 34	100	1, 8, 45
30	3, 49	150	0, 45, 50
35	3, 16½	200	0, 34, 22
40	2, 52	250	0, 27, 30
45	2, 33	300	0, 22, 55
50	2, 17½	400	0, 17, 11
		500	0, 13, 45

Quod ſi ergo numerum lentium multiplicando iam fere ad tantum campum apparentem pertigerimus, is vlterius nullo modo augeri poterit.

Scho-

Scholion 2.

286. Quod ad locum oculi idoneum attinet, eum ideo in O constituimus, vt omnes radios per lentes transmissos accipiat, etiamsi pupilla maxime esset constricta: ex quo patet ob amplitudinem pupillae oculum de hoc loco sine vllo detrimento aliquantillum remoueri posse, ita vt superfluum foret hunc locum nimis sollicite obseruare, nisi forte apertura vltimae lentis fuerit admodum magna. Sin ea autem pupillam non superet, eaque adeo sit minor, manifestum est oculum ei immediate applicatum aeque omnes radios excipere, et eundem campum contueri, ac si in loco idoneo esset constitutus. His igitur casibus, si forte distantia O pro loco oculi prodeat negatiua, nihil de campo apparente perit, dummodo oculus lenti vltimae immediate applicetur. His itaque, quae ad visionem per instrumenta dioptrica in genere pertinent, expeditis, superest, vt inuestigemus, quantum visio ob diuersam radiorum refrangibilitatem turbetur, et quemadmodum hanc perturbationem euitare queamus.

CAPVT

CAPVT VI.

DE CONFVSIONE A DIVERSA RADIORVM INDOLE ORIVNDA.

Problema I.

287.

Si a puncto dato E radii per lentem PP transmittantur definire variationem in loco imaginis F, quae a diuersa radiorum refrangibilitate oritur.

Solutio.

Sit distantia puncti E ante lentem $AE = a$, facierum autem lentis radius anterioris $= f$ posterioris $= g$ et crassities $Aa = v$, quae quantitates sunt constantes. Posita nunc refractionis ratione ex aere in vitrum $= n : 1$, ob diuersam radiorum naturam numerus n erit variabilis, ideoque etiam locus imaginis F post lentem expressae, cuius distantia si ponatur $aF = \alpha$, erit ex supra inuentis

$$\frac{n-1}{f} = \frac{1}{a} + \frac{1n}{k+v} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{1n}{k-v}$$

vbi

vbi quantitas k etiam pro variabili eſt habenda, quia tantum a, f, g et v ſunt conſtantes. Quaeſtio ergo huc redit, vt ſi numerus n differentiali ſuo dn creſcere ſumatur, definiatur differentiale diſtantiae α. Quare differentientur ambae aequationes illae:

$$\frac{dn}{f} = \frac{2dn}{k+v} - \frac{2ndk}{(k+v)^2}$$

$$\frac{dn}{g} = \frac{-d\alpha}{\alpha\alpha} - \frac{2dn}{k-v} + \frac{2ndk}{(k-v)^2}.$$

indeque eliminato dk habebitur

$$\frac{dn(k+v)^2}{f} + \frac{dn(k-v)^2}{g} = 2dn(k+v) - 2dn(k-v) - \frac{d\alpha(k-v)^2}{\alpha\alpha}$$

Reſtituantur pro f et g valores initio poſiti, ac peruenietur ad hanc aequationem:

$$\frac{dn(k+v)^2}{a} + \frac{dn(k-v)^2}{\alpha} + 4vdn + \frac{(n-1)d\alpha}{\alpha\alpha}(k-v)^2 = 0$$

vnde reperitur:

$$\frac{d\alpha}{\alpha\alpha} = \frac{-dn}{(n-1)a}\left(\frac{k+v}{k-v}\right)^2 - \frac{dn}{(n-1)\alpha} - \frac{4vdn}{(n-1)(k-v)^2} \text{ ſeu}$$

$$d\alpha = \frac{-\alpha dn}{n-1}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right)$$

Tum vero cum etiam k ſit quantitas variabilis erit

$$dk = \frac{-(k+v)dn}{n(n-1)}\left(1 + \frac{k+v}{2a}\right)$$

Cum ergo poſuerimus $\frac{k-v}{k+v} = i$, ob $di = \frac{2vdk}{(k+v)^2}$ erit

$$di = \frac{-vdn}{n(n-1)}\left(\frac{1}{a} + \frac{2}{k+v}\right).$$

Coroll. I.

288. Si $n : 1$ denotet rationem refractionis radiorum mediae naturae, vt ſit $n = \frac{31}{20} = 1,55$. erit pro

pro radiis rubris ſeu minime refractis $n = 1,54$, et violaceis $n = 1,56$. quorum valorum diſcrimen a medio, cum ſit $= \frac{1}{155}$, pro differentiali dn haberi poterit.

Coroll. 2.

289. Quare ſi α denotet diſtantiam imaginis a radiis mediis formatae, pro ea quae a rubris formatur, erit $dn = -\frac{1}{155}$ et $\frac{dn}{n-1} = \frac{-1}{55}$. Hinc diſtantia imaginis rubrae poſt lentem erit $= \alpha + \frac{\alpha}{55}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right)$ Diſtantia autem imaginis violaceae poſt lentem erit

$$\alpha - \frac{\alpha}{55}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right).$$

Coroll. 3.

290. Si craſſities lentis euaneſcat, vt ſit $v = 0$, ob variabilitatem numeri n erit

$$d\alpha = \frac{-\alpha dn}{n-1}\left(1 + \frac{\alpha}{a}\right) = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{a} + \frac{1}{\alpha}\right)$$

Ac ſi diſtantia focalis lentis ponatur $= p$, cum ſit $\frac{1}{a} + \frac{1}{\alpha} = \frac{1}{p}$, erit $d\alpha = \frac{-\alpha\alpha dn}{(n-1)p} = \frac{-20\alpha\alpha dn}{11p}$.

Scholion.

291. Hoc ergo modo ob diuerſam radiorum naturam valor diſtantiae α immutatur; vnde ſi ab ea tanquam ab obiecto radii porro ad lentem ſecundam emittantur, fiet etiam reſpectu huius lentis diſtantia obiecti variabilis. Quam ob cauſam in loco imaginis ab ea formatae duplex variatio orietur: id quod deinceps etiam in lentibus ſequentibus multo magis

magis eueniet. Hanc igitur variationem, quae pro quauis lente in loco imaginis nafcitur, in fequente problemate determinemus.

Problema 2.

292. Si locus imaginis F, quae refpectu lentis QQ vicem obiecti gerit, ob diuerfam radiorum naturam ipfe fit variabilis, determinare variationem, quam ob eandem caufam imago fequens in G patietur.

Solutio.

Sit pro radiis mediae naturae, quibus refpondet numerus n diftantia obiecti F ante lentem $BF = b$; imaginisque inde proiectae diftantia poft lentem $bG = \beta$; dum autem n abit in $n + dn$, hae diftantiae ambae b et β capiant fua incrementa differentialia db et $d\beta$. Ad quae inuenienda fit lentis QQ radius faciei anterioris $= f$, pofterioris $= g$, et craffities $Bb = v$, eritque vt ante:

$$\frac{n-1}{f} = \frac{1}{b} + \frac{2n}{k+v} \text{ et } \frac{n-1}{g} = \frac{1}{\beta} - \frac{2n}{k-v}$$

vbi k cum b et β pro variabili eft habenda. Differentiatione ergo inftituta habebitur.

$$\frac{dn}{f} = \frac{-db}{bb} + \frac{2dn}{k+v} - \frac{2n\,dk}{(k+v)^2}$$

$$\frac{dn}{g} = -\frac{d\beta}{\beta\beta} - \frac{2dn}{k-v} + \frac{2n\,dk}{(k-v)^2}$$

vnde eliminato dk, fit

$$\frac{dn(k+v)^2}{f} + \frac{dn(k-v)^2}{g} = \frac{-db}{bb}(k+v)^2 - \frac{d\beta}{\beta\beta}(k-v)^2 + 4v\,dn$$

quae multiplicata per $n-1$, si pro f et g valores dati substituantur prodit

$$\frac{dn(k+v)^2}{b} + 2ndn(k+v) + \frac{dn(k-v)^2}{\mathfrak{g}} - 2ndn(k-v)$$
$$= -\frac{(n-1)db}{bb}(k+v)^2 - \frac{(n-1)d\mathfrak{g}}{\mathfrak{g}\mathfrak{g}}(k-v)^2 + 4(n-1)vdn$$

seu

$$\frac{dn(k+v)^2}{(n-1)b} + \frac{dn(k-v)^2}{(n-1)\mathfrak{g}} + \frac{4vdn}{n-1} + \frac{db}{bb}(k+v)^2 + \frac{d\mathfrak{g}}{\mathfrak{g}\mathfrak{g}}(k-v)^2 = 0$$

Atque hinc elicitur:

$$d\mathfrak{g} = \frac{-\mathfrak{g}\mathfrak{g}db}{bb}\left(\frac{k+v}{k-v}\right)^2 - \frac{\mathfrak{g}dn}{n-1}\left(1 + \frac{\mathfrak{g}}{b}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\mathfrak{g}v}{(k-v)^2}\right)$$

Pro variabilitate autem ipsius k reperietur

$$\frac{dk}{(k+v)^2} = -\frac{db}{2nbb} - \frac{dn}{2n(n-1)b} - \frac{dn}{n(n-1)(k+v)}$$

Quare si ponatur $\frac{k-v}{k+v} = i$, ob $di = \frac{2vdk}{(k+v)^2}$ erit

$$di = \frac{-vdb}{nbb} - \frac{vdn}{n(n-1)b} - \frac{2vdn}{n(n-1)(k+v)}$$

ac si loco k numerus i introducatur, erit

$$d\mathfrak{g} = \frac{-\mathfrak{g}\mathfrak{g}db}{iibb} - \frac{bdn}{n-1}\left(1 + \frac{\mathfrak{g}}{iib} + \frac{(1-i)^2\mathfrak{g}}{iiv}\right) \text{ et}$$

$$di = \frac{-vdb}{nbb} - \frac{vdn}{n(n-1)}\left(\frac{1}{b} + \frac{1-i}{v}\right).$$

Coroll. 1.

293. Inuenta aequatio differentialis etiam hac forma repraesentari potest vt sit

$$\frac{id\mathfrak{g}}{\mathfrak{g}\mathfrak{g}} + \frac{db}{ibb} = \frac{-dn}{n-1}\left(\frac{i}{\mathfrak{g}} + \frac{1}{ib} + \frac{(1-i)^2}{iv}\right)$$

siue

siue restituendo k

$$\frac{d\mathfrak{G}}{\mathfrak{G}\mathfrak{G}}\left(\frac{k-v}{k+v}\right)+\frac{db}{bb}\left(\frac{k+v}{k-v}\right)=\frac{-dn}{n-1}\left(\frac{1}{\mathfrak{G}}\left(\frac{k-v}{k+v}\right)+\frac{1}{b}\left(\frac{k+v}{k-v}\right)+\frac{4v}{kk-vv}\right)$$

vbi haec obseruanda est analogia, vt quemadmodum ad b refertur $\frac{k+v}{k-v}$, ita ad $\mathfrak{G}$ referatur $\frac{k-v}{k+v}$.

Coroll. 2.

294. Si lentis huius crassities euanescat, fit $v=0$, et $i=1$ vbi figura lentis non amplius in computum ingreditur sed sola distantia focalis, vnde variatio in loco imaginis G ita erit comparata, vt sit

$$\frac{d\mathfrak{G}}{\mathfrak{G}\mathfrak{G}}+\frac{db}{bb}=\frac{-dn}{n-1}\left(\frac{1}{\mathfrak{G}}+\frac{1}{b}\right) \text{ ideoque}$$

$$d\mathfrak{G}=\frac{-\mathfrak{G}\mathfrak{G}}{bb}db-\frac{\mathfrak{G}\mathfrak{G}dn}{n-1}\left(\frac{1}{\mathfrak{G}}+\frac{1}{b}\right).$$

Coroll. 3.

295. Si casum a radiis mediae naturae, ad quos formulae hactenus traditae sunt accommodatae, ad radios rubros transferre velimus, poni oportet $dn=-\frac{1}{100}$, sin autem ad radios violaceos $dn=+\frac{1}{100}$.

Problema 3.

296. Si radii ab obiecto E per lentes quotcunque transmittantur, determinare variationem in locis singularum imaginum, quae a diuersa radiorum refrangibilitate proficiscitur.

Solutio.

Retineantur omnes denominationes, quibus in ſuperioribus capitibus ſumus vſi, ac ſint $a, \alpha, b, \beta, c, \gamma$ etc. diſtantiae determinatrices lentium pro radiis mediae naturae. Variata ergo ratione refractionis etiam hae diſtantiae variabuntur, quarum variationes differentialibus indicemus. Cum autem diſtantiae inter binas lentes maneant conſtantes, illae variationes ita erunt comparatae vt ſit

$$d\alpha + db = 0,\ d\beta + dc = 0,\ d\gamma + d.d = 0 \text{ etc.}$$

Cum iam diſtantia obiecti $AE = a$ ſit inuariabilis erit ex problemate primo ſi ponatur $\frac{k-v}{k+v} = i$

$$d\alpha = -db = \frac{-\alpha\alpha dn}{i(n-1)}\left(\frac{i}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right) = \frac{-\alpha\alpha dn}{n-1}\cdot\frac{k+v}{k-v}\left(\frac{k-v}{\alpha(k+v)} + \frac{k+v}{a(k-v)} + \frac{4v}{kk-vv}\right)$$

$$\text{et } di = \frac{-v\,dn}{n(n-1)}\left(\frac{1}{a} + \frac{1-i}{v}\right)$$

Deinde pro ſecunda lente, ad quam referuntur diſtantiae determinatrices b, et β cum arbitraria k' et craſſitie v', vnde fecimus $\frac{k'-v'}{k'+v'} = i'$, habebimus

$$d\beta = -dc = \frac{-\beta\beta db}{i'i'bb} - \frac{\beta\beta dn}{i'(n-1)}\left(\frac{i'}{\beta} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)$$

$$\text{et } di' = \frac{-v'db}{nbb} - \frac{v'dn}{n(n-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

Simili modo pro tertia lente, ad quam referuntur diſtantiae determinatrices c et γ cum arbitraria k'' et craſſitie v'' poſito $\frac{k''-v''}{k''+v''} = i''$ adipiſcemur:

$$d\gamma = -dd = \frac{-\gamma\gamma dc}{i''i''cc} - \frac{\gamma\gamma dn}{i''(n-1)}\left(\frac{i''}{\gamma} + \frac{1}{i''c} + \frac{(1-i'')^2}{i''v''}\right)$$

$$\text{et } di'' = \frac{-v''dc}{ncc} - \frac{v''dn}{n(n-1)}\left(\frac{1}{c} + \frac{1-i''}{v''}\right)$$

atque

atque vlterius progrediendo obtinebimus fequentes formulas:

$$d\delta = -de = \frac{-\delta\delta\, dd}{i'''\, i'''\, dd} - \frac{\delta\delta\, dn}{i'''(n-1)}\left(\frac{i'''}{\delta} + \frac{1}{i'''d} + \frac{(1-i''')^2}{i'''\, v'''}\right)$$

$$\text{et } di''' = \frac{-v'''\, dd}{n\, dd} - \frac{v'''\, dn}{n(n-1)}\left(\frac{1}{d} + \frac{1-i'''}{v'''}\right)$$

vnde haec differentialia facile ad quotcunque lentes extenduntur. Atque fi hic fucceffiue valores differentialium dd, dc, db, iam ante definiti fubftituantur omnia haec differentialia tam diftantiarum determinatricium, quam numerorum i, i', i'', i''' etc. per differentiale dn exprimentur.

Si ratio refractionis pro fingulis lentibus fit diuerfa pro iisque ordine exprimatur numeris, n, n', n'' etc. perfpicuum eft, differentialia hic inuenta fequenti modo expreffum iri:

$$\text{I. } d\alpha = -db = \frac{-\alpha\alpha\, dn}{i(n-1)}\left(\frac{i}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right)$$

$$di = \frac{-v\, dn}{n(n-1)}\left(\frac{1}{a} + \frac{1-i}{v}\right)$$

$$\text{II. } d\beta = -dc = \frac{-\beta\beta\, db}{i'\, i'\, bb} - \frac{\beta\beta\, dn'}{i'(n'-1)}\left(\frac{i'}{\beta} + \frac{1}{i'b} + \frac{(1-i')^2}{i'\, v'}\right)$$

$$di' = \frac{-v'\, db}{n'\, bb} - \frac{v'\, dn'}{n'(n'-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

$$\text{III. } d\gamma = -dc = \frac{-\gamma\gamma\, dc}{i''\, i''\, cc} - \frac{\gamma\gamma\, dn''}{i''(n''-1)}\left(\frac{i''}{\gamma} + \frac{1}{i''c} + \frac{(1-i'')^2}{i''\, v''}\right)$$

$$di'' = \frac{-v''\, dc}{n''\, cc} - \frac{v''\, dn''}{n''(n''-1)}\left(\frac{1}{c} + \frac{1-i''}{v''}\right)$$

etc.

ex quibus formulis etiam mutationes fingularum imaginum ac proinde etiam tandem vltimae imaginis facile

facile definiri poterunt, seu potius loco imaginum angulos, sub quibus eae oculo ad iustam distantiam l positae sint adpariturae, consideremus.

Pro vna lente $\frac{1}{i}\cdot\frac{\alpha}{a}\cdot\frac{z}{l}$

Pro duabus lentibus $\frac{1}{i\,i'}\cdot\frac{\alpha\beta}{a\,b}\cdot\frac{z}{l}$

Pro tribus lentibus $\frac{1}{i\,i'\,i''}\cdot\frac{\alpha\beta\gamma}{a\,b\,c}\cdot\frac{z}{l}$

etc.

quatenus scilicet praeter distantias $a, \alpha; b, \beta; c, \gamma$ etc. etiam litterae i, i', i'', i''' etc. sunt variabiles.

Coroll. 1.

297. Hinc igitur per differentiationem definire licet, quanta mutatio in loco vltimae imaginis, quae obiectum visionis constituit, ob diuersam radiorum refrangibilitatem oriri debeat.

Coroll. 2.

298. Deinde cum etiam magnitudinem cuiusque imaginis supra per distantias determinatrices et numeros i, i', i'' etc. definiuerimus, pro magnitudine imaginis habetur $\frac{1}{i\,i'\,i''\,i'''}\cdot\frac{\alpha\beta\gamma\delta}{a\,b\,c\,d}\cdot\frac{z}{l}$ (189). simili modo mutatio assignari potest, quam magnitudo vltimae imaginis ob diuersam refrangibilitatem radiorum patietur.

Coroll. 3.

299. Cognita autem vtraque mutatione, quam vltima imago tam respectu loci, quam magnitudinis subit,

ſubit, non difficulter colligetur, quanta confuſione ipſa viſio ob diuerſam radiorum refrangibilitatem perturbetur.

Coroll. 4.

300. Si craſſities lentium euaneſcat fiet

$$d\alpha = -db = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

$$d\beta = -dc = -\frac{\beta\beta db}{bb} - \frac{\beta\beta dn}{n-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)$$

$$d\gamma = -dd = \frac{-\gamma\gamma dc}{cc} - \frac{\gamma\gamma dn}{n-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

$$d\delta = -de = \frac{-\delta\delta dd}{dd} - \frac{\delta\delta dn}{n-1}\left(\frac{1}{d}+\frac{1}{\delta}\right)$$

etc.

numeri autem i, i', i'' etc. abeunt in vnitatem nullique mutationi amplius ſunt obnoxii.

Si ergo craſſities lentium euaneſcat, pro diuerſa refractione ſingularum lentium formulae ſuperiores abibunt in ſequentes:

I. $d\alpha = -db = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$

II. $d\beta = -dc = -\frac{\beta\beta db}{bb} - \frac{\beta\beta dn'}{n'-1}\left(\frac{1}{\beta}+\frac{1}{b}\right)$

III. $d\gamma = -dd = \frac{-\gamma\gamma dc}{cc} - \frac{\gamma\gamma dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$

etc.

Ceterum per ſe manifeſtum eſt, quando §. 288 dn vel $+\frac{1}{100}$ vel $-\frac{1}{100}$ ſignificare dicitur, id tantum de illa vitri ſpecie, pro qua eſt refractio radiorum me-

mediorum $n = \frac{31}{20}$ esse intelligendum; et pro aliis vitri speciebus differentialia dn', dn'', dn''', etc. haud mediocriter ab $\frac{1}{100}$ discrepare posse. Quanta autem futura sit haec diuersitas, optandum esset, vt ea potius experimentis, quam ex theoria quapiam definiretur.

Scholion

301. Cum igitur ob diuersam radiorum refrangibilitatem cuique imagini duplex alteratio inducatur, quarum altera eius magnitudinem, altera vero eius locum afficit, duplex inde confusio in visionem infertur. Si enim formulae in superioribus capitibus exhibitae ad radios mediae naturae restringantur, pro quibus est $n = \frac{31}{20}$, posito $dn = -\frac{1}{100}$, ex formulis hic traditis differentialibus locus et magnitudo imaginis a radiis rubris formatae definietur, posito autem $dn = +\frac{1}{100}$,
Tab. III. Fig. 15. locus et magnitudo imaginis violaceae declarabitur. Scilicet si Ii fuerit imago vltima visioni obiecta, quae a radiis mediae naturae formatur, per formulas modo inuentas, prout vel $dn = -\frac{1}{100}$ vel $dn = +\frac{1}{100}$ ponatur, definietur tam imago rubra $R\rho$ quam violacea Vv; atque ex natura differentialium manifestum est cum interualla IR et IV inter se aequalia esse debere, tum etiam differentias $Ii - R\rho$ et $Vv - Ii$, ita vt oculo series innumerabilium imaginum inter extremas $R\rho$ et Vv sitarum simul cernenda offeratur, vnde eo maior confusio oriatur necesse est, quo maior fuerit differentia tam rationе loci quam magnitudinis. Quare haec confusio penitus tolleretur, si eiusmodi lentium dispositio definiri posset, vt tam interuallum RV, quam

quam differentia inter imagines R ϱ et V v ad nihilum redigeretur, quod vtrumque nisi simul praestari queat, confusionem perfecte tollere non licet Verumtamen etiamsi neutri harum conditionum satisfieri possit, tamen dabitur pro oculo eiusmodi locus O, vbi confusio minime sensibilis percipiatur, qui erit in concursu rectae $v\varrho$ productae cum axe: ibi enim omnes extremitates $\varrho\iota v$ communibus radiis cernentur, neque propterea extremitas obiecti colore tincta apparebit. Quare si simul punctum O conueniat cum loco oculi idoneo alia confusio non percipietur, nisi quae inde originem trahit, quod forte imagines extremae R ϱ et V v nimis a distantia iusta discrepent, siquidem media I ι ad distantiam iustam ob oculo fuerit remota. Neque tamen hinc ora obiecti coloribus iridis cincta apparebit, cui confusionis speciei maxime est occurrendum; ideoque ea quae adhuc adfuerit confusio facile tolerari poterit, quae vero etiam omnino tolleretur si modo interuallum RV vel in nihilum redigi vel saltem satis paruum reddi posset. Hinc ergo intelligimus vitium illud, quo obiecta coloribus iridis circumdata saepe repraesentantur, non tam necessario cum instrumentis dioptricis esse coniunctum, vt nullo pacto ab iis separari queat, quamobrem eo magis operae erit pretium, vt inuestigemus quomodo haec instrumenta ab isto vitio liberari possint. Quae tota inuestigatio huc redit, vt determinetur punctum O, vbi recta per terminos imaginum v, ι, ϱ ducta cum axe concurrit hocque punctum cum loco oculi iam supra definito conueniens reddatur, si quidem fieri potest: vnde perspicitur locum oculi O hac proprietate praeditum esse oportere, vt angulus, sub quo vltima imago cernitur ob variabilitatem numero n tributam nullam mutationem patiatur. Tum vero insuper videndum erit, num interualla IR et IV vel ad nihilum reduci, vel minima reddi queant.

Problema 4.

Tab. III. Fig. 12. 302. Proposita vnica lente definire locum oculi, vnde obiectum sine margine colorato cernatur.

Solutio.

Sit obiecti $E\varepsilon$ ante lentem distantia $EA = a$, imago vero per radios mediae naturae in $F\zeta$ repraesentetur, ponaturque $aF = \alpha$. Pro lente vero sit eius crassities $Aa = v$, et quantitas arbitraria $= k$, vnde capiatur $\frac{k-v}{k+v} = i$. Hinc posito $E\varepsilon = z$ erit $F\zeta = \frac{1}{i} \cdot \frac{\alpha}{a} z$ (86): quare si pro loco oculi statuatur distantia $aO = O$, quae est fixa, erit $OF = \alpha - O$, et anguli $FO\zeta$ tangens $= \frac{1}{i} \frac{\alpha}{a} \cdot \frac{z}{\alpha - O}$, quae formula ob diuersam radiorum refrangibilitatem nullam mutationem subire debet. Inde autem quantitates α et i tantum variantur, dum reliquae manent constantes. Quare istius formulae differentiale logarithmicum nihilo aequale positum praebet hanc aequationem

$$-\frac{di}{i} + \frac{d\alpha}{\alpha} - \frac{d\alpha}{\alpha - O} = 0 \quad \text{seu} \quad \frac{-di}{i} - \frac{O\,d\alpha}{\alpha(\alpha - O)} = 0$$

vbi

vbi si valores supra inuenti substituantur, prodit

$$\frac{v\,dn}{in(n-1)}\left(\frac{1}{a}+\frac{1-i}{v}\right)+\frac{O\alpha\,dn}{i(n-1)(\alpha-O)}\left(\frac{i}{\alpha}+\frac{1}{i\alpha}+\frac{(1-i)^2}{iv}\right)=0$$

quae aequatio per $\frac{dn}{i(n-1)}$ diuisa praebet:

$$\frac{v}{n\alpha}+\frac{1-i}{n}+\frac{O\alpha}{\alpha-O}\left(\frac{i}{\alpha}+\frac{1}{i\alpha}+\frac{(1-i)^2}{iv}\right)=0$$

vnde locus oculi definiri poterit: qui si debeat convenire cum supra inuento (238), vbi inuenimus

$$O=\frac{-i\alpha v}{n\alpha-iv},\ \text{erit}$$

$\alpha-O=\frac{n\alpha\alpha}{n\alpha-iv}$ et $\frac{O\alpha}{\alpha-O}=\frac{-iv}{n}$, hincque nostra aequatio per n multiplicata abit in:

$$\frac{v}{a}+1-i-\frac{iiv}{\alpha}-\frac{v}{a}-(1-i)^2=0$$

seu $i-ii-\frac{iiv}{\alpha}=0$, ideoque $i=\frac{\alpha}{\alpha+v}=\frac{k-v}{k+v}$. Quamobrem quantitatem arbitrariam k ita definiri conueniet vt sit $k=2\alpha+v$: et cum sit $i=\frac{\alpha}{\alpha+v}$ pro loco oculi habebimus $O=\frac{-\alpha v}{n\alpha+(n-1)v}=\frac{-20\alpha v}{31\alpha+11v}$ ob $n=\frac{31}{20}$.

Quod si porro hinc variationem in loco imaginis desideremus definiri oportet differentiale $d\alpha$, quod fiet:

$$d\alpha=\frac{-\alpha\alpha\,dn}{i(n-1)}\left(\frac{i}{\alpha}+\frac{1}{i\alpha}+\frac{(1-i)^2}{iv}\right)$$

et pro i posito valore $\frac{\alpha}{\alpha+v}$

$$d\alpha=\frac{-(\alpha+v)dn}{n-1}\left(1+\frac{\alpha+v}{\alpha}\right)$$

qui valor si ad nihilum redigi posset, confusio omnis a diuersa radiorum refrangibilitate oriunda perfecte tolleretur.

 Coroll.

Coroll. 1.

303. Pro lentis ergo constructione quantitas arbitraria k ita accipi debet, vt sit $k = 2a + v$; atque tum oculus in eo loco constitutus, vbi totum campum apparentem percipiat, simul nullam confusionem a diuersa radiorum indole sentiet.

Coroll. 2.

304. Vt autem oculus simul imaginem in distantia iusta aspiciat, oportet sit $\alpha - O = -l$, ideoque $l = \frac{-11\alpha(\alpha+v)}{31\alpha+11v}$. Vnde colligitur: $a = \frac{-l}{2} - \frac{-v}{2} - \surd(\frac{1}{4}ll + \frac{9}{32}vl + \frac{1}{4}vv)$, hincque

$O = \frac{1}{2}l - \frac{1}{2}v - \surd(\frac{1}{4}ll + \frac{9}{32}vl + \frac{1}{4}vv)$ et $k = -l - 2\surd(\frac{1}{4}ll + \frac{9}{32}vl + \frac{1}{4}vv)$

Coroll. 3.

305. Potest vero insuper effici, vt etiam $d\alpha$ euanescat, quod euenit si $a + \alpha + v = 0$, hoc est $a = \frac{1}{2}l - \frac{1}{2}v + \surd(\frac{1}{4}ll + \frac{9}{32}vl + \frac{1}{4}vv)$. Verum cum hoc casu ob $\alpha = -a - v$ imago in ipsum obiectum cadet, ita, vt radii nullam refractionem pati sint censendi, visio per lentem perinde erit comparata atque nudis oculis.

Coroll. 4.

306. Si crassities lentis v plane euanescat, tam ob campum apparentem, quam diuersam radiorum refrangibilitatem fit $O = 0$ hoc ergo casu oculus lenti immediate applicatus nullam confusionem ob diuersam radiorum naturam percipiet. Dum ergo fuerit $\alpha = -l$ visio erit distincta. Scho-

Scholion.

307. Hic scilicet penitus mentem abstrahimus a confusione iam supra determinata, quae a lentium apertura oritur, ideoque aperturam primae lentis vt euanescentem spectamus. Eam igitur hic tantum confusionis speciem contemplamur, quae a diuersa radiorum refrangibilitate originem ducit; quam plerumque tolli obseruauimus, si angulus ad O inuariabilis reddatur; tum enim ora obiecti satis bene terminata conspicietur, neque coloribus iridis cincta. Interim tamen adhuc aliqua confusio sentiri poterit inde oriunda, quod si imago media iustam ab oculo distantiam teneat, imaginum extremarum altera sit nimis propinqua alter animis remota, verum si earum interuallum non sit admodum magnum, confusio haec parum erit sensibilis. Ita hic inuenimus, quod experientia satis comprobatur, si obiecta per vnicam lentem spectemus, ea margine colorato destituta apparere dummodo oculus immediate applicetur quod si quando secus euenire videatur, causa aperturae lentis sine dubio erit tribuenda, cui conditioni rationes hic allegatae refragantur.

Problema 5.

308. Si instrumentum dioptricum duabus instructum sit lentibus, definire locum oculi, vnde obiectum sine margine colorato videatur. Tab. III. Fig. 13.

Solutio

Solutio.

Posita obiecti distantia $AE = a$, sint pro radiis mediae naturae reliquae distantiae determinatrices $aF = \alpha$, $BF = b$ et $bG = \mathfrak{E}$ crassities vero lentium $Aa = v$, $Bb = v'$, et distantiae arbitrariae k et k', ponaturque $\frac{k-v}{k+v} = i$ et $\frac{k'-v'}{k'+v'} = i'$. His positis si magnitudo obiecti $E\varepsilon$ vocetur $= z$, erit imago $G\eta = \frac{1}{ii'} \frac{\alpha\mathfrak{E}}{ab} z$, vnde si oculi distantia ponatur $bO = O$,

ob $GO = \mathfrak{E} - O$, erit anguli $GO\eta$ tangens $\frac{1}{ii'} \cdot \frac{\alpha\mathfrak{E}}{ab} \cdot \frac{z}{\mathfrak{E}-O}$ cuius differentiale logarithmicum nihilo aequatum praebet,

$$-\frac{di}{i} - \frac{di'}{i'} + \frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{d\mathfrak{E}}{\mathfrak{E}} - \frac{d\mathfrak{E}}{\mathfrak{E}-O} = 0$$

quae valoribus supra (296) inuentis substitutis abit in:

$$\frac{v\,dn}{in(1-i)}\left(\frac{1}{a} + \frac{1-i}{v}\right) + \frac{v'\,db}{n'i'bb} + \frac{v'\,dn'}{i'n'(n'-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

$$-\frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{O}{\mathfrak{E}-O}\left(\frac{\mathfrak{E}\,db}{i'i'bb} + \frac{\mathfrak{E}\,dn'}{i'(n'-1)}\left(\frac{i'}{\mathfrak{E}} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)\right) = 0$$

Verum conditio campi exigebat $O = \frac{\mathfrak{E}\mathfrak{E}}{\mathfrak{E} + \frac{1}{ii'} \cdot \frac{\alpha\mathfrak{E}}{ab} z}$, existente

$$\mathfrak{E} = \left(\frac{i'}{i} \cdot \frac{\alpha+b}{a} - \frac{i'bv}{n'a\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{n'ab}\right) z, \quad (245), \text{ vnde fit}$$

$$\frac{O}{\mathfrak{E}-O} = \frac{\mathfrak{E}\mathfrak{E}}{\frac{1}{ii'} \cdot \frac{\alpha\mathfrak{E}\mathfrak{E}}{ab} z} = \frac{ii'ab}{\alpha\mathfrak{E}}\left(\frac{i'}{i} \cdot \frac{\alpha+b}{a} - \frac{i'bv}{na\alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{n'ab}\right)$$

Nunc

Nunc vero eſt $db=\frac{\alpha\alpha dn}{i(n-1)}\left(\frac{i}{\alpha}+\frac{1}{ia}+\frac{(1-i)^2}{iv}\right)$, quem valorem antequam ſubſtituamus, transformemus aequationem noſtram in hanc formam

$$\frac{dn}{n-1}\left(\frac{v}{ina}+\frac{1-i}{in}+\frac{v'}{i'nb}+\frac{1-i'}{i'n}+\frac{\beta O}{i'(\beta-O)}\left(\frac{i'}{\beta}+\frac{1}{i'b}+\frac{(1-i')^2}{i'v'}\right)\right)$$

$$+db\left(\frac{v'}{ni'bb}-\frac{1}{\alpha}-\frac{1}{b}+\frac{\beta O}{i'i'bb(\beta-O)}\right)=0$$

vbi poſterius membrum abit in $-\frac{iv}{n\alpha\alpha}db$, tum vero erit

$$0=\frac{dn}{n-1}\left\{\begin{array}{l}1+\frac{b}{\alpha}+\frac{i'i'b}{\beta}\left(1+\frac{b}{\alpha}\right)+\frac{(1-i')^2b(\alpha+b)}{\alpha v'}+\frac{2-i-i'}{n}\\ -\frac{iv}{n\alpha}-\frac{i'v'}{n\beta}-\frac{ibv}{n\alpha\alpha}-\frac{ii'i'bbv}{n\alpha\alpha\beta}-\frac{i(1-i')^2bbv}{n\alpha\alpha v'}\end{array}\right\}$$

diſtinguendo n' ab n erit

$$\frac{dn}{n-1}\left(\frac{iv}{\alpha\alpha}-\frac{(1-i)}{n}\right)=\frac{dn'}{n'-1}\left\{\begin{array}{l}1+\frac{b}{\alpha}+\frac{i'i'b}{\beta}\left(1+\frac{b}{\alpha}\right)-\frac{ii'i'bbv}{n\alpha\alpha\beta}-\frac{i'v'}{n'\beta}+\frac{1-i'}{n'}\\ -\frac{ibv}{n\alpha\alpha}+\frac{b(1-i')^2}{v'}\left(1+\frac{b}{\alpha}\right)-\frac{i(1-i')^2bbv}{n\alpha\alpha v'}\end{array}\right\}$$

cuius aequationis complicatio obſtat, quominus quicquam commode inde concludi poſſit.

Coroll. I.

309. Si ambae lentes craſſitie careant, vt ſit $v=0$, $v'=0$, et $i=i'=1$, aequatio differentialis prima eſt

$$\frac{d\alpha}{\alpha}-\frac{db}{b}-\frac{O\,d\beta}{\beta(\beta-O)}=0$$

tum vero:

$$d\alpha=-db=\frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{\alpha}+\frac{1}{a}\right) \text{ et}$$

$$d\beta=\frac{-\beta\beta db}{bb}-\frac{\beta\beta dn}{n-1}\left(\frac{1}{\beta}+\frac{1}{b}\right)=\frac{-dn}{n-1}\left(\frac{\alpha\alpha\beta\beta}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\beta\beta\left(\frac{1}{\beta}+\frac{1}{b}\right)\right)$$

 qui-

quibus valoribus ſubſtitutis et per $\frac{dn}{n-1}$ diuiſione facta fit

$$-\alpha\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{a}\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)+\tfrac{O\beta}{\beta-O}\left(\tfrac{\alpha\alpha}{bb}\left(\tfrac{1}{\alpha}+\tfrac{1}{a}\right)+\tfrac{1}{\beta}+\tfrac{1}{b}\right)=0$$

Vnde ſi oculus lenti poſteriori immediate applicaretur vt eſſet

$O=0$, deberet eſſe $\left(\tfrac{1}{\alpha}+\tfrac{1}{a}\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)=0$.

Coroll. 2.

310. Verum in eadem hypotheſi, vt locus oculi congruat cum eo, quem viſio campi exigit, debet eſſe $O=\frac{b\beta(\alpha+b)}{b(\alpha+b)+\alpha\beta}$, vnde fit $\beta-O=\frac{\alpha\beta\beta}{b(\alpha+b)+\alpha\beta}$, ideoque $\frac{O}{\beta-O}=\frac{b(\alpha+b)}{\alpha\beta}=\frac{bb}{\beta}\left(\frac{1}{\alpha}+\frac{1}{b}\right)$:
quo valore ſubſtituto noſtra aequatio erit

$$0=\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)\left(-\alpha\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{a}\right)+bb\left(\tfrac{\alpha\alpha}{bb}\left(\tfrac{1}{\alpha}+\tfrac{1}{a}\right)+\tfrac{1}{\beta}+\tfrac{1}{b}\right)\right)$$

quae reducitur ad hanc formam

$$0=bb\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)\left(\tfrac{1}{\beta}+\tfrac{1}{b}\right)$$

diſtinguendo n' ab n, membra a dn pendentia ſe deſtruunt et oritur

$$0=\tfrac{dn'}{n'-1}bb\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)\left(\tfrac{1}{\beta}+\tfrac{1}{b}\right)$$

quod ita oſtenditur:

Cum aequatio prima differentialis praebeat:

$$\tfrac{d\alpha}{\alpha}-\tfrac{db}{b}-\tfrac{O\,d\beta}{\beta(\beta-O)}=0 \text{ ſiue}$$

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)-\tfrac{O\,d\beta}{\beta(\beta-O)}=0$$

Cum

Cum igitur ex conditione campi apparentis sit

$$\frac{O}{\beta - O} = \frac{bb}{\beta}\left(\frac{1}{\alpha}+\frac{1}{b}\right) \text{ erit}$$

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right) - \frac{bb}{\beta^2}\left(\frac{1}{\alpha}+\frac{1}{b}\right) d\beta = 0$$

ideoque loco $d\beta$ suum valorem sustituendo

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right) - \frac{bb}{\beta^2}\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{\beta\beta\, d\alpha}{OO} - \frac{\beta^2\, dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)\right) = 0$$

vbi membra, quae $d\alpha$ continent, manifesto se destruunt, et tota quaestio ad hanc aequationem perducitur

$$\frac{dn'}{n'-1}\cdot bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\beta}+\frac{1}{b}\right) = 0.$$

Coroll. 3.

311. Quod si ergo huic conditioni satisfieri possit, obiectum sine margine colorato apparebit: praeterea vero confusio penitus tolleretur, si reddi liceret $d\beta = 0$, quod fit per hanc aequationem

$$\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\beta}+\frac{1}{b} = 0 \text{ siue}$$

$$+\frac{dn}{n-1}\cdot\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)$$

$$+\frac{dn'}{n'-1}\left(\frac{1}{\beta}+\frac{1}{b}\right) = 0$$

$$\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)+bb\left(\frac{1}{\beta}+\frac{1}{b}\right) = 0.$$

Coroll. 4.

312. Priori autem aequationi satisfieri nequit nisi fuerit vel $\alpha + b = 0$ vel $\frac{1}{\beta}+\frac{1}{b} = 0$. Illo casu ambae lentes coniungerentur, vt vnicam constituerent, hoc vero posterioris distantia focalis fieret infinita, qui

casus iterum ad casum vnicae lentis rediret; foret enim $O = -\alpha - b$, ob $\beta = -b$, et oculus priori lenti immediate applicari deberet.

Scholion.

313. Si simili modo has inuestigationes ad plures lentes extendere vellemus, non neglecta earum crassitie in formulas plane inextricabiles delaberemur vnde vix quicquam concludi posset. Verum quia in omnibus fere instrumentis dioptricis, praecipue quae pluribus lentibus constant, iis tam exigua crassities tribui solet, vt sine notabili errore pro nihilo haberi possit, tam taediosae indagationi facile supersedere poterimus. Ad quod accedit, quod hic non de summo rigore geometrico agatur, sed contenti esse queamus, dummodo hanc confusionem satis prope cognouerimus: ex quo sufficiet in consideratione plurium lentium earum crassitiem prorsus neglexisse.

Supplementum IV.

Si ratio refractionis in singulis lentibus sit diuersa solutio sequenti modo absoluetur:

I. Prima aequatio differentialis prorsus se habebit, vt in problemate, ita, vt sit

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)+d\beta\left(\frac{1}{\beta}+\frac{1}{c}\right)-\frac{O\,d\gamma}{\gamma(\gamma-O)}=0$$

et quia etiam, vt ante, est

$$\frac{O}{\gamma-O}=\frac{bbcc}{\alpha\beta^2\gamma}\left(1+\frac{\alpha}{b}\right)+\frac{cc}{\beta\gamma}\left(1+\frac{\beta}{c}\right)$$

erit

erit nostra aequatio

$$\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(d\alpha-\frac{bbcc}{\beta^2\gamma^2}d\gamma\right)+\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(d\beta-\frac{cc}{\gamma\gamma}d\gamma\right)=0.$$

II. Nunc autem ratio diuersae refractionis est habenda; vnde in superioribus additamentis inuenimus esse

$$d\alpha=-\frac{\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

$$d\beta=\frac{\beta\beta d\alpha}{bb}-\frac{\beta\beta dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)$$

$$d\gamma=\frac{\gamma\gamma d\beta}{cc}-\frac{\gamma\gamma dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

Hincque ergo fiet

$$d\alpha-\frac{bbcc}{\beta^2\gamma^2}d\gamma=d\alpha-\frac{bbd\beta}{\beta^2}+\frac{bbccdn''}{\beta^2(n''-1)}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

$$=\frac{bb.dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{bbccdn''}{\beta^2(n''-1)}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

Deinde

$$d\beta-\frac{cc}{\gamma\gamma}d\gamma=\frac{ccdn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

III. Ex his ergo nostra aequatio differentialis abibit in hanc formam:

$$\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{bbdn'}{n'-1}\left[\frac{1}{b}+\frac{1}{\beta}\right]+\frac{bbc^2dn''}{\beta^2(n''-1)}\left[\frac{1}{c}+\frac{1}{\gamma}\right]\right)$$

$$+\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(\frac{ccdn''}{n''-1}\left[\frac{1}{c}+\frac{1}{\gamma}\right]\right)$$

siue

$$\frac{dn'}{n'-1}.bb.\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\beta}+\frac{1}{b}\right)$$

$$+\frac{dn''}{n''-1}.cc\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\frac{bb}{\beta\beta}\left(\frac{1}{\alpha}+\frac{1}{b}\right)+\frac{1}{\beta}+\frac{1}{c}\right)=0$$

IV. Pro casu autem illo singulari, quo oculus lenti vltimae immediate, debet adplicari, ob $O=0$, habebitur simpliciter haec aequatio

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)+d\beta\left(\frac{1}{\beta}+\frac{1}{c}\right)=0$$

siue, substituto valore $d\beta$

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)+\tfrac{\beta\beta d\alpha}{bb}\left(\tfrac{1}{\beta}+\tfrac{1}{c}\right)$$
$$-\tfrac{\beta\beta dn'}{n'-1}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)=0.$$
$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}+\tfrac{\beta\beta}{bb}\left[\tfrac{1}{\beta}+\tfrac{1}{c}\right]\right)$$
$$-\tfrac{\beta^2 dn'}{n'-1}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)=0,$$

seu tandem

$$+\tfrac{dn}{n-1}\alpha\alpha\left(\tfrac{1}{b}+\tfrac{1}{a}\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{b}+\tfrac{\beta^2}{b^2}\left[\tfrac{1}{\beta}+\tfrac{1}{c}\right]\right)$$
$$+\tfrac{dn'}{n'-1}\beta\beta\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)=0.$$

V. Hoc modo tantum margo coloratus tollitur; vt autem tota confusio tollatur, quod fit si $d\gamma=0$, insuper satisfieri debet huic aequationi

$$0=\tfrac{dn}{n-1}\cdot\tfrac{\alpha\alpha\beta\beta\gamma\gamma}{bbcc}\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)$$
$$+\tfrac{dn'}{n'-1}\cdot\tfrac{\beta\beta\gamma\gamma}{cc}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)$$
$$+\tfrac{dn''}{n''-1}\cdot\gamma\gamma\cdot\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right).$$

Problema 6.

Tab. III. Fig. 14. 314. Si instrumentum dioptricum tribus constet lentibus, quarum crassities euanescat, eam definire dispositionem, vt oculus in eo loco, quem campus postulat, constitutus obiectum sine margine colorato conspiciat.

Solutio.

Posita ergo distantia obiecti ante lentem obiectiuam $AE=a$, eiusque magnitudine $E\epsilon=z$, vocentur

centur diſtantiae imaginum a radiis mediae naturae formatarum, vt hactenus

$$aF = \alpha,\ BF = b;\ bG = \beta,\ CG = c,\ \text{et}\ cH = \gamma.$$

eritque imago vltima $H\theta = \frac{\alpha\beta\gamma}{abc}z$, et poſita oculi poſt lentem vltimam diſtantia $cO = O$, erit $OH = \gamma - O$ et anguli $HO\theta$ tangens $= \frac{\alpha\beta\gamma}{abc} \cdot \frac{z}{\gamma - O}$, quae debet eſſe invariabilis. Poſito ergo eius differentiali logarithmico $= 0$ habebimus hanc aequationem:

$$\frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{d\beta}{\beta} - \frac{dc}{c} + \frac{d\gamma}{\gamma} - \frac{d\gamma}{\gamma - O} = 0$$

At et formulis ſupra (300) erutis habemus

$$d\alpha = -db = \frac{-dn}{n-1} \cdot \alpha\alpha\left(\frac{1}{a} + \frac{1}{\alpha}\right)$$

$$d\beta = -dc = -\frac{\beta\beta dn}{n-1}\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{a} + \frac{1}{\alpha}\right) + \frac{1}{b} + \frac{1}{\beta}\right)$$

$$d\gamma = -\frac{\gamma\gamma dn}{n-1}\left(\frac{\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a} + \frac{1}{\alpha}\right) + \frac{\beta\beta}{cc}\left(\frac{1}{b} + \frac{1}{\beta}\right) + \frac{1}{c} + \frac{1}{\gamma}\right)$$

vnde noſtra aequatio erit

$$d\alpha\left(\frac{1}{\alpha} + \frac{1}{b}\right) + d\beta\left(\frac{1}{\beta} + \frac{1}{c}\right) - \frac{O\,d\gamma}{\gamma(\gamma - O)} = 0$$

Sed ob campum apparentem ſupra 254 inuenimus:

$$\mathfrak{B} = \beta = b\left(1 + \frac{\alpha}{b}\right)\frac{z}{a}$$

$$\mathfrak{C} = c = \frac{bc}{\beta}\left(1 + \frac{\alpha}{b}\right)\frac{z}{a} + \frac{\alpha c}{b}\left(1 + \frac{\beta}{c}\right)\frac{z}{a}$$

hincque

$$O = \frac{\gamma c}{c + H\theta} \text{ et } \frac{O}{\gamma - O} = \frac{c}{H\theta} \text{ vnde fit ob}$$

$$H\theta = \frac{\alpha\beta\gamma}{bc} \cdot \frac{z}{a}$$

$$\frac{O}{\gamma - O} = \frac{bbcc}{\alpha\beta\beta\gamma}\left(1 + \frac{\alpha}{b}\right) + \frac{cc}{\beta\gamma}\left(1 + \frac{\beta}{c}\right) \text{ ſeu}$$

$$\frac{O}{\gamma(\gamma - O)} = \frac{bbcc}{\beta\beta\gamma\gamma}\left(\frac{1}{\alpha} + \frac{1}{b}\right) + \frac{cc}{\gamma\gamma}\left(\frac{1}{\beta} + \frac{1}{c}\right)$$

Valo-

Valoribus iam his substitutis habebimus:

$$\left(\frac{1}{a}+\frac{1}{b}\right)\left(d\alpha-\frac{bbcc}{\beta\beta\gamma\gamma}d\gamma\right)+\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(d\beta-\frac{cc}{\gamma\gamma}d\gamma\right)=0$$

At est

$$d\alpha-\frac{bbcc}{\beta\beta\gamma\gamma}d\gamma=\frac{dn}{n-1}\left(bb\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{bbcc}{\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)$$

$$d\beta-\frac{cc}{\gamma\gamma}d\gamma=\frac{dn}{n-1}\left(cc\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)$$

Quare facta diuisione per $\frac{dn}{n-1}$ nanciscemur hanc aequationem:

$$bb\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{b}+\frac{1}{\beta}+\frac{cc}{\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)+cc\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(\frac{1}{c}+\frac{1}{\gamma}\right)=0$$

Quodsi vero oculus lenti postremae immediate applicetur, seu sit $O=0$, conditio praescripta hanc postulat aequationem:

$$a\alpha\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\beta\beta\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{1}{b}+\frac{1}{\beta}\right)=0$$

Confusio vero a diuersa radiorum refrangibilitate oriunda perfecte tolletur si praeterea fuerit $d\gamma=0$ seu

$$\frac{a\alpha\beta\beta}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{\beta\beta}{cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{1}{c}+\frac{1}{\gamma}=0.$$

Coroll. 1.

315. Si rationes aperturarum lentium in computum ducantur, eaque pro lente secunda ponatur $=\pi$, pro tertia $=\pi'$ erit $bN'=\frac{\pi b\beta}{b+\beta}$ et $CM''=cN''=\frac{\pi' c\gamma}{c+\gamma}$. Tum vero posito $\frac{x}{a}=\Phi$, erit $G\eta=\frac{\alpha\beta}{ab}a\Phi$, et $H\theta=\frac{\alpha\beta\gamma}{abc}\cdot a\Phi$. Hinc fiet $\frac{O}{\gamma-O}=\frac{cN''}{H\theta}=\frac{\pi' abcc}{\alpha\beta(c+\gamma)a\Phi}$ et $\frac{O}{\gamma(\gamma-O)}=\frac{\pi' abcc}{\alpha\beta\gamma(c+\gamma)}\cdot\frac{1}{a\Phi}$.

Coroll.

Coroll. 2.

316. Quodsi porro vt supra ponatur

$\alpha = Aa,\ \beta = Bb,\ \gamma = Cc$ erit $\frac{O}{\gamma(\gamma-1)} = \frac{\pi'}{ABC(C+1)c\Phi}$;
tum vero $bN' = \frac{\pi Bb}{B+1}$ et $CM'' = \frac{\pi' Cc}{C+1}$: ac $G\eta = ABa\Phi$;
Cum iam sit

$bN' + G\eta : bG = CM'' - G\eta : CG$ erit $\frac{bN' + G\eta}{bG} = \frac{CM'' - G\eta}{CG}$.

ideoque

$\frac{1}{bG} + \frac{1}{CG} = \frac{CM''}{CG \cdot G\eta} - \frac{bN'}{bG \cdot G\eta}$ hoc est $\frac{1}{b} + \frac{1}{c} = \frac{\pi' C}{AB(C+1)a\Phi} - \frac{\pi}{AB(B+1)a\Phi}$
simili vero modo est $\frac{1}{\alpha} + \frac{1}{b} = \frac{\pi B}{A(B+1)a\Phi}$.

Coroll. 3.

317. Per easdem substitutiones fit

$\frac{1}{a} + \frac{1}{\alpha} = \frac{A+1}{Aa}$; $\frac{1}{b} + \frac{1}{\beta} = \frac{B+1}{Bb}$; $\frac{1}{c} + \frac{1}{\gamma} = \frac{C+1}{Cc}$ etc.
hincque:

$\alpha\alpha\ (\frac{1}{a} + \frac{1}{\alpha}) = A(A+1)a$

$\beta\beta\ (\frac{1}{b} + \frac{1}{\beta}) = B(B+1)b$

$\gamma\gamma(\frac{1}{c} + \frac{1}{\gamma}) = C(C+1)c$ sicque porro.

Coroll. 4.

318. His ergo nouis denominationibus introductis differentialia ex §. 300 ita exprimentur

$d\alpha = -db = -\frac{dn}{n-1} . A(A+1)a$

$d\beta = -dc = -BBdb - \frac{dn}{n-1} . B(B+1)b$

$d\gamma = -dd = -CCdc - \frac{dn}{n-1} . C(C+1)c$

quae formulae commodius in calculum introducentur.

Quemadmodum hic nouae formae adhibentur in ſequentibus vſurpandae; ita et pro caſu diuerſae refractionis ſequentibus formulis in poſterum vti licebit:

$$\frac{0}{\gamma(\gamma-\delta)} = \frac{\pi'}{ABC(C+1)a\Phi}$$

$$\frac{1}{\alpha}+\frac{1}{b} = \frac{\pi B}{A(B+1)a\Phi}.$$

$$\frac{1}{\beta}+\frac{1}{c} = \frac{\pi' C}{AB(C+1)a\Phi} - \frac{\pi}{AB(B+1)a\Phi}$$

$$\frac{1}{\gamma}+\frac{1}{d} = \frac{\pi'' D}{ABC(D+1)a\Phi} - \frac{\pi'}{ABC(C+1)a\Phi}$$

etc.

tum vero formulae differentiales erunt

$$d\alpha = -db = \frac{-dn}{n-1}.\, A(A+1)a$$

$$d\beta = -dc = B^2 d\alpha \frac{-dn'}{n'-1}.\, B(B+1)b$$

$$d\gamma = -dd = C^2 d\beta \frac{-dn''}{n''-1} C(C+1)c$$

$$d\delta = -de = D^2 d\gamma \frac{-dn'''}{n'''-1}.\, D(D+1)d$$

etc.

Atque ex his formulis vt margo coloratus euaneſcat, ſatisfieri debet huic aequationi

$$0 = \frac{dn'}{n'-1}.\, \frac{\pi b}{Aa\Phi} + \frac{dn''}{n''-1}.\, \frac{\pi' c}{ABa\Phi}$$

Vt autem haec confuſio penitus tollatur, fieri debet

$$d\gamma = -\frac{dn}{n-1}.\, A(A+1)B^2C^2 a$$
$$-\frac{dn'}{n'-1}.\, B(B+1)C^2.\, b$$
$$-\frac{dn''}{n''-1}.\, C(C+1)c = 0.$$

ſiue

fiue

$$\frac{d\pi}{\pi-1}\cdot\frac{(A+1)a}{A}$$
$$+\frac{d\pi'}{\pi'-1}\cdot\frac{(B+1)b}{AAB}=0$$
$$+\frac{d\pi''}{\pi''-1}\cdot\frac{(C+1)c}{A^2\,B\cdot C}.$$

Problema 7.

319. Si inftrumentum dioptricum quatuor conftet lentibus quarum craffities negligi queat, eam definire difpofitionem vt oculus in eo loco quem campus apparens poftulat, conftitutus obiectum fine confufione a diuerfa radiorum indole oriunda confpiciat. Tab. III. Fig. 15.

Solutio.

Pofita diftantia obiecti ante inftrumentum $AE=a$, eiusque magnitudine $E\varepsilon=z$, vocentur diftantiae imaginum a radiis mediae naturae formatarum vt hactenus.

$aF=\alpha$, $BF=b$; $bG=\mathfrak{b}$, $CG=c$; $cH=\gamma$; $DG=d$; $dI=\delta$

Tum vero ponamus praeterea

$$\alpha=Aa,\ \mathfrak{b}=Bb;\ \gamma=Cc;\ \delta=Dd$$

atque introducantur rationes aperturarum pro fingulis lentibus poft primam, quae fint π pro fecunda QQ, π' pro tertia RR et π'' pro quarta SS, fumto pro campo $\frac{z}{a}=\Phi$. His pofitis erunt imagines $F\zeta=Aa\Phi$; $G\eta=ABa\Phi$; $H\theta=ABCa\Phi$ et $I\iota=ABCDa\Phi=\frac{\alpha\mathfrak{b}\gamma\delta}{abcd}z$.

 Iam

Iam poſita oculi diſtantia poſt inſtrumentum $dO = O$, vt ſit $OI = \delta - O$ erit anguli IOi tangens $\frac{\alpha\beta\gamma\delta}{abcd} \cdot \frac{z}{\delta - O}$, quae cum ob diuerſam radiorum refrangibilitatem immutata manere debeat, differentiata dabit hanc aequationem

$$\frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{d\beta}{\beta} - \frac{dc}{c} + \frac{d\gamma}{\gamma} - \frac{dd}{d} - \frac{O\,d\delta}{\delta(\delta - O)} = 0$$

quae ob $db = -d\alpha$, $dc = -d\beta$ et $dd = -d\gamma$ abit in hanc

$$d\alpha\left(\frac{1}{\alpha} + \frac{1}{b}\right) + d\beta\left(\frac{1}{\beta} + \frac{1}{c}\right) + d\gamma\left(\frac{1}{\gamma} + \frac{1}{d}\right) - \frac{O\,d\delta}{\delta(\delta - O)} = 0$$

Verum modo ante notauimus fore (318)

$$d\alpha = \frac{-dn}{n-1} \cdot A(A+1)a$$

$$d\beta = BB\,d\alpha - \frac{dn}{n-1} \cdot B(B+1)b$$

$$d\gamma = CC\,d\beta - \frac{dn}{n-1} \cdot C(C+1)c$$

$$d\delta = DD\,d\gamma - \frac{dn}{n-1} \cdot D(D+1)d$$

vbi pro b, c, d, valores §. 266 aſſignati ſubſtitui debent. Porro vero iam animaduertimus eſſe (316)

$$\frac{1}{\alpha} + \frac{1}{b} = \frac{1}{Aa\Phi} \cdot \frac{\pi B}{B+1}$$

$$\frac{1}{\beta} + \frac{1}{c} = \frac{1}{ABa\Phi}\left(\frac{\pi' C}{C+1} - \frac{\pi}{B+1}\right)$$

$$\frac{1}{\gamma} + \frac{1}{d} = \frac{1}{ABCa\Phi}\left(\frac{\pi'' D}{D+1} - \frac{\pi'}{C+1}\right).$$

atque

$$\frac{O}{\delta(\delta - O)} = \frac{\pi''}{ABCD(D+1)a\Phi}.$$

Quod

Quodſi iam priores valores in poſterioribus ſucceſſiue ſubſtituantur, habebimus

$$d\alpha = \frac{-dn}{n-1} \cdot A(A+1)a$$

$$d\beta = \frac{-dn}{n-1} \cdot B(AB(A+1)a+(B+1)b$$

$$d\gamma = \frac{-dn}{n-1} \cdot C(AB^2C(A+1)a+BC(B+1)b+(C+1)c)$$

$$d\delta = \frac{-dn}{n-1} \cdot D(AB^2C^2D(A+1)a+BC^2D(B+1)b+CD(C+1)c+(D+1)d)$$

His igitur valoribus in aequatione differentiali ſubſtitutis, et diuiſione per $\frac{dn}{(n-1)\Phi}$ facta aequatio noſtra ſecundum ſingula membra diſtributa erit

$$-\frac{\pi B}{(B+1)} \cdot (A+1)a$$

$$-\frac{1}{A}\left(\frac{\pi' C}{C+1}-\frac{\pi}{B+1}\right)(AB(A+1)a+(B+1)b)$$

$$-\frac{1}{AB}\left(\frac{\pi'' D}{D+1}-\frac{\pi'}{C+1}\right)(AB^2C(A+1)a+BC(B+1)b+(C+1)c)$$

$$+\frac{1}{ABC} \cdot \frac{\pi''}{D+1}(AB^2C^2D(A+1)a+BC^2D(B+1)b+CD(C+1)c+(D+1)d)=0$$

Terna autem priora membra negatiua ſola collecta praebent

$$-\frac{BCD(A+1)\pi''}{D+1}a-\frac{CD(B+1)\pi''}{A(D+1)}b-\frac{D(C+1)\pi''}{AB(D+1)}c$$

$$+\frac{\pi}{A}b+\frac{\pi'}{AB}c$$

quibus ſi quartum addatur prodit

$$\frac{\pi}{A}b+\frac{\pi'}{AB}c+\frac{\pi''}{ABC}d$$

Iam duo hic caſus conſiderari oportet, alterum quo punctum O poſt lentem vltimam cadit, alterum

 vero

vero, quo ob distantiam O negatiuam oculus lenti vltimae immediate applicatur. Pro priori casu, quo distantia $dO = O$ prodit positiua habetur ista aequatio si quidem in aequatione modo inuenta pro b, c, d valores §. 266 inuenti substituantur

$$\frac{(B+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)\pi''}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

Pro casu posteriori, quo distantia O euanescens assumitur facta multiplicatione per $\frac{D+1}{a\ \Phi}$.

$$\frac{BCD(A+1)\pi''}{\Phi}+\frac{CD(B+1)^2\pi''}{B\pi-(B+1)\Phi}+\frac{D(C+1)^2\pi''}{C\pi'-(C+1)(\pi-\Phi)}$$
$$=\frac{(B+1)(D+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)(D+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}.$$

Hoc modo efficitur, vt obiectum sine margine colorato appareat at omnis confusio tolletur si praeterea fuerit

$$AB^2C^2D(A+1)+\frac{ABC^2D(B+1)^2\Phi}{B\pi-(B+1)\Phi}+\frac{ABCD(C+1)^2\Phi}{C\pi'-(C+1)(\pi-\Phi)}+\frac{ABC(D+1)^2\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

seu per ABC diuidendo:

$$BCD(A+1)+\frac{CD(B+1)^2\Phi}{B\pi-(B+1)\Phi}+\frac{D(C+1)^2\Phi}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)^2\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

$$BCD(A+1)+\frac{CD(B+1)b}{Aa}+\frac{D(C+1)c}{ABa}+\frac{(D+1)d}{ABCa}=0$$

Coroll. 1.

320. Si vtrique conditioni satisfieri potest, vt nec locus nec magnitudo imaginis vllam mutationem patiatur locus oculi non amplius in computum ingreditur sed imago, vndecunque cernatur, ab omni confusione prorsus libera erit.

Coroll.

Coroll. 2.

321. Vt ergo hunc fummum perfectionis gradum confequamur, binis his aequationibus fatisfieri oportet.

$$\frac{(B+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)\pi''}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

et

$$A+1+\frac{(B+1)^2\Phi}{B(B\pi-(B+1)\Phi)}+\frac{(C+1)^2\Phi}{BC(C\pi'-(C+1)(\pi-\Phi))}+\frac{(D+1)^2\Phi}{BCD(D\pi''-(D+1)(\pi'-\pi+\Phi))}=0.$$

Coroll. 3.

322. Si porro vt fupra fecimus ponamus

$$\frac{A}{A+1}=\mathfrak{A},\ \frac{B}{B+1}=\mathfrak{B},\ \frac{C}{C+1}=\mathfrak{C} \text{ et } \frac{D}{D+1}=\mathfrak{D}$$

iftae aequationes fequenti modo fimplicius exprimentur:

$$+\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0 \text{ et}$$

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABCD(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0$$

hic prima membra multiplicari debent per $\frac{dn}{n-1}$

fecunda per $\frac{dn'}{n'-1}$

tertia per $\frac{dn''}{n''-1}$

etc.

Coroll. 4.

323. Sin autem non liceat has ambas aequationes adimplere, curandum eft, vt faltem priori, qua magnitudo apparens a confufione liberatur, fatisfiat

fiat hec enim modo obiectum fine margine colorato apparebit vbi quidem ad duos cafus refpici conuenit prout diftantia oculi dO prodierit pofitiua vel negatiua.

Scholion.

324. Introducendis ergo rationibus aperturarum quibus fupra iam commode fumus vfi ad campum apparentem definiendum aequationes etiam iftae confufionem a diuerfa radiorum refrangibilitate oriundam tollentes fatis fiunt fimplices, vt fine moleftia tractari queant; fi quidem craffities lentium negligatur. Hoc ergo modo problema generale, quicunque fuerit lentium numerus, expediri conueniet.

Supplementum V.

Si ratio refractionis in fingulis lentibus difcrepet, prodit primo quidem eadem aequatio differentialis

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)+d\beta\left(\tfrac{1}{\beta}+\tfrac{1}{c}\right)+d\gamma\left(\tfrac{1}{\gamma}+\tfrac{1}{d}\right)\frac{-O\,d\delta}{\delta(\delta-O)}=0$$

in qua ergo cafu $O=0$ vltimum membrum abiici debet.

I) Si autem O habeat valorem pofitiuum, erit, vt ante,

$$\frac{O}{\delta(\delta-O)}=\frac{\pi\nu}{ABCD(D-\phi-1)n\Phi}$$

atque etiam valores $\frac{1}{\alpha}+\frac{1}{b}$; $\frac{1}{\beta}+\frac{1}{c}$; $\frac{1}{\gamma}+\frac{1}{d}$ manent iidem, vt ante.

At

At vero ob diuerſitatem refractionis habebimus

$$d\alpha = \frac{-dn}{n-1}.\,A(A+1)a$$

$$d\beta = BB\,d\alpha - \frac{dn'}{n'-1}.\,B(B+1)b$$

$$d\gamma = CC\,d\beta - \frac{dn''}{n''-1}\,C.\,(C+1)c.$$

$$d\delta = DD\,d\gamma - \frac{dn'''}{n'''-1}.\,D(D+1)d$$

Hinc ergo aequatio noſtra ſucceſſiue ita formetur:

$$\frac{-O\,d\delta}{\delta(\delta-O)} = \frac{-\pi''\,d\delta}{ABCD(D+1)a\Phi}$$

$$= \frac{-\pi''\,D\,d\gamma}{ABC(D+1)a\Phi} + \frac{dn'''}{n'''-1}.\,\frac{\pi''.\,d}{ABC.\,a\Phi}$$

Addatur $d\gamma\left(\frac{1}{\gamma}+\frac{1}{d}\right) = \frac{d\gamma.\,\pi''D}{ABC(D+1)a\Phi} - \frac{\pi'\,d\gamma}{ABC(C+1)a\Phi}$

et prodibit $\frac{-\pi'\,d\gamma}{ABC(C+1)a\Phi} + \frac{dn'''}{n'''-1}.\,\frac{\pi''.\,d}{ABC.\,a\Phi}$

et pro $d\gamma$ ſubſtituto valore

$$\frac{-\pi'\,C\,d\beta}{AB(C+1)a\Phi} + \frac{dn''}{n''-1}.\,\frac{\pi'\,c}{ABa\Phi} + \frac{dn'''}{n'''-1}.\,\frac{\pi''.\,d}{ABC.\,a\Phi}$$

Iam addatur

$$d\beta\left(\frac{1}{\beta}+\frac{1}{c}\right) = \frac{\pi'\,C\,d\beta}{AB(C+1)a\Phi} - \frac{\pi\,d\beta}{AB(B+1)a\Phi}$$

proditque

$$\frac{-\pi\,d\beta}{AB(B+1)a\Phi} + \frac{dn''}{n''-1}.\,\frac{\pi'\,c}{AB.\,a\Phi} + \frac{dn'''}{n'''-1}.\,\frac{\pi''\,d}{ABCa\Phi}$$

et ſubſtituto valore ipſius $d\beta$

$$\frac{-\pi B\,d\alpha}{A(B+1)a\Phi} + \frac{dn'}{n'-1}.\,\frac{\pi.\,b}{A.\,a\Phi} + \frac{dn''}{n''-1}.\,\frac{\pi'\,c}{ABa\Phi} + \frac{dn'''}{n'''-1}.\,\frac{\pi''\,d}{ABC.\,a\Phi}$$

denique addatur $d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right) = \frac{\pi B\,d\alpha}{A(B+1)a\Phi}$

ac aequatio quaeſita, qua margo coloratus euaneſcit, erit

$$\frac{dn'}{n'-1}.\,\frac{\pi b}{Aa\Phi} + \frac{dn''}{n''-1}.\,\frac{\pi'\,c}{ABa\Phi} + \frac{dn'''}{n'''-1}.\,\frac{\pi''\,d}{ABCa\Phi} = 0$$

II) Sin autem O habeat valorem negatiuum : tunc sumi debet O=o et pro eodem scopo aequatio ita formabitur :

Cum sit

$$d\gamma\left(\tfrac{1}{\gamma}+\tfrac{1}{d}\right)=\frac{\pi'' D\, d\gamma}{ABC(D+1)a\Phi}-\frac{\pi' d\gamma}{ABC(C+1)a\Phi}$$

substituto valore $d\gamma$ fiet.

$$CC d\beta\left(\frac{\pi'' D}{ABC(D+1)a\Phi}-\frac{\pi'}{ABC(C+1)a\Phi}\right)$$
$$-\frac{d n''}{n''-1}.\, C(C+1)c\left(\frac{\pi'' D}{ABC(D+1)a\Phi}-\frac{\pi'}{ABC(C+1)a\Phi}\right)$$

Addatur

$$d\beta\left(\tfrac{1}{\beta}+\tfrac{1}{c}\right)=\frac{\pi' C d\beta}{AB(C+1)a\Phi}-\frac{\pi\, d\beta}{AB(B+1)a\Phi}$$

eritque

$$d\beta\left(\frac{CD\pi''}{AB(D+1)a\Phi}-\frac{\pi}{AB(B+1)a\Phi}\right)-\frac{d n''}{n''-1}(C+1)c\left(\frac{\pi'' D}{AB(D+1)a\Phi}-\frac{\pi'}{AB(C+1)a\Phi}\right)$$

et substituto valore ipsius $d\beta$

$$d\alpha\left(\frac{BCD\pi''}{A(D+1)a\Phi}-\frac{B\pi}{A(B+1)a\Phi}\right)$$
$$-\frac{d n'}{n'-1}\left(\frac{(B+1)CDb\pi''}{A(D+1)a\Phi}-\frac{\pi b}{A.a\Phi}\right)-\frac{d n''}{n''-1}(C+1)c\left(\frac{\pi'' D}{AB(D+1)a\Phi}-\frac{\pi'}{AB(C+1)a\Phi}\right)$$

Addatur

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)=\frac{\pi B d\alpha}{A(B+1)a\Phi}$$

eritque

$$-\frac{dn}{n-1}.\left(\frac{(A+1)BCDa\pi''}{(D+1)a\Phi}\right)$$
$$-\frac{dn'}{n'-1}\left(\frac{(B+1)CDb\pi''-(D+1)b\pi}{A(D+1)a\Phi}\right)$$
$$-\frac{dn''}{n''-1}\left(\frac{(C+1)Dc\pi''-(D+1)c\pi'}{AB(D+1)a\Phi}\right)$$

Vnde

Vnde pro casu $O = 0$ aequatio, qua margo coloratus destruitur, erit

$$0 = \frac{adn}{n-1} \cdot (A+1)BCD\pi''$$
$$+ \frac{bdn'}{n'-1} \cdot \left(\frac{(B+1)CD\pi'' - (D+1)\pi}{A}\right)$$
$$+ \frac{cdn''}{n''-1} \left(\frac{(C+1)D\pi'' - (D+1)\pi'}{AB}\right)$$

III) Vt autem praeterea omnis confusio tollatur, insuper reddi oportet $d\delta = 0$; vnde haec aequatio nascitur

$$0 = \begin{cases} \frac{adn}{n-1} \cdot A(A+1)B^2C^2D^2 \\ + \frac{bdn'}{n'-1} \cdot B(B+1)C^2D^2 \\ + \frac{cdn''}{n''-1} \cdot C(C+1)D^2 \\ + \frac{d.dn'''}{n'''-1} \cdot D(D+1) \end{cases}$$

quae per $A^2B^2C^2D^2$ diuisa dat

$$0 = \begin{cases} \frac{adn}{n-1} \cdot \frac{A+1}{A} \\ + \frac{bdn'}{n'-1} \cdot \frac{B+1}{A^2B} \\ + \frac{cdn''}{n''-1} \cdot \frac{C+1}{A^2B^2C} \\ + \frac{d.dn'''}{n'''-1} \cdot \frac{D+1}{A^2B^2C^2D} \end{cases}$$

Circa hanc autem aequationem inprimis notandum est, si omnes lentes pari facultate refringente sint praeditae, ei satisfieri haud posse; ex quo haec aequatio proprie pertinet ad casum, quo diuersae refractiones locum habent.

 Pro-

Problema 8.

325. Si instrumentum dioptricum ex quotcunque lentibus sit compositum, quarum crassitiem negligere liceat, eam determinare dispositionem, vt oculus in eo loco, quem campus apparens postulat positus nullam confusionem sentiat.

Solutio.

Sit obiecti ante lentem primam distantia $AE = a$, eiusque magnitudo $E\epsilon = z$, quae quidem conspici queat, et statuatur $\frac{z}{a} = \Phi$. Deinde sint distantiae imaginum a radiis mediae naturae formatarum vt supra:

$$AE = a;\ BF = b;\ CG = c;\ DH = d;\ EI = e$$
$$a F = \alpha;\ b G = \mathfrak{E};\ c H = \gamma;\ d I = \delta;\ e K = \varepsilon$$

etc.

ac vocemus breuitatis gratia

$$\alpha = Aa,\ \mathfrak{E} = Bb,\ \gamma = Cc,\ \delta = Dd,\ \varepsilon = Ee \text{ etc.}$$

tum vero etiam

$$\frac{A}{A+1} = \mathfrak{A};\ \frac{B}{B+1} = \mathfrak{B};\ \frac{C}{C+1} = \mathfrak{C};\ \frac{D}{D+1} = \mathfrak{D},\ \frac{E}{E+1} = \mathfrak{E} \text{ etc.}$$

Iam apertura primae lentis PP vt euanescente considerata sit ratio aperturae pro reliquis lentibus

$$QQ = \pi;\ RR = \pi';\ SS = \pi'';\ TT = \pi''' \text{ etc.}$$

His

His positis supra vidimus (266) fore praeter $\alpha = \mathrm{A}a$

$$b = \frac{\mathrm{A}a\Phi}{\mathfrak{B}\pi - \Phi} \qquad \beta = \frac{\mathrm{AB}a\Phi}{\mathfrak{B}\pi - \Phi}$$

$$c = \frac{\mathrm{AB}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \qquad \gamma = \frac{\mathrm{ABC}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$d = \frac{\mathrm{ABC}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \qquad \delta = \frac{\mathrm{ABCD}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$e = \frac{\mathrm{ABCD}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \qquad \varepsilon = \frac{\mathrm{ABCDE}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

etc. etc.

Deinde singularum imaginum magnitudo ita se habebit

$$\mathrm{F}\zeta = \mathrm{A}a\Phi;\ \mathrm{G}\eta = \mathrm{AB}a\Phi;\ \mathrm{H}\theta = \mathrm{ABC}a\Phi;\ \mathrm{I}\iota = \mathrm{ABCD}a\Phi$$

etc.

et ipsi aperturarum semidiametri:

Lentis secundae $\mathrm{QQ} = \frac{\mathrm{A}\mathfrak{B}\Phi}{\mathfrak{B}\pi - \Phi}\pi$

Lentis tertiae $\mathrm{RR} = \frac{\mathrm{AB}\mathfrak{C}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}\pi'$

Lentis quartae $\mathrm{S\,S} = \frac{\mathrm{ABC}\mathfrak{D}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\pi''$

Lentis quintae $\mathrm{TT} = \frac{\mathrm{ABCD}\mathfrak{E}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}\cdot\pi'''$

Mutata iam refractionis lege $n:1$ infinite parum, distantiae $\alpha, b, \beta, c; \gamma, d, \delta$ etc. tales mutationes recipiunt.

$$d\alpha = -\frac{dn}{n-1}.\mathrm{A}a(\mathrm{A}+1)$$

$$d\beta = \frac{-dn}{n-1}.\mathrm{AB}a\left((\mathrm{A}+1)\mathrm{B} + \frac{(\mathrm{B}+1)\Phi}{\mathfrak{B}\pi - \Phi}\right)$$

$$d\gamma = \frac{-dn}{n-1}.\mathrm{ABC}a\left((\mathrm{A}+1)\mathrm{BC} + \frac{(\mathrm{B}+1)\mathrm{C}\Phi}{\mathfrak{B}\pi - \Phi} + \frac{(\mathrm{C}+1)\Phi}{\mathfrak{C}\pi' - \pi + \Phi}\right)$$

$$d\delta = \frac{-dn}{n-1}\cdot\mathrm{ABCD}a\left((\mathrm{A}+1)\mathrm{BCD} + \frac{(\mathrm{B}+1)\mathrm{CD}\Phi}{\mathfrak{B}\pi - \Phi} + \frac{(\mathrm{C}+1)\mathrm{D}\Phi}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{(\mathrm{D}+1)\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\right)$$

etc.

 Porro

Porro vero habetur:

$$\frac{1}{\alpha}+\frac{1}{b}=\frac{1}{Aa\Phi}\cdot\mathfrak{B}\pi$$
$$\frac{1}{\beta}+\frac{1}{c}=\frac{1}{ABa\Phi}\left(\mathfrak{C}\pi'-\frac{\mathfrak{B}\pi}{B}\right)$$
$$\frac{1}{\gamma}+\frac{1}{d}=\frac{1}{ABCa\Phi}\left(\mathfrak{D}\pi''-\frac{\mathfrak{C}\pi'}{C}\right)$$
$$\frac{1}{\delta}+\frac{1}{e}=\frac{1}{ABCDa\Phi}\left(\mathfrak{E}\pi'''-\frac{\mathfrak{D}\pi''}{D}\right)$$
etc.

His positis pro quolibet lentium numero seorsim formulas quaesito satisfacientes expediamus: posita distantia oculi post lentem vltimam $=O$.

I. Pro vnica Lente

Habetur haec aequatio differentialis $\frac{-O\,d\alpha}{\alpha(\alpha-O)}=0$, pro quo casu tam magnitudo imaginis quam eius locus manebit inuariatus si fuerit $d\alpha=0$, hoc est $A(A+1)=0$, vnde deberet esse vel $A=0$ vel $A=-1$: quorum prius visio non admittit posterius autem lentem tollit. Tum vero ob campum esse debet $O=0$.

II. Pro duabus lentibus

Habetur haec aequatio differentialis, qua margo coloratus tollitur:

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{O\,d\beta}{\beta(\beta-O)}=0$$

at ob campum apparentem est $\frac{O}{\beta(\beta-O)}=\frac{1}{BB}\left(\frac{1}{\alpha}+\frac{1}{b}\right)$, ita vt habeamus:

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{d\beta}{BB}\left(\frac{1}{\alpha}+\frac{1}{b}\right)=0$$

Verum

Verum eſt $d\mathfrak{C} = BB\,d\alpha - \frac{dn}{n-1} \cdot AB\alpha \cdot \frac{(B+1)\Phi}{\mathfrak{B}\pi - \Phi}$

Quare ſi O habeat valorem poſitiuum erit ob

$$\mathfrak{B}(B+1) = B, \quad \frac{\pi}{\mathfrak{B}\pi - \Phi} = 0$$

Sin autem valor O prodeat negatiuus, quo caſu capitur $O = 0$ erit $\frac{(A+1)\mathfrak{B}\pi}{\Phi} = 0$.

Omnis autem confuſio penitus tolletur, ſi inſuper fuerit

$$\frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} = 0$$

III. Pro tribus Lentibus

Si calculum eodem modo proſequamur, obiectum ſine margine colorato conſpicietur:

1. Si ex campo apparente diſtantia O prodeat poſitiua hanc aequationem adimplendo:

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = 0.$$

2. Si ob diſtantiam O prodeuntem negatiuam capiatur $O = 0$ huic aequationi erit ſatisfaciendum:

$$\frac{\pi'}{\mathfrak{A}\Phi} + \frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} = \frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi - \Phi)}$$

Omnis autem confuſio penitus tolletur ſi fuerit inſuper

$$\frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} = 0.$$

IV. Pro quatuor Lentibus

Vt obiectum ſine margine colorato ſpectetur:

1. Si

1. Si ex campo apparente distantia O prodeat positiua huic aequationi erit satisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0$$

2. Sin autem capiatur $O=0$, huic

$$\frac{\pi''}{\mathfrak{A}\Phi}+\frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}=\frac{\pi}{ABC\mathfrak{D}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'}{ABC\mathfrak{D}(\mathfrak{C}\pi'-\pi+\Phi)}$$

Omnis vero confusio penitus tolletur si fuerit praeterea

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0$$

V. Pro quinque Lentibus

Vt obiectum tantum sine margine colorato conspiciatur

1. Si ex campo apparente distantia O prodeat positiua, huic aequationi erit satisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}+\frac{\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}=0$$

2. Sin autem capiatur $O=0$ huic

$$\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}.$$

$$\frac{1}{ABCD\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right)$$

Omnis autem confusio penitus tolletur, si praeterea satisfiat huic aequationi:

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}=0$$

Atque hinc manifesta est progressio ad maiorem lentium numerum.

Coroll.

Coroll. 1.

326. In casu ergo vnicae lentis licet quidem obiectum a margine colorato liberare neque vero confusionem penitus tollere. Casu autem duarum lentium ne margo quidem coloratus tolli potest, siquidem oculus in eo loco, quem campi apparentis conditio postulat, teneatur.

Coroll. 2.

327. Quodsi vero plures duabus habeantur lentes sufficiens quantitatum numerus adest, quarum determinatione non solum margo coloratus deleri, sed etiam forte omnis confusio penitus auferri posse videtur praecipue si lentium numerus ternarium superet.

Scholion.

328. Quod ergo incommodum a diuersa radiorum natura oriundum adeo graue vel summo Neutono est visum, vt instrumenta dioptrica nullo modo ab eo liberari posse sit arbitratus, id quidem saltem quod ad marginem coloratum attinet ad quem Neutonus inprimis spectabat, iam satis feliciter tolli posse certum est; ita vt saltem ob hanc causam non opus sit ad Telescopia Catoptrica confugere. Hoc autem vitio sublato si praeterea alterum confusionis fontem obstruamus lentes scilicet nullam confusionem parientes adhibendo, nullum est dubium quin instrumenta dioptrica ad summum perfectionis gradum euehi queant.

Quae igitur hactenus particulatim circa singulas horum instrumentorum affectiones proposuimus, ea colligi conueniet, vnde in capite sequente praecepta generalia pro omnium instrumentorum dioptricorum constructione tradere est visum.

Supplementum VI.

Ex iis, quae ante sunt adiecta, poterimus etiam problematis solutionem pro casu exhibere, quo singulae lentes peculiari refractione sunt praeditae, vbi quidem tantum postremae aequationes pro confusione vitanda mutationem quandam postulant; interim tamen etiam priores formulas, quibus locus oculi, quem campus apparens requirit, distinctius repraesentemus.

I. *Distantia Oculi post vltimam lentem* pro quouis lentium numero se habebit, vt sequitur.

Num. lentium	O id est, distantia oculi post lentem vltimam
I.	o
II.	$\frac{A\mathfrak{B}a\pi\Phi}{(\pi-\Phi)(\mathfrak{B}\pi-\Phi)}$ seu $\frac{\mathfrak{B}b\pi}{\pi-\Phi}$
III.	$\frac{AB\mathfrak{C}a\pi'\Phi}{(\pi'-\pi+\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$ seu $\frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi}$
IV.	$\frac{ABC\mathfrak{D}a\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$ seu $\frac{\mathfrak{D}d\pi''}{\pi''-\pi'+\pi-\Phi}$
V.	$\frac{ABCD\mathfrak{E}a\pi'''\Phi}{(\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$ seu $\frac{\mathfrak{E}e\pi'''}{\pi'''-\pi''+\pi'-\pi+\Phi}$

etc.

II.

II. Si valor ipsius O sit positiuus, ad marginem coloratum tollendum sequentes aequationes sunt adimplendae:

Num. lentium	
I.	$o = o$
II.	$o = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi}$
III.	$o = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi}$
IV.	$o = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi} + \frac{ddn'''}{n'''-1} \cdot \frac{\pi''}{ABCa\Phi}$
	etc.

III. Si valor ipsius O prodeat negatiuus, quo casu capi debet $O = o$, ad marginem coloratum tollendum sequentes aequationes sunt adimplendae.

Num. lentium	
I.	$o = o$
II.	$o = \frac{adn}{n-1} \cdot (A+1)B\pi$
III.	$o = \frac{adn}{n-1}(A+1)BC\pi' + \frac{bdn'}{n'-1} \cdot \frac{(B+1)C\pi' - (C+1)\pi}{A}$
IV.	$o = \frac{adn}{n-1} \cdot (A+1)BCD\pi'' + \frac{bdn'}{n'-1}\left(\frac{(B+1)CD\pi'' - (D+1)\pi}{A}\right) + \frac{cdn''}{n''-1}\left(\frac{(C+1)D\pi'' - (D+1)\pi'}{AB}\right)$
	etc.

IV. Vt autem insuper omnis confusio huius generis tollatur, sequentes aequationes sunt adimplendae:

Num. lentium	
I.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A}$
II.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2 B}$
III.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2 B} + \frac{c\,dn''}{n''-1} \cdot \frac{C+1}{A^2 B^2 C}$
IV.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2 B} + \frac{c\,dn''}{n''-1} \cdot \frac{C+1}{A^2 B^2 C} + \frac{d\,dn'''}{n'''-1} \cdot \frac{D+1}{A^2 B^2 C^2 D}$

etc.

Quarum formularum ordo hinc distinctius perspicitur, quam in problemate.

CAPVT

CAPVT VII.

DE CONSTRVCTIONE INSTRVMENTORVM DIOPTRICORVM IN GENERE.

Problema I.

329.

Si instrumentum dioptricum vnica constet lente crassitiei cuiuscunque, PP, definire omnia momenta ad visionem pertinentia. Tab. III. Fig. 12.

Solutio.

Quod primo ad ipsius lentis structuram attinet ponatur obiecti $E\varepsilon$ ante eam distantia $AE = a$, imaginisque $F\zeta$ post eam proiectae $aF = \alpha$; tum vero lentis crassities $Aa = v$ et quantitas arbitraria $= k$, vnde capiatur $\frac{k-v}{k+v} = i$. Hinc facies lentis ita erunt formatae, vt sit existente $n = \frac{31}{20}$ Conf. §. 68.

radius faciei anterioris $= \frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei posterioris $= \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$.

Ponatur porro semidiameter obiecti conspicui $E\varepsilon = z$, et semidiameter aperturae in facie anteriori $AM = x$, in facie autem posteriori semidiameter aperturae non

 fit

ſit minor quam ix. Denique oculi poſt lentem diſtantia vocetur $aO = O$, cuius diſtantia iuſta ſit $= l$. His poſitis ſequentia momenta perpendi oportet.

1. Debet eſſe $O = \alpha + l$ vt diſtantia imaginis viſae naturae oculi conueniat.

2. Conſideranda venit multiplicatio, quae ex ratione definitur, quam diameter obiecti viſus tenet ad eiusdem diametrum viſum ſi nudo oculo in diſtantia data $= b$ ſpectaretur. Quodſi ergo hic exponens multiplicationis ponatur $= m$ erit

$m = \frac{l}{i} \cdot \frac{\alpha b}{a l}$ pro ſitu inuerſo

3. Gradus claritatis determinatur ſemidiametro coni luminoſi, qui a quouis obiecti puncto in oculum immittitur, qui ſi ponatur $= y$ erit

$y = i\, l . \frac{x}{\alpha} = \frac{b x}{m a}$

4. Confuſio inquinans viſionem menſuratur ſemidiametro apparente circuli, qui nudo oculo aeque magnus cernitur, ac ſingula obiecti puncta per lentem ſpectata. Hanc menſuram vocaui (194) ſemidiametrum confuſionis, ad quem definiendum ſi ponatur:

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{\alpha} + \frac{2}{k+v}\right)\left(\frac{1}{i\alpha} + \frac{2}{k-v}\right)^2 + \left(\frac{n}{\alpha} - \frac{2}{k-v}\right)\left(\frac{i}{\alpha} - \frac{2}{k+v}\right)^2\right)$$

erit ſemidiameter confuſionis $= \frac{1}{4} i . \frac{\alpha}{l} x^3 P = \frac{1}{4} i^2 . \frac{m a}{b} x^3 . P.$

5. Vt oculus maximum campum apparentem percipiat, debet eſſe $O = \frac{-i \alpha v}{n \alpha - i v}$

6. Quae

6. Quae diſtantia ſi fuerit poſitiua, ſemidiameter campi ſeu obiecti conſpicui $E\varepsilon = z$, ita pendet ab apertura faciei poſterioris, vt poſito huius ſemidiametro $= a$ ſit $z = \frac{na}{v}.\, a$.

7. Sin autem diſtantia illa pro O aſſignata prodierit negatiua, oculum lenti immediate applicari conueniet, vt ſit $O = o$; tum vero pro a poſito ſemidiametro pupillae ω erit pro campo apparente $z = \frac{na}{v}\omega$.

8. Vt obiectum ſine margine colorato appareat, exiſtente diſtantia O ante inuenta poſitiua, debet eſſe $i = \frac{a}{a+v}$ ſeu $k = 2a + v$.

9. Sin autem ob illam diſtantiam negatiuam prodeuntem capiatur $O = o$, vt margo coloratus euitetur, oportet eſſe $\frac{v}{a} + 1 - i = o$, vnde ſit $k = -2a - v$.

10. Omnis denique confuſio a diuerſa radiorum refrangibilitate oriunda prorſus tolletur, ſi inſuper fieri poſſet $(\alpha + v)(a + \alpha + v) = o$

Coroll. 1.

330. Si exponens multiplicationis m cum gradu claritatis y proponatur, erit $my = \frac{bx}{a}$. quae proprietas ad lentium numerum quantumuis magnum patet. Cum ergo $\frac{b}{a}$ ſit quantitas data, erit x vt my; tum vero y vt $\frac{x}{m}$, ac m vt $\frac{x}{y}$

Coroll. 2.

331. Tam maior ergo multiplicatio quam maior claritatis gradus poſtulat maiorem aperturam. Verum

Verum aucto x confusio augetur in ratione triplicata, siquidem quantitas P maneat eadem: quare si x ex confusione etiamnum tolerabili determinetur, simul quantitas my determinatur.

Coroll. 3.

332. Vt igitur tam multiplicationem m quam claritatem y salua confusione maxime augere liceat, quantitatem arbitrariam k ita definiri conueniet, vt litterae P minimus valor concilietur.

Cum autem posuerimus $\frac{k-v}{k+v}=i$, valor ipsius P fiet minimus, huic aequationi satisfaciendo:

$$2i^4\left(\frac{n}{\alpha}+\frac{1}{v}\right)\left(\frac{1}{\alpha}+\frac{2}{v}\right)^2-\frac{2ni^3}{\alpha v}\left(\frac{1}{\alpha}+\frac{1}{v}\right)-\frac{i^3}{v}\left(\frac{1}{\alpha}+\frac{2}{v}\right)^2$$
$$+\frac{2ni}{av}\left(\frac{1}{a}+\frac{1}{v}\right)+\frac{i}{v}\left(\frac{1}{a}+\frac{2}{v}\right)^2-2\left(\frac{n}{a}+\frac{1}{v}\right)\left(\frac{1}{a}+\frac{1}{v}\right)^2=0$$

vti §. 45 inuenimus.

Coroll. 4.

333. Si loco i valor substituatur suus $\frac{k-v}{k+v}$, aequatio transibit in hanc formam:

$$+\frac{2n}{a^3}(k+v)^4+\frac{(2n+1)}{aav}(k+v)^3(k+3v)+\frac{4(n+2)}{av}(k+v)^2$$
$$-\frac{2n}{a^3}(k-v)^4-\frac{(2n+1)}{a\alpha v}(k-v)^3(k-3v)+\frac{4(n+2)}{av}(k-v)^2+32k=0$$

quae per v multiplicata etiam ad casum accommodari potest quo est $v=0$: tum autem reperitur

$$k=\frac{4(n+2)a\alpha}{(2n+1)(a-\alpha)} \text{ et}$$

$$P=\frac{n}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left((4n-1)\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{a\alpha}\right)$$

Scholion

Scholion

334. Hic non generatim omnem visionem, quae fit per vnicam lentem, considero, sed tantum eam, qua maximus campus apparens conspicitur; quamobrem locum oculi ita definiui, vt ipsi maximus campus adferatur. Sin autem minore campo velimus esse contenti oculus quoque alibi post lentem constitutus obiecta distincte cernere poterit, quemadmodum per lentes satis amplas, quae tabularum vitrearum nomine sunt notae, fieri solet. Hunc autem casum quoniam per se facile expeditur, atque in instrumentis dioptricis magnitudo campi potissimum spectatur, hic non attingo.

Problema 2.

335. Si instrumentum dioptricum duabus constet lentibus cuiuscunque crassitiei definire omnia momenta ad visionem pertinentia. Tab. III. Fig. 13.

Solutio.

Obiecto constituto in $E\epsilon$, eius imagines proiiciantur per istas lentes in $F\zeta$ et $G\eta$, ac ponantur quantitates vtramque lentem determinantes

$AE = a$, $aF = \alpha$, crassities $Aa = v$ et distantia arbitraria $= k$

$BF = b$, $bG = \beta$, crassities $Bb = v'$ et distantia arbitraria $= k'$

ponaturque breuitatis gratia $\frac{k-v}{k+v}=i$ et $\frac{k'-v'}{k'+v'}=i'$. Hinc exiſtente $n=\frac{31}{20}$, conſtructio vtriusque lentis ita ſe habebit,

Radius faciei	anterioris	poſterioris
Pro Lente PP . .	$\frac{(n-1)a(k+v)}{k+v+2na}$ . .	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Pro Lente QQ . .	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$ . .	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$

Sit porro ſemidiameter aperturae lentis primae PP in facie anteriori $=x$, in facie autem poſteriori maior ſit quam ix. Tum vero lentis ſecundae QQ ſemidiameter aperturae in facie anteriori maior eſſe debet quam $i.\frac{bx}{\alpha}$, in poſteriori vero maior quam $ii'.\frac{bx}{\alpha}$. Deinde ſit ſemidiameter obiecti conſpicui $E\epsilon=z$, diſtantia oculi $bO=O$, eiusque diſtantia iuſta $=l$. His poſitis ad ſequentia momenta erit attendendum.

1. Vt oculus imaginem $G\eta$ in diſtantia iuſta conſpiciat, oportet eſſe $O=\beta+l$.

2. Poſita diſtantia $=h$ ad quam multiplicatio referatur, ſit exponens multiplicationis $=m$, ac ſupra inuenimus eſſe oportere $m=\frac{1}{ii'}.\frac{\alpha\beta h}{abl}$ pro ſitu erecto.

3. Denotante y gradum claritatis fiet

$$y=ii'l.\frac{bx}{\alpha\beta} \text{ ideoque } my=\frac{hx}{a}$$

4. Pro confuſione poſito brevitatis ergo

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}+\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$

$$Q=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$

erit-

eritque

semid. confusionis $= \frac{1}{4} i i' \cdot \frac{\mathfrak{b}}{1} \cdot \frac{b}{\alpha} x^3 \left(\frac{1}{i' i'} \cdot \frac{\alpha \alpha}{b b} P + i i \cdot \frac{b b}{\alpha \alpha} Q \right)$.

5. Pro campo apparente definiendo, qui pendet ab apertura singularum facierum lentium, si ponamus

Semidiametrum aperturae

Pro lente PP $\begin{cases} \text{faciei anterioris} = \mathfrak{A} = x \\ \text{faciei posterioris} = \mathfrak{a} \end{cases}$

Pro lente QQ $\begin{cases} \text{faciei anterioris} = \mathfrak{B} \\ \text{faciei posterioris} = \mathfrak{b} \end{cases}$

habebimus sequentes aequationes

$$\mathfrak{a} = \frac{v}{n a} z$$

$$\mathfrak{B} = \left(\frac{1}{i} \cdot \frac{\alpha + b}{a} - \frac{b v}{n a \alpha} \right) z$$

$$\mathfrak{b} = \left(\frac{i'}{i} \cdot \frac{\alpha + b}{a} - \frac{i' b v}{n a \alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{n a b} \right) z$$

ex quibus tribus aequationibus valor ipsius z minimus praebet semidiametrum campi apparentis.

6. Quem campum vt oculus reuera perspiciat, eius locus ita debet sumi, vt sit

$$O = \frac{\frac{i'}{i} \frac{\alpha + b}{a} - \frac{i' b v}{n a \alpha} - \frac{1}{i} \frac{\alpha v'}{n a b}}{\frac{i'}{i} \cdot \frac{\alpha + b}{a} - \frac{i' b v}{n a \alpha} - \frac{1}{i} \cdot \frac{\alpha v'}{n a b} + \frac{1}{\mu'} \cdot \frac{\alpha \mathfrak{b}}{a b}} \cdot \mathfrak{b}$$

si quidem haec distantia fuerit positiua.

7. Sin autem haec distantia prodierit negatiua, oculum lenti QQ immediate applicari conuenit, vt sit $O = o$; tum vero pro campo apparente inue-

 niendo

niendo in aequationibus n°. 5 allatis loco $\mathfrak{C}$ scribatur semidiameter pupillae ω, atque ex iisdem valor ipsius z minimus erutus dabit semidiametrum campi apparentis.

8. Obiectum porro sine margine colorato cernetur, existente distantia O n° 6 inuenta positiua, si huic aequationi satisfiat:

$$0 = 1 + \frac{b}{\alpha} + \frac{i'i'b}{\mathfrak{C}}\left(1 + \frac{b}{\alpha}\right) + \frac{(1-i')^2 b(\alpha+b)}{\alpha v'} + \frac{2-i-i'}{n}$$
$$- \frac{iv}{n\alpha} - \frac{i'v'}{n\mathfrak{C}} - \frac{ibv}{n\alpha\alpha} - \frac{ii'i'bbv}{n\alpha\alpha\mathfrak{C}} - \frac{i(1-i')^2 bbv}{n\alpha\alpha v'}$$

9. Sin autem capiatur distantia $O=0$, obiectum margine colorato carebit, si fuerit

$$0 = \frac{1-i}{in} + \frac{1-i'}{i'n} + \frac{v}{ina} + \frac{v'}{inb} + \alpha\left(1 + \frac{\alpha}{ia} + \frac{(1-i)^2\alpha}{iiv}\right)\left(\frac{v'}{ni'bb} - \frac{1}{\alpha} - \frac{1}{b}\right)$$

10. Omnis autem confusio penitus tolletur, quae quidem a diuersa radiorum refrangibilitate proficiscitur, si huic aequationi satisfacere licuerit $d\mathfrak{C} = 0$, seu huic

$$0 = \frac{\alpha\alpha}{ii'bb}\left(\frac{i}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right) + \frac{i'}{\mathfrak{C}} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}.$$

Coroll. 1.

336. Quia ambarum lentium distantia aB necessario est positiua, oportet sit $\alpha + b > 0$. Tum vero aperturae ita debent esse comparatae vt sit $a > ix$; $\mathfrak{B} > i.\frac{bx}{\alpha}$ et $\mathfrak{C} > ii'.\frac{bx}{\alpha}$.

Coroll. 2.

337. Casu igitur, quo ob valorem ipsius O prodeuntem negatiuum distantia $O = 0$ assumitur, quia

quia tum $b = \omega$ ſtatuitur, etiam eſſe debet $\omega > ii' \frac{bx}{\alpha}$ ſeu $x < \frac{\alpha\omega}{ii'b}$. Scilicet hoc caſu inutile eſſet primam aperturam maiorem ſumere, quia radii non in oculum ingrederentur.

Scholion.

338. Si ſimili modo inſtrumenta dioptrica pluribus lentibus inſtructa proſequi vellemus, in calculos tantopere intricatos delaberemur, vt ex iis nihil fere concludere liceret. Oritur autem haec calculi complicatio a craſſitie lentium, qua neglecta omnia fiunt multo concinniora. Quare ſi pluribus lentibus vti velimus, craſſitiem tam paruam aſſumamus, vt ſine errore pro nihilo haberi poſſit, id quod in praxi etiam ſedulo obſeruari ſolet. Praeterea etiam campi apparentis locique oculi determinatio non capit rigorem geometricum, parumque refert ſi in ea aliquantum aberretur. Similis quoque ratio eſt conditionum, quibus effectus a diuerſa radiorum refrangibilitate oriundus tollitur; ſufficiet enim iis proxime ſatisfeciſſe, cum perfecta huius confuſionis deſtructio ne ſperari quidem poſſit. Quocirca in ſequentibus, vbi inſtrumenta pluribus lentibus inſtructa euoluemus, craſſitiem lentium in calculo penitus praetermittamus; in quo negotio ne eadem toties repetere opus habeamus, problema generale praemittamus in quo omnia momenta generatim tantum exponamus eaque deinceps pro quouis lentium numero accurate deſcribamus.

Problema 5.

339. Si instrumentum dioptricum compositum sit ex lentibus quotcunque quarum crassitiem negligere liceat, elementa eius constructionem continentia exponere, ex quibus deinceps regulae dirigentes constructionem ipsam stabiliri possint.

Solutio.

Obiecto in E constituto ponantur distantiae determinatrices singularum lentium vna cum numeris arbitrariis ad singulas pertinentibus vt sequitur.

Pro Lente prima $EA = a$; $aF = \alpha$; num: arb: $= \lambda$

Pro Lente secunda $FB = b$; $bG = \beta$; num: arb: $= \lambda'$

Pro Lente tertia $GC = c$; $cH = \gamma$; num: arb: $= \lambda''$

Pro Lente quarta $HD = d$; $dI = \delta$; num: arb: $= \lambda'''$

Pro Lente quinta $IE = e$; $eK = \varepsilon$; num: arb: $= \lambda''''$

Pro Lente sexta $KF = f$; $fL = \zeta$; num: arb: $= \lambda'''''$

etc.

ex quibus elementis quomodo singulae lentes debeant formari supra est expositum.

Nunc autem porro ponatur breuitatis gratia:

$\alpha = Aa$; $\beta = Bb$; $\gamma = Cc$: $\delta = Dd$; $\varepsilon = Ee$; $\zeta = Ff$

etc.

tum

tum vero statuatur etiam

$$\frac{A}{A+1}=\mathfrak{A};\ \frac{B}{B+1}=\mathfrak{B};\ \frac{C}{C+1}=\mathfrak{C};\ \frac{D}{D+1}=\mathfrak{D};\ \frac{E}{E+1}=\mathfrak{E};\ \frac{F}{F+1}=\mathfrak{F}$$

etc.

ita vt sint distantiae focales lentium:

$$\mathfrak{A}a;\ \mathfrak{B}b;\ \mathfrak{C}c;\ \mathfrak{D}d;\ \mathfrak{E}e;\ \mathfrak{F}f \quad \text{etc.}$$

Nunc sit semidiameter aperturae primae lentis obiectiuae $=x$, pro reliquis vero lentibus ratio aperturarum litteris π, π', π'', π''', π'''' etc. exponatur; ita vt semidiameter aperturae, cuiusque maior accipi debeat quam secundum has rationes: scilicet capi oportebit:

Semid: apert:

Lentis Secundae	$>\pi\,\mathfrak{B}\,b$
Lentis tertiae	$>\pi'\,\mathfrak{C}\,c$
Lentis quartae	$>\pi''\,\mathfrak{D}\,d$
Lentis quintae	$>\pi'''\,\mathfrak{E}\,e$
Lentis sextae	$>\pi''''\,\mathfrak{F}\,f$ etc.

Quod si iam semidiameter spatii in obiecto conspicui vocetur $=z$, fiatque $\frac{z}{a}=\Phi$, vt sit $z=a\Phi$, atque hinc distantiae determinatrices ita exprimentur, vt sit primo $\alpha=Aa$ tum vero:

$$b=\frac{Aa\Phi}{\mathfrak{B}\pi-\Phi};\qquad \beta=\frac{ABa\Phi}{\mathfrak{B}\pi-\Phi}$$

$$c=\frac{ABa\Phi}{\mathfrak{C}\pi'-\pi+\Phi};\qquad \gamma=\frac{ABCa\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$$

$$d=\frac{ABCa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi};\qquad \delta=\frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$$

$$e=\frac{ABCDa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi};\qquad \varepsilon=\frac{ABCDEa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}$$

$$f=\frac{ABCDEa\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi};\qquad \zeta=\frac{ABCDEFa\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}\quad \text{etc.}$$

Circa

Circa has expressiones primum notandum est, aggregata $\alpha+b$; $\mathfrak{b}+c$; $\gamma+d$; $\delta+e$; $\varepsilon+f$ etc. esse oportere positiua quippe quibus lentium interualla exprimuntur. Erit nempe

Interuallum
Lentium

$$\text{I et II} = \frac{A\mathfrak{B}\alpha\pi}{\mathfrak{B}\pi-\Phi} > 0$$

$$\text{II et III} = \frac{AB\alpha\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)} > 0$$

$$\text{III et IV} = \frac{ABC\alpha\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)} > 0$$

$$\text{IV et V} = \frac{ABCD\alpha\Phi(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)} > 0$$

$$\text{V et VI} = \frac{ABCDE\alpha\Phi(\mathfrak{F}\pi''''-(1-\mathfrak{E})\pi''')}{(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)} > 0$$

etc.

Praeterea vt lentes sequentes omnes radios a prima exceptos transmittant, debet esse

Semid: aperturae

Lentis secundae $> \frac{\Phi}{\mathfrak{B}\pi-\Phi}x$

Lentis tertiae $> \frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi}x$

Lentis quartae $> \frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}x$

Lentis quintae $> \frac{\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}x$

Lentis sextae $> \frac{\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}x$

etc.

Denique

Denique ex §. 214 posito $\mu = 0,938191$ et $\nu = 0,232692$ erunt litterarum maiuscularum P, Q, R, etc. valores

$$P = \frac{\mu}{A^3a^3}(A+1)(\lambda(A+1)^2+\nu A)$$

$$Q = \frac{\mu}{B^3b^3}(B+1)(\lambda'(B+1)^2+\nu B)$$

$$R = \frac{\mu}{C^3c^3}(C+1)(\lambda''(C+1)^2+\nu C)$$

$$S = \frac{\mu}{D^3d^3}(D+1)(\lambda'''(D+1)^2+\nu D)$$

$$T = \frac{\mu}{E^3e^3}(E+1)(\lambda''''(E+1)^2+\nu E)$$

$$V = \frac{\mu}{F^3f^3}(F+1)(\lambda'''''(F+1)^2+\nu F)$$

etc.

His ita constitutis denominationibus, sit O distantia oculi post lentem vltimam, cuius distantia iusta ponatur $= l$. Deinde statuatur exponens multiplicationis $= m$, relatus ad distantiam b, ita vt diameter obiecti per instrumentum visi m vicibus maior cernatur, quam si idem obiectum à nudo oculo in distantia $= b$ aspiceretur: multiplicationi autem adiungi conuenit situm indicando, vtrum obiectum situ erecto an inuerso sit appariturum. Porro gradus claritatis y denotet semidiametrum coni luminosi, qui a quouis obiecti puncto in oculum transmittitur posito semidiametro pupillae $= \omega$. Semidiametrum denique confusionis voco semidiametrum apparentem circulorum qui in nudo oculo aeque magni depinguntur, atque singula obiecti puncta per instrumentum in oculo.

Coroll. 1.

340. Quia pro apertura ſingularum lentium geminos limites inuenimus, ſemidiameter aperturae cuiusque conuenientiſſime aggregato amborum limitum aequalis aſſumitur, vel ſaltem non minor. Vterque autem limes etſi forte alter prodeat negatiuus, affirmatiue accipi debet.

Coroll. 2.

341. Hinc ergo ſequentes conſequimur formulas pro ſingularum lentium aperturis.

Semidiam: apert:

Lentis ſecundae $= \frac{\mathfrak{A}\mathfrak{B}a\pi \pm x}{\mathfrak{B}\pi - \phi} \cdot \Phi$

Lentis tertiae $= \frac{\mathfrak{A}\mathfrak{B}\mathfrak{C}a\pi' \pm x}{\mathfrak{C}\pi' - \pi + \phi} \cdot \Phi$

Lentis quartae $= \frac{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}a\pi'' \pm x}{\mathfrak{D}\pi'' - \pi' + \pi - \phi} \cdot \Phi$

Lentis quintae $= \frac{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}\mathfrak{E}a\pi''' \pm x}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \phi} \cdot \Phi$

Lentis ſextae $= \frac{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}\mathfrak{E}\mathfrak{F}a\pi'''' \pm x}{\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \pi - \phi} \cdot \Phi$

etc.

Coroll. 3.

342. Ex diſtantiis binis determinatricibus cum numero arbitrario quaelibet lens facile conſtruitur; id quod pro lente prima ex ſuperioribus repetamus. Nempe ſi ſit $\lambda > 1$ lens ſimplex ſatisfaciet cuius conſtructio poſito breuitatis gratia:

$$\varrho = 0,190781;\ \sigma = 1,627401 \text{ et } \tau = 0,905130$$

ita

ita se habet:

anterioris — posterioris

radius faciei . . $\frac{a\alpha}{\rho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}$; $\frac{a\alpha}{\rho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}$

seu numeris substitutis si sit $\lambda=1+v$

$$\text{radius faciei}\begin{cases}\text{anterioris}=\dfrac{a\alpha}{+0,190781\alpha+1,627401a\pm0,905133(a+\alpha)\sqrt{v}}\\[2ex]\text{posterioris}=\dfrac{a\alpha}{+1,627401\alpha+0,190781a\mp0,905133(a+\alpha)\sqrt{v}}\end{cases}.$$

Coroll. 4.

343. Sin autem sit $\lambda<1$, sed tamen $>0,191827$ lens est duplicanda seu ex duabus simplicibus componenda, quarum constructio, si ponatur $\lambda=0,191827+v$ ita se habet

Pro Lente radius faciei

$$\text{priori}\begin{cases}\text{anterioris}=\dfrac{a\alpha}{-0,622915\alpha+0,113700a\pm0,905133(a+\alpha)\sqrt{v}}\\[2ex]\text{posterioris}=\dfrac{a\alpha}{+1,532010\alpha+0,055390a\mp0,905133(a+\alpha)\sqrt{v}}\end{cases}$$

$$\text{posteriori}\begin{cases}\text{anterioris}=\dfrac{a\alpha}{+0,055390\alpha+1,532010a\pm0,905133(a+\alpha)\sqrt{v}}\\[2ex]\text{posterioris}=\dfrac{a\alpha}{+0,113700\alpha-0,622915a\mp0,905133(a+\alpha)\sqrt{v}}\end{cases}$$

Coroll. 5.

344. At si sit $\lambda=0,042165+v$, lente utendum est triplicata ita construenda:

Pro lente radius

Priori $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{-0,854153\alpha + 0,542467a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+1,500214\alpha + 0,063594a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Media $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{-0,415280\alpha + 1,021340a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+1,021340\alpha - 0,415280a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Posteriori $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{+0,063594\alpha + 1,500214a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+0,542467\alpha - 0,854153a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Coroll. 6.

345. Si denique fit $\lambda = -0,010216 + v$ lens facienda est quadruplicata ita construenda:

Pro lente radius faciei

Prima $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{-1,089770\alpha + 0,406450a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+1,484315\alpha + 0,047655a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Secunda $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{-0,670615\alpha + 0,766035a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+1,125160\alpha - 0,311460a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Tertia $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{-0,311460\alpha + 1,125160a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+0,766035\alpha - 0,670615a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Quarta $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{+0,047655\alpha + 1,484315a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+0,406450\alpha - 1,089770a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Scholion

Scholion.

346. Quae in his corollariis de constructione lentis primae sunt allata, mutatis litteris a et α ad reliquas lentes facile accommodari manifestum est quarum loco, si vsus requirat, vt numeri λ', λ'' etc. sint vnitate minores, etiam lentes siue duplicatae siue triplicatae siue adeo quadruplicatae adhiberi debebunt. Ceterum hae formulae ad lentes quotcunque patent ita vt proposito lentium numero quocunque tantum literae, quae ad lentes sequentes pertinerent, sint omittendae. In his autem denominationibus statim introduximus campum apparentem numero Φ contentum, cum sit semidiameter spatii in obiecto conspicui $z = a\Phi$. Interim tamen campum apparentem non ad lubitum augere licet, siquidem is multiplicationis ratione et numeris π, π', π'' etc. determinatur. Hi autem numeri semper infra $\frac{1}{2}$ imo $\frac{1}{3}$ subsistunt, ac plerumque $\frac{1}{4}$ vel adeo $\frac{1}{5}$ superare nequeunt; quandoque etiam minores accipi debent, id quod formulae coroll. 2. quouis casu declarabunt, quibus semidiameter aperturae cuiusque lentis exprimitur. Quare retentis his denominationibus momenta constructionis pro quouis lentium numero determinato expendamus.

Problema 4.

347. Si instrumentum dioptricum duabus constet lentibus, quarum crassitiem negligere liceat, definire momenta quibus constructio continetur.

Solutio.

Cum hic lens fecunda fit vltima, momenta conftructionis ita fe habebunt.

1. Vt oculus imaginem in diftantia iufta confpiciat, debet effe $O = \mathfrak{b} + l$ ideoque

$O = l + \frac{ABa\Phi}{\mathfrak{B}\pi - \Phi}$. et ex conditione campi apparentis

$O = \frac{\mathfrak{B}b\pi}{\pi - \Phi}$.

2. Exponens multiplicationis m ad diftantiam b relatus praebet hanc aequationem

$m = AB\frac{b}{l}$ pro fitu erecto

fin autem fitus inuerfus defideretur, numerus m negatiue eft capiendus.

3. Pro gradu claritatis femper eft $y = \frac{bx}{ma}$.

4. Pro campo apparente habebitur haec aequatio: $\pi - \Phi = \frac{-ma\Phi}{b}$ vnde fit $\Phi = \frac{-\pi b}{ma - b}$.

5. Si diftantia O hinc prodeat negatiua, capi debet $O = 0$ feu $\frac{ABa\Phi}{\mathfrak{B}\pi - \Phi} = -l$ tum vero π eiusmodi valorem induit, vt apertura lentis ocularis non fuperet aperturam pupillae; fiet nempe $\pi\mathfrak{B}b = \omega$, hincque

$\Phi = \frac{\mathfrak{B}\pi\omega}{A\mathfrak{B}\pi + \omega}$. vel $(B+1)\omega = -\pi l$.

6. Semidiameter confufionis fequenti formula exprimetur.

$\frac{Bbb}{4Aal}x^3\left(\frac{AAaa}{bb}P + \frac{bb}{AAaa}Q\right)$

quae

quae factis substitutionibus abit in hanc:

$$\frac{\mu m x^3}{4aab}\left(\frac{(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\Phi(B+1)(\lambda'(B+1)^2+\nu B)}{A^2 a(\mathfrak{B}\pi-\Phi)}\right)$$

7. Vt obiectum sine margine colorato appareat, si distantia O fuerit positiua, necesse est sit:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}=0$$

8. Sin autem capiatur $O=0$, fieri debet $\frac{(A+1)\mathfrak{B}\pi}{\Phi}=0$ seu $\frac{\pi}{\mathfrak{A}\Phi}=0$.

9. Omnis autem confusio a diuersa radiorum refrangibilitate oriunda penitus tolletur si insuper fuerit

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}=0$$

10. Praeterea autem erit distantia lentium

$\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$ quae debet esse positiua.

11. Denique Semidiameter aperturae lentis ocularis est;

$$\frac{A\mathfrak{B}a\pi'\pm\pi}{\mathfrak{B}\pi-\Phi}\cdot\Phi$$

vbi signorum $\pm$ id capi debet, quod valorem praebet maximum siue positiuum siue negatiuum.

Coroll. 1.

348. Cum sit (n°. 4) $\Phi=\frac{-\pi b}{ma-b}$, et ob multiplicationem $B=\frac{ml}{Ab}$, hincque $\mathfrak{B}=\frac{ml}{ml+Ab}$; habebitur pro loco oculi $O=\frac{-Abl(ma-b)}{mmal+Abb}$.

Tum

Tum vero eſt $\mathfrak{B}\pi-\Phi=\frac{ml\pi}{ml+Ab}+\frac{\pi b}{ma-b}=\frac{\pi(mmal+Abb)}{(ma-b)(ml+Ab)}$, vnde fit lentium diſtantia $\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}=\frac{mAa(ma-b)}{mmal+Abb}l$.

Coroll. 2.

349. Cum ſit $\frac{A}{A+1}=\mathfrak{A}$ et $A=\frac{\mathfrak{A}}{1-\mathfrak{A}}$ ſemidiameter confuſionis etiam ita exprimi poteſt vt ſit

$$\frac{\mu m x^3}{4aab}\left(\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3}+\frac{\Phi^3(\lambda'+\nu\mathfrak{B}(1-\mathfrak{B}))}{A^3\mathfrak{B}^3(\mathfrak{B}\pi-\Phi)}\right).$$

Coroll. 3.

350. Cum ſit lentium diſtantia $=\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$, obiectum ſine coloribus cerni nequit, niſi lentium diſtantia euaneſcat; ſiquidem oculus in eo loco, quem campus apparens poſtulat, teneatur. Et quoniam huic conditioni ſatisfieri nequit, multo minus tota confuſio ex n°. 9 tolli poterit.

Scholion.

351. Hic ſtatim ab inſtrumentis duabus lentibus inſtructis incepi, quoniam lens vnica facillime expeditur. Primum enim neceſſe eſt vt ſit $O=l+\alpha=l+Aa$. Tum vero multiplicatio praebet $m=\frac{b}{a}$ pro ſitu erecto, et campus apparens non terminatur ſumendo $Aa+l=0$, ſeu $O=0$, ita vt oculus lenti immediate debeat applicari. Semidiameter vero confuſionis erit

$=\frac{a}{4l}x^3.P=\frac{-Aax^3}{4l}.P=-\frac{1}{4}x^3P=\frac{-max^3}{4b}.P$ ob $l=-Aa$ et $ma=b$

Quare ſemidiameter confuſionis erit

$$\frac{\mu m x^3}{4aab}\left(\frac{(A+1)(\lambda(A+1)^2+\nu A)}{A^3}\right)=\frac{\mu m x^3}{4aab}\cdot\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3}.$$

Pro

Pro gradu claritatis autem habetur vt semper $y = \frac{bx}{m\,a}$.

Obiectum porro hoc casu sine margine colorato cernetur, tota autem confusio tolli nequit, nisi sit $\mathfrak{a} = 0$ seu $A = -1$, id quod scopo lentium repugnat. Quare ad considerationem plurium lentium progrediamur.

Problema 5.

352. Si instrumentum dioptricum tribus instructum sit lentibus, quarum crassities tam sit parva, vt negligi queat, definire cuncta momenta ad constructionem dirigendam necessaria.

Solutio.

Hic igitur lens tertia erit vltima seu ocularis, ideoque momenta sequenti modo se habebunt:

1. Vt oculo imago in distantia iusta spectanda offeratur debet esse $O = \gamma + l$, ac loco γ valore substituto

$O = l + \frac{ABC\,a\,\Phi}{\mathfrak{E}\pi' - \pi + \Phi}$ et ex conditione campi apparentis

$$O = \frac{\mathfrak{E}\,c\,\pi'}{\pi' - \pi + \Phi}.$$

2. Exponens multiplicationis m ad distantiam b relatus praebet hanc aequationem:

$$m = -\frac{ABCb}{l} \text{ pro situ erecto.}$$

vnde si situs inuersus desideretur, numerum m negatiue accipi conuenit.

3. Pro gradu claritatis habemus, vt semper, $y = \frac{bx}{ma}$.

4. Pro campo autem apparente definiendo habemus hanc aequationem

$$\Phi = \frac{-\pi + \pi'}{ma - b} \cdot b.$$

5. At si distantia O hinc prodeat negatiua, vt capi oporteat $O = 0$, ideoque $\frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi} = -l$ campus apparens definiri debet ex hac aequatione $\pi'\mathfrak{C}c = \omega$ seu $\frac{AB\mathfrak{C}a\Phi\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = \omega$ vel $(C+1)\omega = -\pi' l$.

6. Semidiameter confusionis vero ita exprimetur:

$$\frac{Ccc'}{4ABal} x^3 \left(\frac{AABBaa}{cc} P + \frac{BBb^4}{AAaacc} Q + \frac{cc}{AABBaa} R \right)$$

quae ob $\mathfrak{z} = \frac{-m}{ABCb}$ abit signo mutato in hanc:

$$\frac{\mu m x^3}{4aab} \begin{cases} + \frac{(A+1)(\lambda(A+1)^2 + \nu A)}{A^3} \\ + \frac{\Phi(B+1)(\lambda'(B+1)^2 + \nu B)}{A^3 B^3 (\mathfrak{B}\pi - \Phi)} \\ + \frac{\Phi(C+1)(\lambda''(C+1)^2 + \nu C)}{A^3 B^3 C^3 (\mathfrak{C}\pi' - \pi + \Phi)}. \end{cases}$$

7. Vt obiectum sine margine colorato appareat, si quidem distantia O prodierit positiua, esse oportet.

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = 0$$

8. Sin autem ob istam distantiam prodeuntem negatiuam capiatur $O = 0$, margo coloratus euanescet faciendo:

$$\frac{\pi'}{\mathfrak{A}\Phi} + \frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} = \frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi - \Phi)}$$

9. Quod

9. Quod ſi marginem coloratum tollere liceat, praeterea viſio ab omni confuſione liberabitur, ſi huic aequationi ſatisfiat.

$$\frac{1}{a}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}=0.$$

10. Denique effici debet, vt diſtantia lentium fiat poſitiua vnde habebitur:

Interuallum
Lentium

I et II $=\frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi-\Phi}>0$

II et III $=\frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0$

11. Tandem numeros π et π' ita accipi oportet, vt aperturae lentium non fiant nimis magnae: eſt vero

Semidiameter
aperturae

Lentis ſecundae $=\frac{A\mathfrak{B}a\pi+x}{\mathfrak{B}\pi-\Phi}.\Phi$

Lentis tertiae $=\frac{AB\mathfrak{C}a\pi'+x}{\mathfrak{C}\pi'-\pi+\Phi}.\Phi.$

Coroll. I.

353. Cum ſit $C=\frac{-ml}{ABb}$, ideoque $\mathfrak{C}=\frac{-ml}{ABb-ml}$; et $\Phi=\frac{-\pi+\pi'}{ma-b}.b$, ſi hi valores ſubſtituantur erit pro loco oculi.

$$O=\frac{ABbl(ma-b)\pi'}{(mmal-ABbb)\pi'+ma(ABb-ml)\pi}.$$

Coroll. 2.

354. Semidiameter confusionis etiam ita exprimi potest vt sit

$$\frac{\mu m x^3}{4 a a b}\left(\frac{\lambda + \nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3} + \frac{\Phi(\lambda' + \nu\mathfrak{B}(1-\mathfrak{B}))}{A^3\mathfrak{B}^3(\mathfrak{B}\pi - \Phi)} + \frac{\Phi(\lambda'' + \nu\mathfrak{C}(1-\mathfrak{C}))}{A^3 B^3 \mathfrak{C}^3(\mathfrak{C}\pi' - \pi + \Phi)}\right)$$

cuius formae cohaerentia cum praecedentibus iam ita est manifesta, vt ad plures lentes facile extendi queat.

Problema 6.

355. Si instrumentum dioptricum quatuor lentibus sit instructum, quarum crassities in computum duci non mereatur, definire omnia momenta ad constructionem dirigendam necessaria.

Solutio.

Quia hic lens quarta est vltima.

1. Vt oculo imago postrema in distantia iusta offeratur debet esse $O = \delta + l$, ideoque

$O = l + \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$ et ex conditione campi apparentis

$O = \frac{\mathfrak{D}d\pi''}{\pi'' - \pi' + \pi - \Phi}$.

2. Exponens multiplicationis m ad distantiam b relatus, si obiectum vt hactenus situ erecto exhiberi assumatur, erit

$$m = +ABCD.\ \frac{b}{r} \text{ pro situ erecto.}$$

sin autem situs desideretur inuersus numerus m negatiue est accipiendus.

3. Pro

3. Pro gradu claritatis perpetuo habetur $y = \frac{bx}{ma}$.

4. Campus autem apparens definiri debet ex hac aequatione

$$\Phi = \frac{\pi + \pi' - \pi''}{ma - b} b$$

hinc enim $a\Phi$ praebebit femidiametrum fpatii in obiecto confpicui, fiquidem diftantia O fuerit pofitiua.

5. Sin autem diftantia O prodeat negatiua quo cafu capi oportet $O = o$, feu $\frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = -l$ quantitas Φ determinari debet ex hac aequatione $\pi''\mathfrak{D}d = \omega$ feu hac

$$\frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = \omega \text{ fiue } (D+1)\omega = -\pi'' l.$$

6. Semidiameter confufionis ita reperietur expreffus:

$$\frac{\mu m x^3}{4aab}\left\{\begin{array}{l} + \frac{(A+1)(\lambda(A+1)^2 + \nu A)}{A^3} \\ + \frac{\Phi(B+1)(\lambda'(B+1)^2 + \nu B)}{A^3 B^3 (\mathfrak{B}\pi - \Phi)} \\ + \frac{\Phi(C+1)(\lambda''(C+1)^2 + \nu C)}{A^3 B^3 C^3 (\mathfrak{C}\pi' - \pi + \Phi)} \\ + \frac{\Phi(D+1)(\lambda'''(D+1)^2 + \nu D)}{A^3 B^3 C^3 D^3 (\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} \end{array}\right.$$

7. Vt obiectum fine margine colorato appareat, fiquidem diftantia O fit pofitiua effe oportet:

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = o.$$

8. Verum fi acceperimus $O = o$, margo coloratus euanefcet, huic aequationi fatisfaciendo.

$$\frac{\pi''}{\mathfrak{A}\Phi} + \frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} = \frac{\pi}{ABC\mathfrak{D}(\mathfrak{B}\pi - \Phi)} + \frac{\pi'}{ABC\mathfrak{D}(\mathfrak{C}\pi' - \pi + \Phi)}.$$

9. Omnis

9. Omnis autem confuſio a diuerſa radiorum natura oriunda penitus tolletur, ſi ſatisfieri liceat huic aequationi:

$$\frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} + \frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} = 0.$$

10. Denique efficiendum eſt vt diſtantiae lentium ſint poſituae vnde oriuntur formulae ſequentes:

Interuallum
Lentium

I et II $= \frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi - \Phi} > 0$

II et III $= \frac{ABa\Phi(\mathfrak{C}\pi' - (1 - \mathfrak{B})\pi)}{(\mathfrak{B}\pi - \Phi)(\mathfrak{C}\pi' - \pi + \Phi)} > 0$

III et IV $= \frac{ABCa\Phi(\mathfrak{D}\pi'' - (1 - \mathfrak{C})\pi')}{(\mathfrak{C}\pi' - \pi + \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} > 0.$

11. Aperturae autem ſingularum lentium ita ſunt capiendae vt ſit

Semid: apert:

Lentis ſecundae $= \frac{A\mathfrak{B}a\pi + x}{\mathfrak{B}\pi - \Phi} \cdot \Phi$

Lentis tertiae $= \frac{AB\mathfrak{C}a\pi' + x}{\mathfrak{C}\pi' - \pi + \Phi} \cdot \Phi$

Lentis quartae $= \frac{ABC\mathfrak{D}a\pi'' + x}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \cdot \Phi.$

Coroll. I.

356. Cum ſit ob multiplicationem $D = \frac{ml}{ABCb}$ ideoque $\mathfrak{D} = \frac{ml}{ABCb + ml}$ et $\Phi = \frac{-\pi + \pi' - \pi''}{ma - b} \cdot b$; pro loco oculi habebitur haec expreſſio:

$$O = \frac{-ABCbl(ma - b)\pi''}{(mmal + ABCbb)\pi'' - ma(ABCb + ml)(\pi' - \pi)}.$$

Coroll.

Coroll. 2.

357. Quia hic tot occurrunt litterae determinandae, dubium nullum esse videtur, quin formulis sub n^{o}. 7 vel 8 et 9 contentis satisfieri queat, et certum saltem est, id fieri posse, si plura adhibeantur mediorum refringentium genera.

Problema 7.

358. Si instrumentum dioptricum quinque lentibus sit instructum, quarum crassities ob paruitatem negligi queat, definire omnia momenta constructionem dirigentia.

Solutio.

Cum hic lens quinta vltimum locum teneat; habebimus.

1. Pro loco oculi, vt imago visa ipsi in iusta distantia offeratur debet esse $O = \varepsilon + l$ ideoque
$O = l + \frac{ABCDE\alpha\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$ et ex conditione campi apparentis
$O = \frac{\mathfrak{E}\, e\, \pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi}$.

2. Exponens multiplicationis m ad distantiam $= b$ relatus, si quidem obiectum situ erecto sit spectandum

$$m = -ABCDE.\frac{b}{r}.$$

pro situ autem inuerso numerus m negatiue capi debet.

3. Pro

3. Pro gradu claritatis eſt vt hactenus $y = \frac{bx}{ma}$.

4. Campus apparens definiri debet ex hac aequatione

$$\Phi = \frac{-\pi + \pi' - \pi'' + \pi'''}{ma - b} \, b$$

ſiquidem diſtantia illa O prodierit poſitiua.

5. Sin autem haec diſtantia prodeat negatiua vt capiatur $O = 0$; ſeu $\frac{ABCDEa\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = -l$, campus apparens non ex formula n^o. 4 ſed ex hac $\pi''' \mathfrak{E} e = \omega$ definiri debet, vnde fit

$$\frac{ABCD\mathfrak{E}a\pi'''\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = \omega \text{ ſeu } (E+1)\omega = -\pi''' l$$

6. Semidiameter confuſionis ab apertura lentium oriundae ſequenti modo exprimitur,

$$\frac{\mu m x^3}{4aab} \left\{ \begin{array}{l} + \frac{(A+1)(\lambda(A+1)^2 + \nu A)}{A^3} \\ + \frac{\Phi(B+1)(\lambda'(B+1)^2 + \nu B)}{A^3 B^3 (\mathfrak{B}\pi - \Phi)} \\ + \frac{\Phi(C+1)(\lambda''(C+1)^2 + \nu C)}{A^3 B^3 C^3 (\mathfrak{C}\pi' - \pi + \Phi)} \\ + \frac{\Phi(D+1)(\lambda'''(D+1)^2 + \nu D)}{A^3 B^3 C^3 D^3 (\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} \\ + \frac{\Phi(E+1)(\lambda''''(E+1)^2 + \nu E)}{A^3 B^3 C^3 D^3 E^3 (\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)} \end{array} \right.$$

7. Vt obiectum ſine margine colorato appareat, ſiquidem diſtantia O fuerit poſitiua, huic aequationi erit ſatisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} + \frac{\pi'''}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = 0.$$

8. At

8. At ſi diſtantia O ſit negatiua, capiaturque O = o pro hoc ſcopo obtinendo oportet ſit

$$\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$

$$=\frac{1}{ABCD\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right).$$

9. Omnis autem confuſio a diuerſa radiorum natura oriunda penitus tolletur, ſi fieri poſſit

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}=0$$

10. Tum vero efficiendum eſt, vt interualla lentium omnia ſint poſitiua, vnde oriuntur formulae ſequentes.

Interuallum
Lentium

I. et II. $=\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}>0$

II. et III. $=\frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0$

III. et IV. $=\frac{ABCa\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}>0$

IV. et V. $=\frac{ABCDa\Phi(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}>0$

11. Aperturae denique ſingularum lentium ita ſunt capiendae, vt ſit:

Semid. apert:

Lentis ſecundae $=\frac{A\mathfrak{B}a\pi+x}{\mathfrak{B}\pi-\Phi}\Phi$

Lentis tertiae $=\frac{AB\mathfrak{C}a\pi'+x}{\mathfrak{C}\pi'-\pi+\Phi}\Phi$

Lentis quartae $=\frac{ABC\mathfrak{D}a\pi''+x}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\Phi$

Lentis quintae $=\frac{ABCD\mathfrak{E}a\pi'''+x}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\Phi.$

Corollarium.

359. Ob multiplicationem ergo erit:

$$\mathrm{E} = \frac{-ml}{\mathrm{ABCD}b} \text{ et } \mathfrak{E} = \frac{-ml}{\mathrm{ABCD}b - ml}$$

vnde oritur:

$$\mathrm{O} = \frac{\mathrm{ABCD}bl(ma-b)\pi'''}{(mmal - \mathrm{ABCD}bb)\pi''' + ma(\mathrm{ABCD}b - ml)(\pi'' - \pi' + \pi)}.$$

Scholion.

360. Hactenus suppoſui oculum vel in eo loco teneri, quem viſio campi apparentis poſtulat, vel lenti poſtremae immediate applicari; quorum vtroque caſu efficiendum eſt, vt imago vltima oculo ad diſtantiam iuſtam offeratur. Hic quidem locum oculi, quem campus praebet ſtatim coniunxi cum iuſta imaginis ſpectandae diſtantia; ſed etiam ſine reſpectu ſiue ad hanc diſtantiam ſiue ad multiplicationem habito definiri poteſt. Ex ſuperioribus enim §. 271 habemus

Pro caſu

vnius lentis $\mathrm{O} = 0$

duarum lentium $\mathrm{O} = \frac{\mathrm{A}\mathfrak{B}a\pi\Phi}{(\pi-\Phi)(\mathfrak{B}\pi-\Phi)}$

trium lentium $\mathrm{O} = \frac{\mathrm{AB}\mathfrak{C}a\pi'\Phi}{(\pi'-\pi+\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$

quatuor lentium $\mathrm{O} = \frac{\mathrm{ABC}\mathfrak{D}a\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$

quinque lentium $\mathrm{O} = \frac{\mathrm{ABCD}\mathfrak{E}a\pi'''\Phi}{(\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$

etc.

Verum

Verum si nolimus ad campum apparentem respicere semper eiusmodi locum oculo assignare licet, vnde obiectum sine margine colorato cernatur. Haud abs re autem fore arbitror hunc locum determinasse, quia ex eius distantia a loco oculi ob rationem campi assumto facilius iudicare licebit, quanto margine cinctum apparere debeat obiectum si oculus in loco isto posteriori teneatur. Si enim aequationi n^o. 7 appositae satisfieri nequeat, nil aliud intelligimus, nisi obiectum non sine margine colorato esse apparitiurum at si insuper constet ille locus, vnde obiectum sine huiusmodi margine spectari posset, facilius quantitatem istius marginis colligere poterimus.

Problema 8.

361. Si instrumentum dioptricum ex lentibus quotcunque constet, quarum crassitiem negligere liceat, eum pro oculo locum assignare, vnde obiectum sine margine colorato conspiciatur.

Solutio.

Ponatur haec oculi post lentem vltimam distantia $= \Omega$, atque ex aequationibus §. 325 datis, si pro O scribatur Ω haec ipsa distantia Ω, quam quaerimus, elici potest. Scilicet pro quouis lentium numero erit vt sequitur.

 1. Pro

I. Pro vnica Lente.

Habetur haec aequatio $\frac{\Omega\, d\alpha}{\alpha(\alpha-\Omega)}=0$ vnde fit $\frac{\Omega}{\alpha(\alpha-\Omega)}=0$, seu $\frac{\Omega}{\alpha-\Omega}=0$. Verum etiam campus apparens exigit $O=0$, vnde hoc casu ambo loca oculi O et Ω conveniunt, neque margo coloratus est pertimescendus.

II. Pro duabus Lentibus

Hoc casu reperta est ista aequatio

$$\frac{\Omega\, d\beta}{\beta(\beta-\Omega)}=d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)$$

vnde substitutis valoribus supra assignatis oritur

$$\frac{\Omega}{\beta(\beta-\Omega)}=\frac{\pi}{\mathfrak{A}\Phi}:(B+1)ABa\left(\tfrac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}\right)$$

seu si ponamus

$$Y=\frac{\pi}{\mathfrak{A}\Phi} \text{ et } Z=\tfrac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}$$

erit

$$\frac{\Omega}{\beta-\Omega}=\frac{Y}{AB(B+1)aZ}.\ \beta=\frac{Y}{(B+1)Z}\cdot\frac{\Phi}{\mathfrak{B}\pi-\Phi}.$$

III. Pro tribus Lentibus.

Eodem modo si ponamus

$$Y=\frac{\pi'}{\mathfrak{A}\Phi}+\frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}-\frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi-\Phi)} \quad \text{et}$$

$$Z=\tfrac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}$$

erit pro casu trium lentium

$$\frac{\Omega}{\gamma-\Omega}=\frac{Y}{ABC(C+1)aZ}.\ \gamma=\frac{Y}{(C+1)Z}\cdot\frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$$

vbi

vbi notandum eſt $Y=0$ et $Z=0$ eſſe eas ipſas aequationes, quae in ſuperioribus problematibus ſub numeris 8 et 9 retulimus.

IV. Pro quatuor lentibus

Si iam ponatur:

$$Y=\frac{\pi''}{\mathfrak{A}\Phi}+\frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}-\frac{1}{ABC\mathfrak{D}}\left(\frac{\pi}{(\mathfrak{B}\pi-\Phi)}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}\right)$$

$$Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$

erit:

$$\frac{\Omega}{\delta-\Omega}=\frac{Y}{ABCD(D+1)aZ}\delta$$

hincque $\frac{\Omega}{\delta-\Omega}=\frac{Y}{(D+1)Z}\cdot\frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$.

V. Pro quinque Lentibus.

Poſito ſecundum aequationes n^o. 8 et 9 exhibitas

$$Y=\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)})$$

$$-\frac{1}{ABCD\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right)$$

$$Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$

$$+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$$

habebimus

$$\frac{\Omega}{\varepsilon-\Omega}=\frac{Y}{ABCDE(E+1)aZ}\cdot\varepsilon \text{ vel}$$

$$\frac{\Omega}{\varepsilon-\Omega}=\frac{Y}{(E+1)Z}\cdot\frac{\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}.$$

Lex harum formularum pro quouis maiori lentium numero hinc ſatis eſt perſpicua.

Coroll. 1.

362. Cum igitur ſupra pro quouis lentium numero exhibuerimus aequationes n°. 8. et n°. 9. formula n°. 8. dabit valorem Y et formula n°. 9. valorem ipſius Z, quibus cognitis locus oculi, vbi margo coloratus euaneſcit, facile innoteſcit.

Coroll. 2.

363. Si praeterea ex aequatione n°. 7. expoſita ponatur

$$X = \frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} + \frac{\pi'''}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

erit quidem pro caſu quinque lentium

$$Y = \frac{\pi'''}{\Phi} Z - \frac{1}{ABCDE} X$$

Pro quatuor lentibus $Y = \frac{\pi''}{\Phi} Z - \frac{1}{ABCD} X$

Pro tribus lentibus $Y = \frac{\pi'}{\Phi} Z - \frac{1}{ABC} X$

et pro duabus lentibus $Y = \frac{\pi}{\Phi} Z - \frac{1}{AB} X$.

Scholion.

364. Quo facilius haec oculi loca littera Ω deſignata cum praecedentibus littera O deſignatis comparare liceat, notandum eſt eſſe.

Pro caſu

Vnius Lentis $\frac{O}{\alpha - O} = 0$

Duarum Lentium $\frac{O}{\beta - O} = \frac{\pi}{(B+1)(\mathfrak{B}\pi - \Phi)}$

Trium Lentium $\frac{O}{\gamma - O} = \frac{\pi'}{(C+1)(\mathfrak{C}\pi' - \pi + \Phi)}$

Quatuor Lentium $\frac{O}{\delta - O} = \frac{\pi''}{(D+1)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$

Quinque Lentium $\frac{O}{\varepsilon - O} = \frac{\pi'''}{(E+1)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$ etc.

Quare

Quare vt ambo loca congruant retentis pro quouis lentium numero litteris Y et Z modo adhibitis, oportet ſit

Pro vna Lente $0 = 0$
Pro duabus Lentibus $\frac{Y}{Z} = \frac{\pi}{\Phi}$
Pro tribus Lentibus $\frac{Y}{Z} = \frac{\pi'}{\Phi}$
Pro quatuor Lentibus $\frac{Y}{Z} = \frac{\pi''}{\Phi}$
Pro quinque Lentibus $\frac{Y}{Z} = \frac{\pi'''}{\Phi}$ etc.

vnde generatim ſit $X = 0$, quae eſt ipſa aequatio n°. 7. allata. Haec igitur ſunt principia generalia, ex quibus conſtructio inſtrumentorum dioptricorum erit perficienda, quae quidem inſtrumenta hic potiſſimum ad viſionem accommodaui. Nihilo vero minus ad repraeſentationem obiectorum in camera obſcura ſuper Tabula alba adhiberi poſſunt, vbi praeter iam data praecepta tenendum eſt, has effigies etiam ſine margine colorato eſſe apparituras, ſi aequationibus n°. 7. ſatisfiat, omnem vero confuſionem a diuerſa radiorum natura oriundam penitus tolli, ſi inſuper aequationibus n°. 9 ſatisfiat.

Supplementum VII.

Si ratio refractionis in ſingulis lentibus fuerit diverſa et ſecundum lentium ordinem litteris n, n', n'', n''' etc. exhibeatur, inde pro lentium conſtructione

I°. Litterae reſpondentes ϱ, σ, τ et pro confuſione

fusione litterae μ et ν conuenienter definiri debent secundum formulas supra datas §. 55

$$\varrho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1 \, ; \; \sigma = \frac{1}{2(n-1)} - \frac{1}{n+2} + 1$$

$$\tau = \tfrac{1}{3}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right) \sqrt{(4n-1)}$$

ita, vt sit

$$\varrho + \sigma = \frac{1}{n-1} \text{ et } \sigma - \varrho = \frac{-2}{n+2} + 2 = \frac{2(n+1)}{n+2}.$$

Porro autem

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{1}{8(n-1)^2}$$

$$\nu = \frac{4(n-1)^2}{4n-1};$$

vnde intelligitur, quemadmodum ex ratione refractionis n' litterae respondentes ϱ', σ', τ', μ', ν' definiri debeant.

II°. Quod iam ad elementa occasione campi apparentis introducta π, π', π'' etc. attinet; distantiae determinatrices lentium inde eodem modo manent determinatae, vt supra, ita, vt non opus sit, eas formulas hic transscribere; interim tamen meminisse iuuabit; formulas primitiuas, vnde illae sunt natae, quae sunt

1°. $\frac{\mathfrak{B}\pi - \Phi}{\Phi} = \frac{Aa}{b}$

2°. $\frac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = \frac{ABa}{c}$

3°. $\frac{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}{\Phi} = \frac{ABCa}{d}$

4°. $\frac{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}{\Phi} = \frac{ABCDa}{e}$

etc.

III.

III. Verum valores litterarum P, Q, R etc. ob diuerſas refractiones mutantur, vt ſequitur.

$$P = \frac{\mu}{A^3 a^3}(A+1)(\lambda(A+1)^2+\nu A)$$

$$Q = \frac{\mu'}{B^3 b^3}(B+1)(\lambda'(B+1)^2+\nu' B);$$

$$R = \frac{\mu''}{C^3 c^3}(C+1)(\lambda''(C+1)^2+\nu'' C);$$

$$S = \frac{\mu'''}{D^3 d^3}(D+1)(\lambda'''(D+1)^2+\nu''' D);$$

etc.

IV. Diſtantiae lentium etiam manent, vt ante, vna cum ſemidiametris aperturarum ſingularum lentium; at radii binarum facierum vtriusque lentis ita ſunt ad praeſentem caſum accommodandae, vt pro lente cuius refractio eſt n', ei reſpondentes litterae ϱ', σ', et τ' vſurpari debeant, ſimilique modo etiam pro ſequentibus lentibus, quarum refractio litteris n'', n''' etc. indicatur.

His praemiſſis ſingula momenta, quae in conſtructione inſtrumentorum dioptricorum ſunt obſeruanda, ſequenti modo repraeſentabimus.

I. Pro loco oculi ſeu eius diſtantia poſt vltimam lentem $= O$

habebimus pro ſingulis lentium numeris ſequentes determinationes:

Num. lentium.	
I.	$O = o = \alpha + l$; ideoque $\alpha = -l = Aa$
II.	$O = \frac{\mathfrak{B}b\pi}{\pi - \Phi} = \beta + l$; ideoque $\beta = \frac{\mathfrak{B}b\pi}{\pi - \Phi} - l = Bb$; $l = \frac{-Bb(\mathfrak{B}\pi - \Phi)}{\pi - \Phi}$
III.	$O = \frac{\mathfrak{C}c\pi'}{\pi' - \pi + \Phi} = \gamma + l$; hinc $\gamma = \frac{\mathfrak{C}c\pi'}{\pi' - \pi + \Phi} - l = Cc$;
	hinc $l = -\frac{Cc(\mathfrak{C}\pi' - \pi + \Phi)}{\pi' - \pi + \Phi}$
IV.	$O = \frac{\mathfrak{D}d\pi''}{\pi'' - \pi' + \pi - \Phi} = \delta + l$; hinc $\delta = \frac{\mathfrak{D}d\pi''}{\pi'' - \pi' + \pi - \Phi} - l$;
	adeoque $l = \frac{-Dd(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}{\pi'' - \pi' + \pi - \Phi}$
V.	$O = \frac{\mathfrak{E}e\pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi} = \varepsilon + l$; hinc $\varepsilon = \frac{\mathfrak{E}e\pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi} - l$;
	adeoque $l = -\frac{Ee(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}{\pi''' - \pi'' + \pi' - \pi + \Phi}$
	etc.

II. Pro ratione multiplicationis ad diſtantiam b relata

habebimus pro quolibet lentium numero ſequentes valores litterae m.

I.	$m = -A\frac{b}{l}$	Pro ſitu erecto, ſi m hinc prodierit poſitiuum; pro ſitu autem inuerſo, ſi m obtinuerit valorem negatiuum.
II.	$m = +AB\frac{b}{l}$	
III.	$m = -ABC\frac{b}{l}$	
IV.	$m = +ABCD\frac{b}{l}$	
V.	$m = -ABCDE\frac{b}{l}$	
	etc.	

III.

III. Pro gradu claritatis y

ex ſuperioribus liquet, eum ſemper pari modo exprimi; quantuſcunque fuerit lentium numerus; perpetuo enim erit $y = \frac{bx}{ma}$

IV. Pro Campo apparente ſi quidem O habeat valorem poſitiuum,

eius ſemidiameter Φ pro quouis lentium numero ſequenti modo definietur:

Num. lentium.	
I.	$\Phi = \infty$ ſeu indefinitum
II.	$\Phi = \frac{-\pi b}{ma - b}$
III.	$\Phi = \frac{-\pi + \pi'}{ma - b} . b$
IV.	$\Phi = \frac{-\pi + \pi' - \pi''}{ma - b} . b$
V.	$\Phi = \frac{-\pi + \pi' - \pi'' + \pi'''}{ma - b} . b$ etc.

(IV.) Pro Campo apparente,

ſi diſtantia O fuerit negatiua, quo caſu oculus lenti vltimae immediate adplicari debet, Φ definietur ex aequationibus ſequentibus.

Num. lentium.	
I.	$\Phi = \infty$ indefinitum
II.	$\frac{A \mathfrak{B} a \Phi}{\mathfrak{B}\pi - \Phi} = \omega$
III.	$\frac{A B \mathfrak{C} a \Phi}{\mathfrak{C}\pi' - \pi + \Phi} = \omega$
IV.	$\frac{A B C \mathfrak{D} a \Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = \omega$
V.	$\frac{A B C D \mathfrak{E} a \Phi}{\pi''' - \pi'' + \pi' - \pi + \Phi} = \omega$. etc.

V. Pro semidiametro confusionis habebuntur sequentes expressiones

I. $\frac{m x^3}{4aab} \cdot \frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}$

II. $\frac{m x^3}{4aab}\left(\frac{\mu(A+1)(\lambda(A+1)^2\nu A)}{A^3}\right)+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B (\mathfrak{B}\pi-\Phi)}\Phi)$

III. $\frac{m x^3}{4aab}\left\{\begin{array}{l}\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B (\mathfrak{B}\pi-\Phi)}\Phi \\ \qquad +\frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3 (\mathfrak{C}\pi'-\pi+\Phi)}\Phi]\end{array}\right.$

IV. $\frac{m x^3}{4aab}\left\{\begin{array}{l}\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3 (\mathfrak{B}\pi-\Phi)}\Phi \\ \frac{\mu''(C+1)(\lambda''(C+1)^2\nu'' C)}{A^3 B^3 C^3 (\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\frac{\mu'''(D+1)(\lambda'''(D+1)^2+\nu''' D)}{A^3 B^3 C^3 D^3 (\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\Phi\end{array}\right.$

V. $\frac{m x^3}{4aab}\left\{\begin{array}{l}\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3 (\mathfrak{B}\pi-\Phi)}\Phi \\ +\frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3 (\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\frac{\mu'''(D+1)(\lambda'''(D+1)^2+\nu''' D)}{A^3 B^3 C^3 D^3 (\mathfrak{D}\pi''-\pi'+\pi+\Phi)}\Phi \\ \qquad +\frac{\mu''''(E+1)(\lambda''''(E+1)^2+\nu'''' E)}{A^3 B^3 C^3 D^3 E^3 (\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}\Phi\end{array}\right.$

etc.

VI. Pro tollendo margine colorato,

si distantia O prodierit positiua, obiectum sine margine colorato apparebit, satisfaciendo sequentibus aequationibus:

Num. lentium.

I. $0=0$

II. $0=\frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi}$

III. $0=\frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi}+\frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi}$

VI. $0=\frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi}+\frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi}+\frac{ddn'''}{n'''-1} \cdot \frac{\pi''}{ABCa\Phi}$ etc.

(VI.)

(VI). Pro tollendo margine colorato.

Sin autem distantia O prodierit negatiua, margo coloratus euanescet, si sequentibus aequationibus satisfiat

$$\text{I.}\quad 0=0$$

$$\text{II.}\quad 0=\frac{a\,dn}{n-1}(A+1)B\pi$$

$$\text{III.}\quad 0=\frac{a\,dn}{n-1}(A+1)BC\pi'+\frac{b\,dn'}{n'-1}\frac{(B+1)C\pi'-(C+1)\pi}{A}$$

$$\text{IV.}\quad 0=\frac{a\,dn}{n-1}(A+1)BCD\pi''+\frac{b\,dn'}{n'-1}\left(\frac{(B+1)CD\pi''-(D+1)\pi}{A}\right)+\frac{c\,dn''}{n''-1}\left(\frac{(C+1)D\pi''-(D+1)\pi'}{AB}\right)$$

$$\text{V.}\quad 0=\frac{a\,dn}{n-1}(A+1)BCDE\pi'''+\frac{b\,dn'}{n'-1}\left(\frac{(B+1)CDE\pi'''-(E+1)\pi}{[illegible]}\right)+\frac{c\,dn''}{n''-1}\left(\frac{(C+1)DE\pi'''-(E+1)\pi'}{AB}\right)+\frac{d\,dn'''}{n'''-1}\left(\frac{(D+1)E\pi'''-(E+1)\pi''}{ABC}\right)$$

etc.

VII. Pro tollenda confusione omni

insuper sequentibus aequationibus est satisfaciendum

$$\text{I.}\quad 0=\frac{a\,dn}{n-1}\cdot\frac{A+1}{A}$$

$$\text{II.}\quad 0=\frac{a\,dn}{n-1}\cdot\frac{A+1}{A}+\frac{b\,dn'}{n'-1}\cdot\frac{B+1}{A^2B}$$

$$\text{III.}\quad 0=\frac{a\,dn}{n-1}\cdot\frac{A+1}{A}+\frac{b\,dn'}{n'-1}\cdot\frac{B+1}{A^2B}+\frac{c\,dn''}{n''-1}\cdot\frac{C+1}{A^2B^2C}$$

$$\text{IV.}\quad 0=\frac{a\,dn}{n-1}\cdot\frac{A+1}{A}+\frac{b\,dn'}{n'-1}\cdot\frac{B+1}{A^2B}+\frac{c\,dn''}{n''-1}\cdot\frac{C+1}{A^2B^2C}+\frac{d\,dn'''}{n'''-1}\cdot\frac{D+1}{A^2B^2C^2D}$$

etc.

 Distan-

Diſtantiae autem lentium et ſemidiametri aperturarum perinde definiuntur, ac ſupra; tantum notetur inſuper, formulas pro ſemidiametro confuſionis exhibitas, niſi penitus ad nihilum redigi queant, aequales poni debere formulae $\frac{1}{4k^3}$, exiſtente circiter $k = 40$ (§. 193) vel adhuc minore, prout circumſtantiae poſtulauerint.

Supplementum VIII.

De Lentibus obiectiuis perfectis.

In Cap. III. nullas alias lentes obiectiuas commemorauimus, niſi quae [illegible] vitri ſpecie et ex principio minimi ſunt paratae, neque tales lentes ex diuerſis vitri ſpeciebus conficere conuenit, quoniam eae ad confuſionem ex diuerſa radiorum refrangibilitate oriundam, tollendam ſunt ineptae. Cum igitur iſta confuſio aliter tolli nequeat, niſi diuerſae materiae refringentes adhibeantur; hic adcuratius perpendamus, quemadmodum ex diuerſis mediis diaphanis eiusmodi lentes conſtrui queant, in quibus non ſolum prior confuſio ex apertura lentium, ſed etiam poſterior ex diuerſa radiorum refrangibilitate oriunda ad nihilum redigi queat, cuiusmodi lentes compoſitas merito *perfectas* appellare licebit.

I. Primo igitur examinemus, quomodo poſteriori confuſioni ſit occurrendum, quando quotcunque lentes inuicem iungantur, ita, vt earum diſtantiae quaſi euaneſcant. Quem in finem conſiderentur formulae ſupra fupra datae tam pro margine colorato vitando, quam pro confufione penitus tollenda N°. VI, VII. At ex aequatione N°. VI. patet, marginem coloratum euanefcere, fi litterae π, π', π'' etc. fint $=0$, quod quidem fponte euenit, fi lentes immediate coniungantur; quare in genere nullus margo coloratus eft metuendus, ftatim atque interualla omnia lentium euanefcunt.

II. Vt autem aequationibus N°. VII. fatisfiat, pro cafu duarum lentium immediate iunctarum habemus.

$$0=\frac{a\,dn}{n-1}\cdot\frac{A+1}{A}+\frac{b\,dn'}{n'-1}\cdot\frac{B+1}{A^2B}$$
$$0=\frac{a\,dn}{n-1}\cdot\frac{a+\alpha}{\alpha}+\frac{b\,dn'}{n'-1}\cdot\frac{(b+\beta)\alpha^2}{\alpha^2\cdot\beta}$$
$$0=\frac{dn}{n-1}\cdot\frac{a+\alpha}{a\alpha}+\frac{b\,dn'}{n'-1}\cdot\frac{b+\beta}{\alpha^2\beta};\ \text{feu ob } \alpha=-b$$
$$0=\frac{dn}{n-1}\cdot\frac{a+\alpha}{a\alpha}+\frac{dn'}{n'-1}\cdot\frac{b+\beta}{b\beta},$$
$$0=\frac{dn}{n-1}\cdot\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{dn'}{n'-1}\cdot\left(\frac{1}{b}+\frac{1}{\beta}\right)$$

vbi duo hi coëfficientes denotant reciproca diftantiarum focalium primae et fecundae lentis.

III. Nunc infuper effici debet, vt prior confufio ab apertura lentium oriunda ad nihilum redigatur; vbi obferuare debemus, hoc cafu effe debere $\alpha+b=0$; $\beta+c=0$; $\gamma+d=0$ etc. pro numero lentium, quas coniungere velimus. Vnde femidiameter confu-

confusionis ex lentibus iunctis ortae euanescet, si reddatur

$$0 = \frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3} - \frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3\,B^3}$$

$$+ \frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3\,B^3\,C^3} \text{ etc.}$$

quoniam supra iam vidimus, fore $\pi = 0$, $\pi' = 0$ etc. atque hinc pro quouis numero lentium inuicem iungendarum constructionem lentium perfectarum petere debemus; quas inuestigationes hic suscipiamus.

De lentibus perfectis ex duabus lentibus compositis.

Quoniam hoc casu distantiae determinatrices ipsius lentis compositae sunt a et β; statuamus

$$\frac{1}{a}+\frac{1}{\alpha} = f\left(\frac{1}{a}+\frac{1}{\beta}\right) \text{ et } \frac{1}{b}+\frac{1}{\beta} = g\left(\frac{1}{a}+\frac{1}{\beta}\right) \text{ siue}$$

$$\frac{a\alpha}{a+\alpha} = \frac{1}{f}\left(\frac{a\beta}{a+\beta}\right); \quad \frac{b\beta}{b+\beta} = \frac{1}{g}\cdot\frac{a\beta}{a+\beta};$$

vnde pro prima conditione implenda sequitur:

$0 = \frac{dn}{n-1}\cdot f + \frac{dn'}{n'-1}\cdot g$. Porro ob $\frac{1}{\alpha} = \frac{f-1}{a} + \frac{f}{\beta}$ et $\frac{1}{b} = \frac{g}{a} + \frac{g-1}{\beta}$ et $\alpha + b = 0$, erit $\frac{f+g-1}{a} + \frac{f+g-1}{\beta} = 0$, seu $f+g=1$.

Sit breuitatis gratia $\frac{dn}{n-1} = \zeta$ et $\frac{dn'}{n'-1} = \eta$, vt sit $\zeta f + \eta g = 0$; ex qua aequatione cum altera $f+g=1$ coniuncta vtraque littera f et g determinabitur, ita, vt sit

$$f = \frac{\eta}{\eta-\zeta} \text{ et } g = -\frac{\zeta}{\eta-\zeta}$$

deinde

deinde cum fit

$$A = \frac{\alpha}{a} = \frac{\beta}{\beta(f-1)+\alpha f} = \frac{\beta}{\alpha f - \beta g} = \frac{(\eta-\zeta)\beta}{\alpha\eta+\beta\zeta} \text{ et}$$

$$B = \frac{\beta}{b} = \frac{g\beta+(g-1)\alpha}{\alpha} = \frac{g\beta - f\alpha}{\alpha} = \frac{-\zeta\beta - \eta\alpha}{(\eta-\zeta)\alpha};$$

$$\text{ergo } A+1 = \frac{\eta(\alpha+\beta)}{\alpha\eta+\beta\zeta} \text{ et } B+1 = \frac{-\zeta(\alpha+\beta)}{\alpha(\eta-\zeta)};$$

vnde pro confusione priori tollenda habebimus hanc aequationem

$$0 = \frac{\mu\lambda.\eta^3(\alpha+\beta)^3}{(\eta\alpha+\zeta\beta)^3} + \frac{\mu\nu\eta(\eta-\zeta)(\alpha+\beta)\beta}{(\eta\alpha+\zeta\beta)^2}$$
$$\frac{-\mu'\lambda'.\zeta^3(\alpha+\beta)^3}{(\eta\alpha+\zeta\beta)^3} + \frac{\mu'\nu'\zeta(\eta-\zeta)(\alpha+\beta)\alpha}{(\eta\alpha+\zeta b)^2}$$

quae reducitur ad hanc

$$0 = \frac{(\alpha+\beta)^2}{\eta\alpha+\zeta\beta}(\mu\lambda\eta^3 - \mu'\lambda'\zeta^3) + (\eta-\zeta)(\mu\nu\eta\beta + \mu'\nu'\zeta\alpha)$$

Breuius etiam ex §. 214 deducitur haec aequatio, cui satisfieri debet $P+Q=0$ ob $a=-b$, existente pro hoc casu

$$P = \mu f\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)\left(\lambda\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)^2 + \tfrac{\nu}{\alpha}\left(\tfrac{f-1}{\alpha}+\tfrac{f}{\beta}\right)\right) \text{ et}$$

$$Q = \mu'.g\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)\left(\lambda' g^2\left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)^2 + \tfrac{\nu'}{\beta}\left(\tfrac{g}{\alpha}+\tfrac{g-1}{\beta}\right)\right).$$

ex qua aequatione siue λ, siue λ' inueniri potest, dummodo hae litterae non minores prodeant vnitate; quamobrem quosdam eiusmodi casus euoluamus et ad praxin accommodemus.

IV. Tria autem mediorum diaphanorum genera hic potissimum contemplabimur, quorum primum sit aqua pluuia, pro qua est $n=\frac{4}{3}=1,3333$; secundum vitrum ordinarium, pro quo est $n=\frac{31}{20}=1,5500$ et tertium vitrum Cryſtallinum, anglice Flint-Glaſs, pro quo eſt $n=\frac{8}{5}=1,6000$; factoque calculo inuenimus, pro litteris graecis inde pendentibus ſequentes valores:

	Aqua	Vitrum commune	Vitrum Cryſtall.
n	$=1,3333$	$=1,5500$	$=1,6000$
ϱ	$=0,8000$	$=0,1907$	$=0,1111$
σ	$=2,2000$	$=1,6274$	$=1,5555$
τ	$=1,2490$	$=0,9051$	$=0,8606$
μ	$=1,9500$	$=0,9381$	$=0,8333$
ν	$=0,1025$	$=0,2326$	$=0,2666$
$\mu\nu$	$=0,1999$	$=0,2184$	$=0,2221.$

V. Quod autem ad valores differentialium dn attinet, optandum eſſet, vt ii per experimenta accuratiſſime definirentur; interim tamen duae ſequentes hypotheſes ex theoria deductae perpendi mererentur, quandoquidem *Newtoniana*, ex qua dn ipſi $n-1$ foret proportionale, ad confuſionem tollendam eſſet inepta. Prima autem hypotheſis, quam dudum propoſueram, facit dn ipſi $n.\log.n$ proportionale; altera autem ex theoria attractionis deducta dat dn ipſi $\frac{nn-1}{n}$ proportionale; atque hinc valores formulae $\frac{dn}{n-1}$ pro vtraque hypotheſi euoluamus:

I^ma Hypotheſis $dn=n\log.n$

	Aqua	Vitrum commune	Vitrum Cryſtall.
dn	0,1665	0,2950	0,3265
$\frac{dn}{n-1}$	0,4995	0,5364	0,5442

II^da

II^{de} Hypothesis $dn = \frac{nn-1}{n}$

	Aqua	Vitrum commune	Vitrum Cryſtall.
$\frac{dn}{n-1}$	1, 7500	1, 6452	1, 6250

VI. Verum neutra harum hypotheſium cum iis experimentis, quae Celeb. *Dollondus* circa prismata inſtituit, ſubſiſtere poteſt. Vſus nempe eſt duobus prismatibus, quorum alterum ex vitro communi, erat paratum, angulo refringente exiſtente triginta graduum; alterum autem ex vitro chryſtallino confectum continebat angulum 19°; hisque duobus prismatibus inuerſe iunctis obſeruauit, in ſpectro inde ad parietem proiecto iridis colores nullos adparere; vnde concluſit, (ſi n pro ratione refractionis vitri communis, n' vero pro refractione vitri chryſtallini aſſumatur) fore $dn : dn' = 2 : 3$, quam rationem vocat rationem diſperſionis; quo experimento admiſſo ſequitur, $\frac{dn}{n-1} : \frac{dn'}{n'-1} = 8 : 11$; ita, vt pro noſtro calculo foret $\zeta = 8$ et $\eta = 11$, quae ratio ab allatis hypotheſibus enormiter aberrat. Prima enim dat $\zeta : \eta = 0,5364 : 0,5442$ hoc eſt proxime, vt $68 : 69$. Altera autem praebet $\zeta : \eta = 1,6452 : 1,6250$ vel proxime, vti $81 : 80$, vnde patet, hypotheſin poſteriorem non poſſe ſubſiſtere, quia ex ea ſequeretur $\zeta > \eta$; prior vero tantum adhuc diſcrepat, vt cum experimento neutiquam conciliari poſſit; ex quo merito ancipites haeremus, quomodo nos in calculo hoc gerere debeamus.

amus. Interim experimentum *Dollondi* nonnullis adhuc difficultatibus premi videtur, quas hic ob oculos ponere visum est.

Digressio de refractione vitri chrystallini secundum experimenta *Dollondi*.

Tab. III. Fig. 17. I. Quo clarius theoriam, cui haec experimenta innituntur, ob oculos ponamus, rem aliquanto generalius, quam *Dollondus* fecit, consideremus. Repraesentet ergo triangulum ABC prisma triangulare ex vitro communi paratum, cuius angulus ad C, quem *Dollondus* 30° assumit, vocetur $=f$; alterum vero prisma ex vitro chrystallino factum DBE ita inuerse priori sit adplicatum, vt sit angulus $CBD=g$, ipseque angulus huius prismatis $DBE=h$ vbi in experimento examinando erat $g=0$ et $h=19^\circ$ tum vero refractio radiorum mediorum in prismate priori sit vt $m:1$, in posteriori vero vt $n:1$, ita vt facultates radios dispergendi nobis per differentialia dm et dn repraesententur. Quibus positis *Dollondus* se obseruasse affirmat, si radius solis per haec duo prismata transmittatur, spectrum inde in parietem proiectum nullis coloribus inquinari. Vnde conclusit, fore $dm:dn=2:3$, quam conclusionem adcuratius examinemus.

II. Hic primum notari conuenit, angulum, sub quo radius solis Op per faciem prioris lentis AC immittitur, non definiri, quasi perinde sit, sub quocunque

cunque angulo haec immerſio fiat, id quod pleniorem expoſitionem poſtulat. Ponamus igitur angulum $OPA = a$, qui eſt complementum anguli incidentiae et anguli ſequentes quauis refractione orti ſint p, q, r, t, u, v, e et radius iterum emergens VZ; ex quo ſtatim perſpicitur, fore $q = p + f$, $r = g + t$ et $u = v + b$; tum vero ex ratione refractionis intelligimus I) Coſ. $a = m$. Coſ. p. II) Coſ. $r = m$. Coſ. q. III) Coſ. $t = n$. Coſ. u. IV) Coſ. $e = n$. Coſ. v.

III. Quia iam hoc tranſitu nulla colorum diſperſio obſeruata eſſe perhibetur, neceſſe eſt, vt in emerſione V omnes radii colorati redditi ſint inter ſe paralleli, ideoque angulus e conſtans perinde ac primus angulus a, etiamſi rationes refractionis m et n ſuis differentialibus dm et dn augeantur; quippe qua mutatione tantum anguli p, q, r, t, u, v, fiunt variabiles.

IV. Nunc igitur primo hanc angulorum mutationem in primo priſmate euoluamus, differentiando ſcilicet aequationes ſupra notatas; vnde erit

$dq = dp$ deinde $o = dm.$ coſ. $p - m\,dp.$ ſin. p ſeu

$dp = \frac{dm.\,\text{coſ}.\,p}{m.\,\text{ſin}.\,p} = dq$;

tum vero $-dr.$ ſin. $r = dm.$ coſ. $q - m\,dq.$ ſin. q ſeu

$-dr.$ ſin. $r = dm.$ coſ. $q - \frac{dm.\,\text{coſ}.\,p.\,\text{ſin}.\,q}{\text{ſin}.\,p}$

$= \frac{dm}{\text{ſin}.\,p}(\text{coſ}.\,q\ \text{ſin}.\,p - \text{coſ}.\,p.\ \text{ſin}.\,q) = \frac{dm.\,\text{ſin}.\,(p-q)}{\text{ſin}.\,p}$

hincque tandem $dr = \frac{dm.\,\text{ſin}.\,f}{\text{ſin}.\,p.\,\text{ſin}\,r}$ ob $p - q = -f$

 Simili

Simili modo euoluantur refractiones per alterum prisma, incipiendo ab angulo constante e vbi aequatio

$$\text{cof. } e = n.\ \text{cof. } v \text{ dat } 0 = dn.\ \text{cof. } v - n\,dv.\ \text{fin. } v \text{ fiue}$$

$$dv = \frac{dn.\ \text{cof. } v}{n.\ \text{fin. } v} = du \text{ ob } u = v + b;$$

deinde vero aequatio cof. $t = n.$ cof. u dabit

$$-dt.\ \text{fin. } t = dn.\ \text{cof. } u - n\,du.\ \text{fin. } u = dn.\ \text{cof. } u - \frac{dn.\ \text{cof. } v.\ \text{fin. } u}{\text{fin. } v} = \frac{dn}{\text{fin. } v}$$

$$(\text{cof. } u.\ \text{fin. } v - \text{cof. } v.\ \text{fin. } u) = \frac{dn}{\text{fin. } v}.\ \text{fin. } (v - u)$$

ergo ob $u - v = b$ habebitur $dt = +\frac{dn.\ \text{fin. } b}{\text{fin. } v.\ \text{fin. } t}$; Cum autem fit

$$r = g + t, \text{ erit } dr = dt; \text{ ergo } \frac{dm.\ \text{fin. } f}{\text{fin. } g.\ \text{fin. } r} = \frac{dn.\ \text{fin. } b}{\text{fin. } v.\ \text{fin. } t}$$

ex qua aequatione haec relatio inter dm et dn concluditur

$$dm : dn = \frac{\text{fin. } b}{\text{fin. } v.\ \text{fin. } t} : \frac{\text{fin. } f}{\text{fin. } g.\ \text{fin. } r}$$

$$dm : dn = \frac{\text{fin. } g.\ \text{fin. } r}{\text{fin. } f} : \frac{\text{fin. } v.\ \text{fin. } t}{\text{fin. } b}$$

quae proportio generaliter locum habet, quoties angulus ad e prodit constans

V. Iam hanc conclusionem ad *Dollondi* experimentum adplicemus et quia ibi est $g = 0$, hincque

$r = t$ erit ratio inter vtramque dispersionem feu

$$dm : dn = \frac{\text{fin. } f}{\text{fin. } g} : \frac{\text{fin. } v}{\text{fin. } b} = \frac{CQ}{PQ} : \frac{BU}{VU};$$

ex qua aequatione iam fatis clare elucet, rationem $\frac{dm}{dn}$ ab angulo incidentiae a neutiquam esse independentem, vti *Dollondus* fuppofuisse videtur; ita, vt etfi

diffu-

diffusio colorum pro certo quodam angulo a euanescens sit deprehensa, hanc conclusionem neutiquam ad omnes angulos incidentiae extendere liceat. Quoniam igitur hic angulus a *Dollondo* non est assignatus, ex hoc experimento nihil certi determinari poterit.

VI. Vt autem hic nullum dubium relinquatur, aliquot casus praecipuos pro angulo a calculo euolvamus, vt adpareat, quanta diuersitas inde in rationem $\frac{dm}{dn}$ ingrediatur, dum tamen plus vna vera esse nequit. Quare cum *Dollondus* inuenisset pro vtraque vitri specie a se adhibita I. $m = 1,53$ et II) $n = 1,58$, tum vero angulos $f = 30°$ et $b = 19°$ sequentia exempla hinc expediamus:

Exemplum I.

Angulum a ita constitutum concipiamus, vt fiat q angulus rectus, ideoque etiam r, t et u recti; erit sin. $p =$ cos. f et sin. $v =$ cos. b, ex quo coligitur $dm : dn =$ cotang. f : cotang. b; siue $\frac{dm}{dn} = 0,59639$, quae iam minor est, quam $\frac{2}{3}$

Exemplum II.

Sit $a = 90°$, qui forte est ipse casus *Dollondi*; atque hinc inuenientur sequentes anguli $p = 90°$; $q = 120°$ $r = 139°, 53' = t$. $u = 118° 57'$. $v = 99° 57'$. $e = 105° 50'$. Vnde sequitur $\frac{dm}{dn} = \frac{\sin. p . \sin. b.}{\sin s . \sin. v.} = 0, 6615$. ideoque proxime $dm : dn = 2 : 3$ vt habet *Dollondus*.

Cete-

Ceterum hinc iam euidens eſt, hanc rationem pendere etiam ab angulo incidentiae, quae tamen in hoc experimento non eſt commemorata.

Si ergo hanc rationem cum *Dollondo* aſſumamus, fiet porro $\frac{dm}{m-1} : \frac{dn}{n-1} = 116 : 159 = 7 : 10$, ita, vt futurum ſit pro lentibus obiectiuis duplicatis, quibus *Dollondus* vtitur, $\zeta = 7$, $\eta = 10$, quarum conſtructionem hic ex noſtris principiis inueſtigemus.

Lens Obiectiua Clariſſ. *Dollondi* duplicata prior, anteriore lente ex vitro communi, poſteriore vero ex vitro chryſtallino parata.

Cum igitur ſit $n = 1,53$ et $n' = 1,58$, $\zeta = 7$, $\eta = 10$, erit ſtatim $f = \frac{\eta}{\eta - \zeta} = \frac{10}{3}$ et $g = \frac{-\zeta}{\eta - \zeta} = -\frac{7}{3}$;

vnde diſtantiae determinatrices vtriusque lentis erunt

Pro prima a, et $\alpha = \frac{(\eta - \zeta)ab}{\eta a + \zeta b}$

Pro poſteriori b et $\beta = \frac{-(\eta - \zeta)ab}{\eta a + \zeta b} = -\alpha$.

Dum ipſius lentis duplicatae diſtantiae determinatrices ſunt a et b.

Tantum igitur reſtat, vt pro his binis lentibus numeri arbitrarii λ et λ' definiantur; et quoniam quaeſtio circa lentes obiectiuas verſatur, ſtatuamus ſtatim $a = \infty$, vt adhuc reſolui debeat haec aequatio

$$0 = \mu\lambda\eta^3 - \mu'\lambda'\zeta^3 + \eta(\eta - \zeta)\zeta\mu'\nu'.$$

ſeu $\mu'\lambda'\zeta^3 = \mu\lambda\eta^3 + \eta\zeta(\eta - \zeta)\mu'\nu'$. quae ob $\zeta = 7$ et $\eta = 10$ abit in hanc

$$343 \cdot \mu'\lambda' = 1000 \cdot \mu\lambda + 210 \cdot \mu'\nu'. \qquad \text{II.}$$

II. Vt igitur *Dollondum* fequamur, pro his valoribus $n = 1,53$ et $n' = 1,58$ litteras inde deriuatas quaerere debemus.

Eruitur autem ex formulis fuperioribus

$\mu = 0,9875$	$\mu' = 0,8724$
$l\mu = 9.9945449.$	$l\mu' = 9.9407397$
$\nu = 0,2194$	$\nu' = 0,2529$
$l.\nu = 9.3413418$	$l.\nu' = 9.4030044$
$l.\mu\nu = 9.3358867.$	$l.\mu'\nu' = 9.3437441.$

Hinc coëfficientes trium terminorum noftrae aequationis computentur.

$l.\,343 = 2,5352941$	$l.\,1000 = 3,0000000$	$l.\,210 = 2,3222193$
$l.\,\mu' = 9.9407397$	$l.\,\mu = 9.9945449$	$l.\,\mu'\nu' = 9.3437441$
2.4760338	2.9945449	1.6659634
Subtrah. - - - -	2.4760338	2.4760338
	0.5185111	9.1899296
	$3,3000$	$0,1548$

ita, vt fit

$$\lambda' = 3,3000\,\lambda + 0,1548$$

quare fumto

$$\lambda = 1, \text{ fit } \lambda' = 3,4548; \text{ ergo}$$

$$\lambda' - 1 = 2,4548, \text{ et log. } \sqrt{\lambda' - 1} = 0,1950080.$$

III. Iam ad radios facierum harum lentium definiendos, cum fit

$$a = \infty \text{ et } a = \frac{(n-\zeta)e}{\eta} \text{ et } b = -\frac{(\eta-\zeta)e}{\eta}$$

 erit

erit pro lente priore radius faciei

anterioris $= \frac{\alpha}{\sigma} = \frac{5\,\mathfrak{B}}{10\,\sigma}$; posterioris $= \frac{\alpha}{\varrho} = \frac{5\,\mathfrak{B}}{10\,\varrho}$.

Pro lente posteriore radius faciei

$$\text{anter.} = \frac{-5\,\mathfrak{B}}{10\,\varrho' - 5\,\sigma' \pm 7\,\tau' \sqrt{\lambda' - 1}}$$

$$\text{poster.} = \frac{-5\,\mathfrak{B}}{10\,\sigma' - 5\,\varrho' \mp 7\,\tau' \sqrt{(\lambda' - 1)}}$$

quare pro duplici refractione $n = 1,53$ et $n' = 1,58$ valores litterarum ϱ, σ, τ quaeri oportet, qui ita se habebunt

$n = 1,\ 53$	$n' = 1,\ 58$
$\varrho = 0,\ 2266$	$\varrho' = 0,\ 1413$
$\sigma = 1,\ 6602$	$\sigma' = 1,\ 5827$
$\tau = 0,\ 9251$	$\tau' = 0,\ 8775.$

IV. Cum igitur $\mathfrak{B}$ denotet distantiam focalem ipsius lentis duplicatae, quam deinceps littera P indicabimus, pro lente priori ex vitro communi subviridi, cuius refractio est $n = 1,53$, reperiemus vtramque faciem; erit radius faciei.

Pro prima lente

anterioris $= 0,\ 1807.\ P$; posterioris $= 1,\ 3239.\ P.$

Pro altera ex vitro chrystallino, cuius refractio $n = 1,58$ facies ita erunt comparatae.

Pro secunda lente

$$\text{anterioris} = \frac{-5\,P}{-5,3351 \pm 5,6240}$$

$$\text{posterioris} = \frac{-5\,P}{15,4031 \mp 5,6240}$$

quia

quia nunc radios curuaturae minores euitari conuenit, signorum ambiguorum ea sunt sumenda, quae denominatores minores producunt; id quod fit, si signa superiora valeant; hinc ergo obtinebimus sequentes determinationes:

radius faciei anterioris $= \frac{-3P}{6,2889} = 0,4770. P$

posterioris $= \frac{-3P}{5,7791} = -0,5191 P$

quae lens obiectiua capax est aperturae, cuius semidiameter est $= 0,0452 P$.

Lens obiectiua duplicata altera, anteriore lente ex vitro chrystallino, posteriori ex vitro communi parata.

I. Cum igitur hic sit

$n = 1,58$ et $n' = 1,53$ erit $\zeta = 10$ et $\eta = 7$; hinc

$f = \frac{\eta}{\eta - \zeta} = -\frac{7}{3}$. et

$g = -\frac{\zeta}{\eta - \zeta} = \frac{10}{3}$; Hincque

$a = \frac{-10 \cdot \alpha g}{7\alpha + 10 g} = -\frac{10 g}{7}$ ob

$\alpha = \infty$, et $b = \frac{10 g}{7}$.

Confusio autem primi generis vt euanescat, fieri debet

$$0 = 343 \mu\lambda - 1000 \mu'\lambda' - 210 \mu'\nu' \text{ seu}$$

$$343 \mu\lambda = 1000 \mu'\lambda' + 210 \mu'\nu'.$$

II. Huius aequationis terni coëfficientes iam quaerantur:

$l.\ 343 = 2,5352941.$	$l.\ 1000 = 3,0000000$	$l.\ 210 = 2,3222193$
$l.\ \mu = 9,9407397$	$l.\ \mu' = 9,9945449$	$l.\ \mu'\nu' = 9\ 3358867$
2.4760338	2.9945449	1.6581060
	2.4760338	2.4760338
	0.5185111	9.1820722
	$3,3000$	$0,1520$

ita, vt fit

$$\lambda = 3,3000\,\lambda' + 0,1520 \text{; quare fumto}$$

$$\lambda' = 1 \text{ fiet } \lambda = 3.4520 \text{ ac propterea}$$

$$\lambda - 1 = 2,4520 \text{ et } \log. \surd(\lambda - 1) = 0,3895205.$$

III. Cum nunc fit

$$a = \infty \text{ et } \alpha = \frac{-3\mathfrak{B}}{7} \text{ et } b = \tfrac{3}{7}\mathfrak{B}$$

erunt radii facierum vtriusque lentis

Pro lente priore radius faciei

anterioris $= \frac{-3\mathfrak{B}}{7(\sigma \pm \tau\surd(\lambda-1))}$; pofter. $= \frac{-3\mathfrak{B}}{7\varrho \pm 7\tau\surd(\lambda-1)}$

Pro lente pofteriore radius faciei

anterioris $= \frac{3\mathfrak{B}}{7\varrho' + 3\sigma'}$; pofterioris $= \frac{3\mathfrak{B}}{7\sigma' + 3\varrho'}$

eft autem

$$\varrho = 0,1413;\ \sigma = 1,5827;\ \tau = 0,8775.$$

$$\varrho' = 0,2266;\ \sigma' = 1,6602:$$

quibus

quibus ſubſtitutis erit pro priore lente radius faciei

$$\text{anter.}\ \frac{-3\mathfrak{b}}{7,0789 \pm 5,6187};\ \text{poſter.} = \frac{-3\mathfrak{b}}{4,8887 \mp 5,6187}$$

valeant autem ob rationes ſupra allegatas ſigna inferiora, eritque radius faciei:

$$\text{anterioris} = \frac{-3\mathfrak{b}}{1,4602} = -2,0545\ P$$

$$\text{poſterioris} = \frac{-3\mathfrak{b}}{10,5074} = -0,2828\ P.$$

Pro lente autem poſteriore erit radius faciei

$$\text{anterioris} = \frac{3\mathfrak{b}}{6,5661} = 0,4568\ P$$

$$\text{poſterioris} = \frac{3\mathfrak{b}}{12,3012} = 0,2438\ P.$$

Inter quos quatuor radios minimus eſt 0, 2438 P vnde haec lens duplicata aperturam admittere poteſt, cuius ſemidiameter $= 0,0609$. P ita, vt haec lens maiorem admittat aperturam, quam praecedens.

De Lentibus obiectiuis ex tribus lentibus compoſitis.

I. Sit pro prima lente ratio refractionis $= n$, pro ſecunda $= n'$, pro tertia $= n''$, deinde diſtantiae determinatrices primae a et α, ſecundae b et $\mathfrak{b}$ tertiae c et γ cum numeris arbitrariis λ, λ', λ'' eritque $\alpha + b = 0$, et $\mathfrak{b} + c = 0$, eruntque a et γ di-

 ſtantiae

ſtantiae determinatrices ipſius lentis triplicatae. Iam vero ſtatuatur

$$\tfrac{1}{a} + \tfrac{1}{\alpha} = f\left(\tfrac{1}{a} + \tfrac{1}{\gamma}\right);$$
$$\tfrac{1}{b} + \tfrac{1}{\beta} = g\left(\tfrac{1}{a} + \tfrac{1}{\gamma}\right);$$
$$\tfrac{1}{c} + \tfrac{1}{\gamma} = b\left(\tfrac{1}{a} + \tfrac{1}{\gamma}\right);$$

fietque

$$\tfrac{1}{\alpha} = -\tfrac{f-1}{a} + \tfrac{f}{\gamma};\ \alpha = \frac{a\gamma}{\gamma(f-1)+af} = -b$$
$$\tfrac{1}{\beta} = \tfrac{g}{a} + \tfrac{g}{\gamma} + \tfrac{f-1}{a} + \tfrac{f}{\gamma}$$
$$\tfrac{1}{\beta} = \tfrac{g+f-1}{a} + \tfrac{g+f}{\gamma} = -\tfrac{1}{c}$$

ac proinde

$$\frac{-f-g+1}{a} - \frac{f-g+1}{\gamma} = \frac{b}{a} + \frac{b}{\gamma}$$

vnde ſequitur fore

$$\frac{-f-g-b+1}{a} - \frac{(f+g+b-1)}{\gamma} = 0$$

hinc $f+g+b=1$.

II. Conditio vero confuſionem colorum penitus tollens poſtulat, vt ſupra vidimus, hanc aequationem

$$0 = \tfrac{dn}{n-1}\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right) + \tfrac{dn'}{n'-1}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right) + \tfrac{dn''}{n''-1}\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right);$$

quae ponendo

$$\tfrac{dn}{n-1} = \zeta;\ \tfrac{dn'}{n'-1} = \eta;\ \tfrac{dn''}{n''-1} = \vartheta$$

abit in hanc formam $0 = \zeta f + \eta g + \vartheta b$.

III. Vt autem confuſio ab apertura pendens deſtruatur, huic aequationi ſatisfieri debet:

$$0 = \frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3} - \frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3} + \frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3}.$$

vbi

vbi notetur esse

$$\frac{A+1}{A}=af\left(\frac{1}{a}+\frac{1}{\gamma}\right) \text{ et}$$

$$\frac{B+1}{B}=bg\left(\frac{1}{a}+\frac{1}{\gamma}\right) \text{ et}$$

$$\frac{C+1}{C}=ch\left(\frac{1}{a}+\frac{1}{\gamma}\right);$$

vnde termini priores continentes λ erunt

$$a^2\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2(\mu\lambda f^2+\mu'\lambda' g^2+\mu''\lambda'' h^2)$$

termini vero tres posteriores fiunt

$$a^2\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\frac{\mu\nu f}{a\alpha}+\frac{\mu'\nu' g}{b\beta}+\frac{\mu''\nu'' h}{c\gamma}\right)$$

adeoque prodit haec aequatio

$$0=\mu\lambda f^2+\mu'\lambda' g^2+\mu''\lambda'' h^2$$

$$+\frac{1}{\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2}\left(\frac{\mu\nu f}{a\alpha}+\frac{\mu'\nu' g}{b\beta}+\frac{\mu''\nu'' h}{c\gamma}\right)$$

vbi notandum est, ex praecedentibus esse

$$\frac{1}{a\alpha}=\frac{1}{a}\left(\frac{f-1}{a}+\frac{f}{\gamma}\right);$$

$$\frac{1}{b\beta}=-\left(\frac{f-1}{a}+\frac{f}{\gamma}\right)\left(\frac{g+f-1}{a}+\frac{f+g}{\gamma}\right)$$

$$\frac{1}{c\gamma}=-\frac{1}{\gamma}\left(\frac{f+g-1}{a}+\frac{f+g}{\gamma}\right)$$

IV. Quodsi nunc ponamus $a=\infty$, sequentes aequationes habebuntur resoluendae:

$$a=\frac{\gamma}{f} \text{ et } b=\frac{-\gamma}{f};\ \frac{1}{\beta}=\frac{g+f}{\gamma} \text{ seu}$$

$$\beta=\frac{\gamma}{f+g} \text{ et } c=\frac{-\gamma}{f+g}.$$

Quibus

Quibus adiungi debent hae aequationes:

$$f+g+b=1\text{ , et }\zeta f+\eta g+\vartheta b=0$$ et

pro omni confusione tollenda

ob $\frac{1}{a\alpha}=0$; $\frac{1}{b\beta}=\frac{-1(f+g)}{\gamma^2}$; $\frac{1}{c\gamma}=-\frac{f+g}{\gamma^2}$

habebitur haec aequatio

$$0=\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' b^3$$
$$-(f+g)(\mu'\nu' fg+\mu''\nu''.b).$$

V. Hic quidem assumsimus, omnes tres materias, ex quibus hae lentes sunt confectae, esse diuersas; in praxi autem nulla ratio suadet, vt tres diversas vitri species adhibeamus; sed potius primam et tertiam lentem ex eadem materia parari conueniet; ex quo consequimur statim

$$n''=n;\ \mu''=\mu;\ \nu''=\nu\text{ et }\vartheta=\zeta,$$

ita, vt sit

$$f+b+\tfrac{\eta}{\zeta}.g=0;$$

quae ab altera subtracta dat

$$g-\tfrac{\eta}{\zeta}.g=1;\text{ hincque }g=\frac{\zeta}{\zeta-\eta}$$

vnde porro fit

$$f+b=-\frac{\eta}{\zeta-\eta}.$$

Ponamus hunc in finem

$$f-b=\frac{x}{\zeta-\eta}$$ eritque

$$f=\frac{x-\eta}{2(\zeta-\eta)}\text{ et }b=\frac{-x-\eta}{2(\zeta-\eta)}$$

ex

ex quibus valoribus prodit

$$a = \frac{2\gamma(\zeta-\eta)}{x-\eta}, b = \frac{-2\gamma(\zeta-\eta)}{x-\eta};$$

$$\mathfrak{b} = \frac{2\gamma(\zeta-\eta)}{x+2\zeta-\eta}; c = \frac{-2\gamma(\zeta-\eta)}{x+2\zeta-\eta}.$$

VI. Aequatio ergo, cui adhuc satisfieri oportet, erit

$$0 = \tfrac{1}{4}\mu\lambda(x-\eta)^3 + 2\mu'\lambda'\zeta^3 - \tfrac{1}{4}\mu\lambda''(x+\eta)^3$$
$$-\tfrac{1}{2}(x+2\zeta-\eta)(\mu'\nu'(x-\eta)\zeta - \mu\nu(x+\eta)(\zeta-\eta))$$

ex qua aequatione vel vna litterarum λ vel etiam numerus x definiri potest; quo posteriori casu hoc commodi consequeremur, vt singulae litterae $\lambda = 1$ statui possent sicque singulae lentes vtpote ex principio minimi deductae facillime construi possent; hanc autem inuestigationem non aliter, nisi in exemplis particularibus suscipere licebit.

Ponamus igitur

$$\lambda = 1, \lambda' = 1 : \lambda'' = 1 \text{ fietque}$$

$$0 = -\tfrac{1}{2}\mu(3\eta x^2+\eta^3) + 2\mu'\zeta^3 - \tfrac{1}{2}\mu'\nu'\zeta(x^2+2(\zeta-\eta)x-\eta(2\zeta-\eta))$$
$$+\tfrac{1}{2}\mu\nu(\zeta-\eta)(x^2+2\zeta x+\eta(2\zeta-\eta))$$

quae aequatio reducitur ad hanc

$$x^2(-3\mu\eta - \mu'\nu'\zeta + \mu\nu(\zeta-\eta))$$
$$+2x.(\zeta-\eta)\zeta(\mu\nu-\mu'\nu') - \mu\eta^3 + 4\mu'\zeta^3 + \mu'\nu'\eta\zeta(2\zeta-\eta) + \mu\nu.\eta(\zeta-\eta)(2\zeta-\eta) = 0.$$

VII. Adplicemus haec ad illas duas species vitri, quibus *Dollondus* vsus est, ac duo casus euoluendi occurrunt. Primus igitur casus esto, quo tam prima, quam tertia lens ex vitro chrystallino conficitur; media autem ex vitro communi subuiridi; ita, vt sit $n = n'' = 1,58$ et $n' = 1,53$, ideoque $\zeta = 10$ et $\eta = 7$ tum vero $\mu = 0,8724$, $\nu = 0,2529$ et $l.\,\mu\nu = 9.3437441$ deinde $\mu' = 0,9875$; $\nu' = 0,2194$ et $l.\,\mu'\nu' = 9.3358867$ vnde superior aequatio reducetur ad hanc formam

$$+19,8255\,x^2 = 0,2340\,x + 4177,6081, \text{ siue}$$

$$x^2 = 0.0118\,x + 210,7190$$

cuius resolutio praebet

$$x = 0,0059 \pm \sqrt{210,7190}$$

$$= 0,0059 \pm 14,5161$$

vnde ambo valores ipsius x sunt

$$x = 14,5220$$

$$x = -14,5102$$

Lens obiectiua triplicata perfecta prior cuius lens prima et tertia ex vitro chrystallino, pro quo $n = 1,58$, media vero ex vitro communi subuiridi, $n = 1,53$, est confecta.

I. Ob duplicem valorem ipsius x duas etiam eiusmodi lentes exhibere poterimus; sit igitur primo $x = 14,5220$ et quia est $\zeta = 10$ et $\eta = 7$, erit $f = 1,2537$; $g = 3,3333$, $b = -3.5870$

$$\frac{1}{a} = \frac{1,2537}{\gamma};\ \frac{1}{b} = \frac{-1,2537}{\gamma};\ \frac{1}{c} = \frac{4,5870}{\gamma} \text{ et } \frac{1}{d} = \frac{-4,5870}{\gamma};$$

quodsi

quodsi iam harum trium lentium radios facierum anteriorum ponamus F, F', F'', posteriorum vero

G, G', G'' ob $\lambda = \lambda' = \lambda'' = 1$,

habebimus

$$\frac{1}{F} = \frac{\varrho}{\alpha} + \frac{\sigma}{\alpha} = \frac{\sigma}{\alpha} = \frac{1,2537 \cdot \sigma}{\gamma}; \quad \frac{1}{G} = \frac{\varrho}{\alpha} + \frac{\sigma}{\alpha} = \frac{\varrho}{\alpha} = \frac{1,2537\,\varrho}{\gamma} \text{ et}$$

$$\frac{1}{F'} = \frac{\varrho'}{b} + \frac{\sigma'}{\beta}; \quad \frac{1}{G'} = \frac{\varrho'}{\beta} + \frac{\sigma'}{b}$$

et substitutis valoribus

$$\frac{1}{F'} = \frac{-1,2537 \cdot \varrho'}{\gamma} + \frac{4,5870 \cdot \sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{4,5870 \cdot \varrho'}{\gamma} - \frac{1,2537 \cdot \sigma'}{\gamma}$$

simili modo pro lente tertia

$$\frac{1}{F''} = \frac{\varrho}{c} + \frac{\sigma}{\gamma}; \quad \frac{1}{G''} = \frac{\varrho}{\gamma} + \frac{\sigma}{c}$$

$$\frac{1}{F''} = \frac{-4,5870 \cdot \varrho + \sigma}{\gamma}; \quad \frac{1}{G''} = \frac{\varrho - 4,5870 \cdot \sigma}{\gamma}$$

existente

$$\varrho = 0,\ 1413;\ \sigma = 1,\ 5827 \text{ et}$$
$$\varrho' = 0,\ 2266;\ \sigma' = 1,\ 6602$$

ac si ipsius lentis triplicatae distantia focalis ponatur $= P$, vt hic sit $\gamma = P$, reperiuntur valores sequentes

$\frac{1}{F} = \frac{1,9843}{P}$, siue $F = +0,\ 5039.\ P.$

$\frac{1}{G} = \frac{0,1771}{P}$, hincque $G = +5,\ 6450.\ P.$

$\frac{1}{F'} = \frac{7,3312}{P}$, siue $F' = +0,\ 1364\ P.$

$\frac{1}{G'} = \frac{-1,0420}{P}$, hincque $G' = -0,\ 9597.\ P.$

$\frac{1}{F''} = \frac{0,9346}{P}$, hincque $F'' = +1,\ 0699.\ P.$

$\frac{1}{G''} = \frac{-7,1186}{P}$, hincque $G'' = -0,\ 1404.\ P.$

II. Inter hos ſex radios cum minimus ſit $0,1364.\,P$, eius pars quarta $= 0,0341.\,P$ dat ſemidiametrum maximae aperturae, cuius haec lens triplicata eſt capax. Contra vero haec lens hac praerogatiua gaudet, quod eius conſtructio ob $\lambda = 1$; $\lambda' = 1$; $\lambda'' = 1$ in praxi minimae difficultati ſit obnoxia.

Altera lens triplicata, pro qua pariter, vt ante, lens prima et tertia ex vitro chryſtallino, media vero ex vitro communi ſubviridi paratur.

I. Sumatur iam

$$x = -14,5102, \text{ ob } \zeta = 10, \text{ et } \eta = 7 \text{ erit}$$

$$f = -3,5850, g = 3,3333; h = +1,2517.$$

$$\frac{1}{a} = \frac{-3,5850}{\gamma}; \quad \frac{1}{b} = \frac{3,5850}{\gamma}$$

$$\frac{1}{\mathfrak{b}} = \frac{-0,2517}{\gamma}; \quad \frac{1}{c} = +\frac{0,2517}{\gamma}$$

vnde pro radiis ſingularum facierum prodibunt ſequentes formulae:

$$\frac{1}{F} = \frac{\sigma}{a} = \frac{-3,5850.\sigma}{\gamma}; \quad \frac{1}{G} = \frac{\rho}{a} = \frac{-3,5850.\rho}{\gamma}$$

$$\frac{1}{F'} = \frac{\rho'}{b} + \frac{\sigma'}{\mathfrak{b}} = \frac{3,5850.\rho' - 0,2517.\sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{\rho'}{\mathfrak{b}} + \frac{\sigma'}{b} = \frac{-0,2517\,\rho' + 3,5850\,\sigma'}{\gamma}$$

$$\frac{1}{F''} = \frac{\rho}{c} + \frac{\sigma}{\gamma} = \frac{0,2517\,\rho + \sigma}{\gamma}$$

$$\frac{1}{G''} = \frac{\rho}{\gamma} + \frac{\sigma}{c} = \frac{\rho + 0,2517.\sigma}{\gamma}$$

vnde

vnde ſequentes valores determinati pro his radiis inveniuntur:

$\frac{1}{F} = -\frac{5,6729}{P}$ hincque $F = -0,1762.\ P.$

$\frac{1}{G} = -\frac{0,5065}{P}$ hincque $G = -1,9741.\ P.$

$\frac{1}{F'} = \frac{0,3945}{P}$ hincque $F' = +2,5349\ P.$

$\frac{1}{G'} = \frac{5,8948}{P}$, hincque $G' = +0,1696.\ P.$

$\frac{1}{F''} = \frac{1,6142}{P}$, hincque $F'' = +0,6194.\ P.$

$\frac{1}{G''} = \frac{0,5396}{P}$ hincque $G'' = +1,8532.\ P.$

quorum ſex radiorum minimus eſt 0,1696 P, cuius pars quarta 0,0424. P dat ſemidiametrum aperturae maximae, cuius haec lens triplicata eſt capax. Ideoque haec lens obiectiua praecedenti eſt anteferenda.

VIII. Secundum §. VII. adhuc alius caſus euolui debet, quo lens prima et tertia ex vitro communi ſubuiridi $n = n'' = 1,53$; media autem ex Chryſtallino $n = 1,58$ parari poteſt; ſicque erit $\zeta = 7$, $\eta = 10$. Vnde cum fiat

$\mu = 0.9875$, $l.\ \mu\nu = 9.3358867$

$\mu' = 0.8724$. $l.\ \mu'\nu' = 9.3437441.$

aequatio noſtra pro confuſione tollenda ſequentem induet formam:

$$31,8198\ x^2 = +0,1638\ x + 245,2121 \text{ ſiue}$$

$$x^2 = 0,0051.\ x + 7,7063$$

 cuius

cuius resolutio dat

$$x = 0,0025 \pm 2,7760$$

vnde bini valores ipsius x erunt

$$\text{I. } x = 2,7785$$

$$\text{II. } x = -2.7735.$$

Lens triplicata perfecta prior, cuius lens prima et tertia ex vitro communi, $n = 1,53$. media ex chryſtallino, $n = 1,58$ eſt confecta.

Cum igitur ſit

$x = 2,7785$ et $\zeta = 7, \eta = 10$ erit

$f = +1,2036;\ g = -2,3333.\ h = 2,1297;$

hincque

$$\frac{1}{a} = \frac{1,2036}{\gamma},\ \frac{1}{b} = \frac{-1,2036}{\gamma},\ \frac{1}{c} = \frac{-1,1297}{\gamma},\ \frac{1}{c} = \frac{1,1297}{\gamma}$$

ex quibus colligitur

$$\frac{1}{F} = \frac{\sigma}{a} = \frac{1,2036.\sigma}{\gamma};\quad \frac{1}{G} = \frac{\rho}{a} = \frac{1,2036\,\rho}{\gamma}$$

$$\frac{1}{F'} = \frac{\rho'}{b} + \frac{\sigma'}{c} = \frac{-1,2036\,\rho' - 1,1297\,\sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{\rho'}{c} + \frac{\sigma'}{b} = \frac{-1,1297.\rho' - 1,2036.\sigma'}{\gamma}$$

$$\frac{1}{F''} = \frac{\rho}{c} + \frac{\sigma}{\gamma} = \frac{1,1297.\rho + \sigma}{\gamma}$$

$$\frac{1}{G''} = \frac{\sigma}{c} + \frac{\rho}{\gamma} = \frac{1,1297.\sigma + \rho}{\gamma}$$

vbi

vbi $\varrho = 0,2266$; $\sigma = 1,6602$

$\varrho' = 0,1413$; $\sigma' = 1,5827$

Facto igitur calculo obtinebimus:

$\frac{1}{F} = \frac{1,9982}{P}$ hincque $F = 0,5004.\ P.$

$\frac{1}{G} = \frac{0,2727}{P}$ hincque $G = 3,6665.\ P.$

$\frac{1}{F'} = \frac{-1,9580}{P}$ hincque $F' = -0,5107.\ P.$

$\frac{1}{G'} = \frac{-2,0645}{P}$ hincque $G' = -0,4843.\ P.$

$\frac{1}{F''} = \frac{1,9161}{P}$ hincque $F'' = 0,5219.\ P.$

$\frac{1}{G''} = \frac{2,1021}{P}$ hincque $G'' = 0,4757.\ P.$

inter quos radios cum minimus sit 0, 4757 P eius pars quarta 0, 1189. P dat semidiametrum aperturae, quam haec lens admittit; ideoque haec lens duabus praecedentibus est anteferenda.

Lens triplicata perfecta altera, cuius lens prima et tertia ex vitro communi $n = 1,53$ media ex chrystallino, $n = 1,58$ est parata.

Sumatur iam ex binis ipsius x valoribus alter negatiuus $x = -2,7735$; et cum sit $\zeta = 7$, $\eta = 10$, inuenitur

$f = +2,1289$; $g = -2,3333$ $h = +1,2044$;

hincque

$\frac{1}{a} = \frac{2,1280}{\gamma}$; $\frac{1}{b} = \frac{-2,1289}{\gamma}$

$\frac{1}{\beta} = \frac{-0,1044}{\gamma}$; $\frac{1}{c} = \frac{0,1044}{\gamma}$

ex

ex quibus consequimur

$$\frac{1}{F} = \frac{\sigma}{\alpha} = \frac{2,1289 \cdot \sigma}{\gamma}; \quad \frac{1}{G} = \frac{\rho}{\alpha} = \frac{2,1289 \cdot \rho}{\gamma}$$

$$\frac{1}{F'} = \frac{\rho'}{b} + \frac{\sigma'}{g} = \frac{-2,1285 \cdot \rho' - 0,1044 \, \sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{\rho'}{g} + \frac{\sigma'}{b} = \frac{-0,1044 \cdot \rho' - 2,1285 \cdot \sigma'}{\gamma}$$

$$\frac{1}{F''} = \frac{\rho}{c} + \frac{\sigma}{\gamma} = \frac{0,1044 \, \rho + \sigma}{\gamma}$$

$$\frac{1}{G''} = \frac{\sigma}{c} + \frac{\rho}{\gamma} = \frac{0,1044 \, \sigma + \rho}{\gamma}.$$

Vnde facto calculo pro radiis facierum singularum lentium sequentes inuenientur radii.

$$\frac{1}{F} = \frac{3,5344}{P}; \text{ seu } F = 0,2829. \ P.$$

$$\frac{1}{G} = \frac{0,4824}{P}; \text{ seu } G = 2,0729. \ P.$$

$$\frac{1}{F'} = \frac{-0,4660}{P}; \text{ seu } F' = -2,1459. \ P.$$

$$\frac{1}{G'} = \frac{-3,3841}{P}; \text{ seu } G' = -0,2955. \ P.$$

$$\frac{1}{F''} = \frac{1,6838}{P}; \text{ seu } F'' = 0,5938. \ P.$$

$$\frac{1}{G''} = \frac{0,3999}{P}; \text{ seu } G'' = 2,5006. \ P.$$

Inter quos radios minimus est 0, 2829. P, cuius pars quarta 0, 0707. P, dat semidiametrum aperturae maximae, cuius haec lens triplicata est capax.

IX. Hae quatuor lentes triplicatae ideo prae ceteris, quas proponere liceret, sunt commendandae, quod in iis assumsimus $\lambda = 1$; $\lambda' = 1$; $\lambda'' = 1$. Quam ob caussam in praxi facillime construi possunt, ex iisdem vero principiis etiam constructio eiusmodi lentium com-

compoſitarum deduci poſſet qua inter binas lentes vitreas aqua aliudue fluidum includeretur, ita, vt lens media ex fluido conſtaret: tum autem praeter conditiones ante tractatas duae nouae eſſent implendae; ſcilicet vt radii facierum mediae lentis aequales eſſent ſtatuendi et contrarii radiis facierum internarum primae ac tertiae lentis. Ob has igitur nouas conditiones non amplius liceret numeros λ, λ' et λ'' vnitati aequales aſſumere; qua poſitione conſtructioni practicae quam maxime conſulitur; deinde vero etiam radii facierum tam parui prodirent, vt tales lentes nimis exiguam aperturam eſſent admiſſurae; quam ob cauſſam operae pretium haud videtur, in earum conſtructionem adcuratius inquirere.

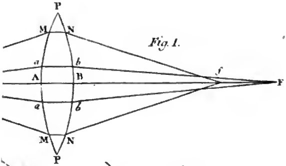

Fig. 1.

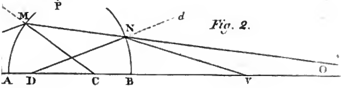

Fig. 2.

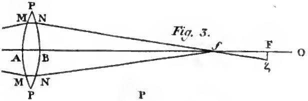

Fig. 3.

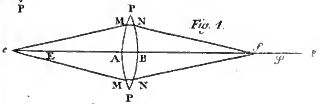

Fig. 4.

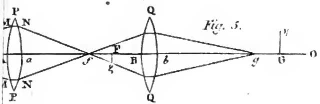

Fig. 5.

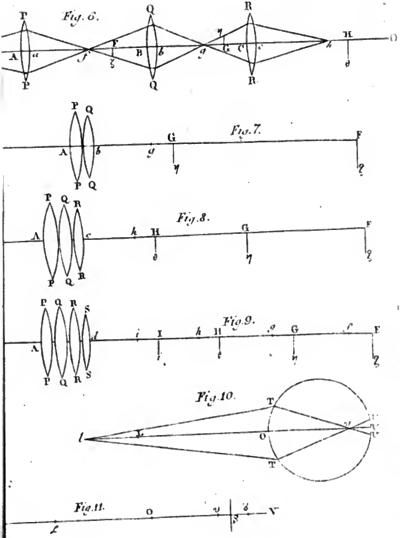
Fig. 6.
P
A
a
P
f
F
ζ
Q
B
b
Q
g
η
G
C
c
R
R
h
H
θ
O
Fig. 7.
P Q
A
b
P Q
g
G
η
F
ζ
Fig. 8.
P Q R
A
c
P Q R
h
H
θ
G
η
F
ζ
Fig. 9.
P Q R S
A
d
P Q R S
i
I
ι
h
H
θ
g
G
η
f
F
ζ
Fig. 10.
T
l
L
O
T
Fig. 11.
l
O
v
s
V

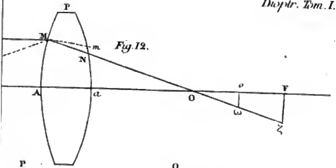

Fig. 12.

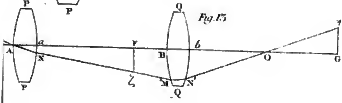

Fig. 13.

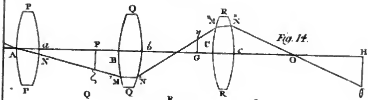

Fig. 14.

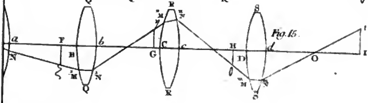

Fig. 15.

Fig. 16.

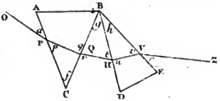

Fig. 17.